Compounded interest formula:	$y = a(1+\frac{r}{n})^{nt}$

Given the # of times interest will be compounded, indicate the n value.

annually	n = 1	semi- annually	n = 2
monthly	n = 12	quarterly	n = 4
daily	n = 365	weekly	n = 52

Consider this: You have $2500 to invest over the course of the next 20 years. Determine which of these options will yield the higher balance. Round each value to two decimal places. Show your work.

<u>Option 1</u> The neighborhood bank offers savings accounts paying 5.89% compounded daily. $r = .0589$ $n = 365$ $y = 2500\left(1+\frac{.0589}{365}\right)^{(365\times20)}$ $ 8118.91	<u>Option 2</u> The credit unit offers $6\frac{1}{8}$ % interest compounded semi-annually. $r = .06125$ $n = 2$ $y = 2500\left(1+\frac{.06125}{2}\right)^{(2\times20)}$ $ 8355.39
<u>Option 3</u> A money market account pays 6.03% compounded monthly. $r = .0603$ $n = 12$ $y = 2500\left(1+\frac{.0603}{12}\right)^{(12\times20)}$ $ 8325.06	<u>Option 4</u> A tax sheltered annuity account pays $5\frac{7}{8}$ % compounded quarterly. $r = .05875$ $n = 4$ $y = 2500\left(1+\frac{.05875}{4}\right)^{(4\times20)}$ $ 8026.47

OPTION 2 is best! (The interest rate has more impact than the number of times compounding)

Solving for the starting amount:

EXAMPLE How much should you deposit into an account that pays 7% compounded quarterly if you wish to have $50,000 at the end of 20 years?

$50{,}000 = a\left(1+\frac{.07}{4}\right)^{(4\times20)}$ $a = \frac{50{,}000}{\left(1+\frac{.07}{4}\right)^{80}} \approx \$12{,}480.06$

You try: The parents of a newborn begin a trust fund the day their child is born. How much should they deposit into a fund paying $5\frac{1}{4}$% quarterly if they wish for the child to have $100,000 the day they turn 21?

$100{,}000 = a\left(1+\frac{.0525}{4}\right)^{(4\times21)}$

$a = \frac{100{,}000}{\left(1+\frac{.0525}{4}\right)^{84}} \approx \$33{,}443.01$

The natural base "e"

What is e?

Use a calculator to evaluate the expression $(1+\frac{1}{n})^n$. Write all decimal places (do not round).

n = 1 ______________________________

n = 10 ______________________________

n = 100 ______________________________

n = 1000 ______________________________

n = 10,000 ______________________________

n = 100,000 ______________________________

n = 1,000,000 ______________________________

n = 10,000,000 ______________________________

n = 100,000,000 ______________________________

Describe what is happening as we choose larger and larger values for n.

Which mathematical concept does this make you think of?

Like pi, the natural base "e" is an irrational number. It can be approximated to 2.718. Because of its natural limiting value, it is often used as the base of the exponential function in realistic applications.

Operating with Base e Functions use e^x on your calculator!

EX1 The growth of a bacteria colony is a petri dish can be modeled by the equation $y = 75e^{.4t}$, where t is the time in hours. Approximate the number of bacteria after two full days. Same as 48 hours

$y = 75e^{.4(48)} \approx 1.63 \times 10^{10}$

EX 2 Suppose the amount of radioactive substance remaining in a 100 milligram sample after t years can be modeled by $A = 100e^{-0.01653t}$. How much is remaining after 24 years?

≈ 67.3 mg

Applications Problems Involving **Continuous Compounding**

There is a special growth formula that we use for problems involving *continuous compounding*. This formula is $y = pe^{rt}$, where y = final amount, p = starting amount, r = growth rate (in decimal form), and t = time.

Jim invests $2000 at 6% interest for 10 years. Find his balance if his interest is compounded

		What do you observe happens as n increases without bound?
Annually	________	
Semi-annually	________	
Quarterly	________	
Monthly	________	
Weekly	________	the value approaches a limit
Daily	________	
Hourly	________	
Minutely	________	
Secondly	________	use compounded interest formula for "CONTINUOUS"

$y = pe^{rt}$ (y: final amount; p: initial amount; r: rate; t: time)

Apply.

1. Your grandparents deposited $500 into a trust fund for you the year you were born. If the bank paid 6.2% interest compounded continuously, what is the account balance today?

$y = 500e^{.062 \times 17} \approx \$1434.55$

2. Nalani plans to buy a new car in 5 years. How much should you deposit into a bank paying 7% continuous interest if she plans on spending $20,000 for her car?

$20{,}000 = pe^{.07 \times 5}$ ↑ isolate p by dividing

$p = \frac{20{,}000}{e^{.07 \times 5}} \approx \$14{,}093.76$

3. Nick is depositing money into an account paying 5¾% interest compounded continuously. How long will it take for his money to double? we do not have a way to solve yet; use tables in calculator to estimate

Double: $2 = 1e^{.0575t}$ (starting: 1)

Put $y = e^{.0575x}$ into calculator; you should see; scroll down until you see y = 2

x	y
0	1

about 12 ~ 13 years

The natural base "e"

What is e?

Use a calculator to evaluate the expression $(1+\frac{1}{n})^n$. Write all decimal places (do not round).

n = 1 ______________________________

n = 10 ______________________________

n = 100 ______________________________

n = 1000 ______________________________

n = 10,000 ______________________________

n = 100,000 ______________________________

n = 1,000,000 ______________________________

n = 10,000,000 ______________________________

n = 100,000,000 ______________________________

Describe what is happening as we choose larger and larger values for n.

Which mathematical concept does this make you think of?

Like pi, the natural base "e" is an ________________________ number. It can be approximated to ____________. Because of its natural limiting value, it is often used as the base of the exponential function in realistic applications.

Operating with Base e Functions

EX1 The growth of a bacteria colony is a petri dish can be modeled by the equation $y = 75e^{.4t}$, where t is the time in hours. Approximate the number of bacteria after two full days.

EX 2 Suppose the amount of radioactive substance remaining in a 100 milligram sample after *t* years can be modeled by $A = 100e^{-0.01653t}$. How much is remaining after 24 years?

Applications Problems Involving **Continuous Compounding**

There is a special growth formula that we use for problems involving *continuous compounding*. This formula is $y = pe^{rt}$, where y = final amount, p = starting amount, r = growth rate (in decimal form), and t = time.

Jim invests $2000 at 6% interest for 10 years. Find his balance if his interest is compounded

Annually	________________	What do you observe happens as n increases without bound?
Semi-annually	________________	
Quarterly	________________	
Monthly	________________	
Weekly	________________	
Daily	________________	
Hourly	________________	
Minutely	________________	
Secondly	________________	

Apply.

1. Your grandparents deposited $500 into a trust fund for you the year you were born. If the bank paid 6.2% interest compounded continuously, what is the account balance today?

2. Nalani plans to buy a new car in 5 years. How much should you deposit into a bank paying 7% continuous interest if she plans on spending $20,000 for her car?

3. Nick is depositing money into an account paying 5¾% interest compounded continuously. How long will it take for his money to double?

$y=a(1+r)^t$ $y=a(1-r)^t$ $y=a\left(1+\frac{r}{n}\right)^{nt}$ $y=pe^{rt}$

All Mixed Up

Extra Practice with Exponential Growth and Decay

Read each problem carefully! Determine whether each problem should be modeled using the exponential growth, decay, or compounded growth formula.

1. You drink a beverage with 120 mg of caffeine. Each hour, the amount of caffeine in your system decreases by 12%. How much caffeine will be left in your system after 30 minutes? After 4 hours?

$y=120(1-.12)^{.5}$

≈ 112.6 mg after 30 minutes

$y=120(1-.12)^4$

≈ 72.0 mg after 4 hours

2. You need to have $40,000 for a down payment on a home in 5 years. The bank will pay 5½% compounded daily. How much should you deposit in order to meet your goal?

n=365

$40{,}000=a\left(1+\frac{.055}{365}\right)^{(365\times5)}$

$a=\frac{40{,}000}{\left(1+\frac{.055}{365}\right)^{(365\times5)}}$

$30,383.51

3. A principle of $2000 is deposited into an account that pay 6¼% interest compounded monthly. How much will you have earned on the account in 2 years?

this is the interest!

2265.56 − 2000

$265.56

$y=2000\left(1+\frac{.0625}{12}\right)^{(12\times2)}$

y = $2265.56 is the balance

4. Six students return from spring break with a viral contagion. The CDC estimates that the virus will spread at a rate of 12.8% each day. How many students will be infected after 3 days? After 3 weeks? How many days will it take for the entire school population of NCHS to be infected? (2700 students)

r=12.8%

calculator estimate using tables

$y=6(1+.128)^3$

8 will be sick after 3 days

$y=6(1+.128)^{21}$

75 will be sick after 3 weeks

It will take 51 days

5. A gas-guzzling SUV purchased for $42,000 in 2006 was sold in 2012 for $20,000. Estimate the rate of depreciation of the vehicle.

$20{,}000=42{,}000(1-r)^{11}$ (a = 42,000, y = 20,000)

$\frac{20{,}000}{42{,}000}=(1-r)^{11}$

$\left(\frac{20{,}000}{42{,}000}\right)^{\frac{1}{11}}=1-r$

$0.934\ldots=1-r$

$-.065\ldots=-r$

$r\approx6.5\%$

6. Andy and Jeff have decided to raise rabbits to make fur coats. They pool their money and are able to purchase 10 rabbits for breeding. If the rabbit population increases at a rate of 40% each year, estimate the number of rabbits they will have after 1 month. After 6 months. After 2 years.

$y=10(1+.4)^{\frac{1}{12}}$ ← 1 month (theoretical)

there will still be only 10 rabbits after 1 month

$y=10(1+.4)^{.5}$ ↑ 6 months

there will be about 12 rabbits

$y=10(1+.4)^2$

there will be about 20 rabbits

7. In 1962, my parents invested $250 into an account that paid 6% interest compounded quarterly. If they have not touched the account in all these years, how much money is in the account today? How much interest have they earned?

$y=250\left(1+\frac{.06}{4}\right)^{(4\times55)}$

$y\approx\$6614.08$

This is balance − principle

$6614.08 − 250.00

$6364.08 interest

8. The population of a small town since 1960 can by modeled by the function $P = 65{,}000(.918)^t$, where **t** is the number of years since 1960.

A. What was the population in 1960? 65,000

B. Is the population increasing or decreasing? decreasing; the base is less than 1

C. What was the population in 1976? $y = 65{,}000(.918)^{16} \approx 16{,}534$

D. Estimate the population of the town today. $y = 65{,}000(.918)^{57} \approx 495$

9. The equation $y = 2500e^{-.0965t}$ models the population of a species of saltwater fish since the year 2015.

A. Is the population of this species increasing or decreasing? How do you know? By what percentage?

Decreasing; the exponent is negative; by 9.65%

B. Approximate the current population of this species.

2061

10. You are investing your savings in an account with quarterly compounding. You estimate that your money will double in ten years. If this is true, what is the interest rate on the account?

↑ use ratios!

$2 = 1\left(1 + \frac{r}{4}\right)^{(4 \times 10)}$

$2 = 1\left(1 + \frac{r}{4}\right)^{40}$

$(2)^{\frac{1}{40}} = 1 + \frac{r}{4}$

$1.017.. = 1 + \frac{r}{4}$

$.017.. = \frac{r}{4}$

$r \approx 7.0\%$

11. Find the interest rate needed on an account that compounds semi-annually if you wish to quadruple your investment by the year 2050.

↑ use ratios

$4 = 1\left(1 + \frac{r}{2}\right)^{(2 \times 33)}$

$4 = \left(1 + \frac{r}{2}\right)^{66}$

$(4)^{\frac{1}{66}} = 1 + \frac{r}{2}$

$1.02.. = 1 + \frac{r}{2}$

$.02.. = \frac{r}{2}$

$r \approx 4.2\%$

12. If Christopher Columbus invested a nickel with the Native American Bank when he first arrived, how much would his account be worth if he was given 6.09% compounded semi-annually? 2017–1492 = 525 years

$y = .05\left(1 + \frac{.0609}{2}\right)^{(2 \times 525)}$

$\$2.38 \times 10^{12}$

13. How much more will you earn on an account that pays 5⅞% compounded quarterly for 10 years than you will on an account that compounds semi-annually for the same period of time given an initial investment of $25,000?

$y = 25{,}000\left(1 + \frac{.05875}{4}\right)^{(4 \times 10)}$

$44,795.30

$y = 25{,}000\left(1 + \frac{.05875}{2}\right)^{(2 \times 10)}$

$44,607.96

you will earn $187.34 more with quarterly compounding

All Mixed Up — Extra Practice with Exponential Growth and Decay

Read each problem carefully! Determine whether each problem should be modeled using the exponential growth, decay, or compounded growth formula.

1. You drink a beverage with 120 mg of caffeine. Each hour, the amount of caffeine in your system decreases by 12%. How much caffeine will be left in your system after 30 minutes? After 4 hours?

2. You need to have $40,000 for a down payment on a home in 5 years. The bank will pay 5½% compounded daily. How much should you deposit in order to meet your goal?

3. A principle of $2000 is deposited into an account that pay 6¼% interest compounded monthly. How much will you have earned on the account in 2 years?

4. Six students return from spring break with a viral contagion. The CDC estimates that the virus will spread at a rate of [illegible].8% each day. How many students will be infected after 3 days? After 3 weeks? How many days will it take for the entire school population of NCHS to be infected? (2700 students)

5. A gas-guzzling SUV purchased for $42,000 in 2006 was sold in 2012 for $20,000. Estimate the rate of depreciation of the vehicle.

6. Andy and Jeff have decided to raise rabbits to make fur coats. They pool their money and are able to purchase 10 rabbits for breeding. If the rabbit population increases at a rate of 40% each year, estimate the number of rabbits they will have after 1 month. After 6 months. After 2 years.

7. In 1962, my parents invested $250 into an account that paid 6% interest compounded quarterly. If they have not touched the account in all these years, how much money is in the account today? How much interest have they earned?

8. The population of a small town since 1960 can by modeled by the function $P = 65{,}000(.918)^t$, where **t** is the number of years since 1960.

A. What was the population in 1960?

B. Is the population increasing or decreasing?

C. What was the population in 1976?

D. Estimate the population of the town today.

9. The equation $y = 2500e^{-.0965t}$ models the population of a species of saltwater fish since the year 2015.

A. Is the population of this species increasing or decreasing? How do you know? By what percentage?

B. Approximate the current population of this species.

10. You are investing your savings in an account with quarterly compounding. You estimate that your money will double in ten years. If this is true, what is the interest rate on the account?

11. Find the interest rate needed on an account that compounds semi-annually if you wish to quadruple your investment by the year 2050.

12. If Christopher Columbus invested a nickel with the Native American Bank when he first arrived, how much would his account be worth if he was given 6.09% compounded semi-annually?

13. How much more will you earn on an account that pays 5⅞% compounded quarterly for 10 years than you will on an account that compounds semi-annually for the same period of time given an initial investment of $25,000?

Compounded interest formula: $y = a(1 + \frac{r}{n})^{nt}$

Given the # of times interest will be compounded, indicate the n value.

annually	n = ________	semi- annually	n = ________
monthly	n = ________	quarterly	n = ________
daily	n = ________	weekly	n = ________

Consider this: You have $2500 to invest over the course of the next 20 years. Determine which of these options will yield the higher balance. Round each value to two decimal places. Show your work.

<u>Option 1</u> The neighborhood bank offers savings accounts paying 5.89% compounded daily.	<u>Option 2</u> The credit unit offers $6\frac{1}{8}$ % interest compounded semi-annually.
<u>ption 3</u> A money market account pays 6.03% compounded monthly.	<u>Option 4</u> A tax sheltered annuity account pays $5\frac{7}{8}$ % compounded quarterly.

Solving for the starting amount:

EXAMPLE How much should you deposit into an account that pays 7% compounded quarterly if you wish to have $50,000 at the end of 20 years?

You try: The parents of a newborn begin a trust fund the day their child is born. How much should they deposit into a fund paying $5\frac{1}{4}$% quarterly if they wish for the child to have 00,000 the day they turn 21?