Exponential Growth $y = a(1 + r)^t$

y = the final amount
a = the starting amount
r = growth rate/interest rate change to decimal → divide by 100
t = time

Things to know/when to use: Values are growing, increasing; this is for money problems if interest is compounded
$n=1 \rightarrow$ "annually"

Ex 1: Finding the final amount

You purchased a home in 1969 for $19,000. The home as appreciated at a rate of 8.5% each year. What is the value of the home today?

$t = 2017 - 1969 = 48$
$a = 19{,}000$
$r = .085$

$y = 19000(1 + .085)^{48}$

$953,632.48

EX 2: Finding the initial amount

You have a $\boxed{\text{balance}}$ of $2983.76 in your bank account. If the bank has been paying you 5.85% annual interest since 2010, what did you deposit into the account?

y, a, $n = 1$, $t = 7$

$$\frac{2983.76}{\boxed{(1+.0585)^7}} = \frac{a\,\cancel{\boxed{(1+.0585)^7}}}{\cancel{(1+.0585)^7}}$$

$2004.14

EX 3: Finding the growth rate

A painting bought at a flea market in 2001 sold at auction last year for $250,000. Estimate the appreciation rate of the painting.

for $30 ← a
2016 ↑ y
↑ r

$$\frac{250{,}000}{30} = \frac{30}{30}(1+r)^{15}$$

$$\left(\frac{250{,}000}{30}\right)^{\frac{1}{15}} = \left((1+r)^{15}\right)^{\frac{1}{15}}$$

$1.825\ldots = 1 + r$
$.825 \approx r$
$\times 100$
82.5%

Exponential Decay $y = a(1 - r)^t$

y = the final amount
a = the starting amount
r = the decay rate/depreciation rate
t = time

Things to know/when to use: when there is a decrease / depreciation / decay / loss / diminish

EX 1: Finding the final amount

A car purchased in 2007 for $29,100 has depreciated at a rate of 8.1% annually. What is the value of the car today?

$y = 29{,}100(1 - .081)^{10}$

$12,503.98

EX 2: Finding the initial amount

You are interested in purchasing a car at the used car lot for $11,500. The car is 12 years old and has been losing value at 10.4% each year. What was the original value of the car?

↑ a

$$\frac{11{,}500}{\boxed{(1-.104)^{12}}} = \frac{a\,\cancel{\boxed{(1-.104)^{12}}}}{\cancel{(1-.104)^{12}}}$$

$42,953.81

EX 3: Finding the decay rate

↑ r

There are approximately 1000 of a rare species of bird remaining in the world today. (y) There were an estimated 4800 of this species in 1970. Estimate the decay rate of this species.

↗ a

$1000 = 4800(1 - r)^{47}$

ISOLATE THE BASE

$$\left(\frac{1000}{4800}\right)^{\frac{1}{47}} = \left((1-r)^{47}\right)^{\frac{1}{47}}$$

$.967\ldots = 1 - r$
$-.0328\ldots = -r$
$.0328 \approx r$
$\boxed{3.28\%}$

Compounded Growth $y = a\left(1+\frac{r}{n}\right)^{nt}$	Continuous Growth $y = pe^{rt}$

y = the final amount
a = the starting amount
r = the growth/interest rate
n = # times interest is compounded per year
t = time
	y = the final amount
	P = the starting amount same thing as a
	e = natural base (use base e on your calculator!)
	r = growth interest rate
	t = time

Read the problem carefully to determine the value of n: use simple formula *annually* n = 1 *monthly* n = 12 *semi-annually* n = 2 *weekly* n = 52 *quarterly* n = 4 *daily* n = 365	Things to know/when to use: you must read word "continuous"; you can approximate e to 2.718 if necessary
EX 1: Finding the final amount Find the balance of an account that has been earning 6¼% interest compounded $\boxed{\text{monthly}}$ for 5 years given the initial deposit was $2000. y n=12 $y = 2000\left(1 + \frac{.0625}{12}\right)^{5 \cdot 12}$ $2731.46	EX 1: Finding the final amount Your grandparents opened a savings account in 1970 with $7500. The account has been earnings 5½% compounded continuously. What is the balance of the account today? $y = 7500e^{(.055 \cdot 47)}$ $99,474.67
EX 2: Finding the initial amount Your account balance today is $5496.92. If you opened the account in 2009 and the account is paying 5.75% compounded $\boxed{\text{quarterly}}$ n=4 how much did you deposit into the account? y $5496.92 = a\boxed{\left(1 + \frac{.0575}{4}\right)^{4 \cdot 8}}$ ← divide both sides by this thing! $\left(1 + \frac{.0575}{4}\right)^{4 \cdot 8}$ $3481.50	EX 2: Finding the initial amount An account that has been earning 6.19% interest compounded continuously for 20 years has a current balance of $130,000. What was the initial deposit amount? P (same as a) $130{,}000 = P\boxed{e^{.0619 \cdot 20}}$ $\boxed{e^{.0619 \cdot 20}}$ $\boxed{e^{.0619 \cdot 20}}$ $37,695.26
EX 3: Multi-step problems Assume that you have $10,000 to invest for the next 20 years. How much more will you earn in an account that offers 5⅞% compounded semi-annually than one that compounds annually at the same rate? - Calculat each balance, then subtract $y = 10{,}000\left(1 + \frac{.05875}{2}\right)^{2 \cdot 20}$ semi-annual $31,837.92 If n=1 $y = 10{,}000\,(1+.05875)^{20}$ $31,323.37 $514.55	EX 3: Using x/y tables to estimate In 1492, Christopher Columbus invested a nickel with the Native American Credit Union. If the account has been paying 4% interest compounded continuously since then, in what year did Columbus become a millionaire? P = .05 $y = .05e^{.04t}$ 1492 + 421 1913

All Mixed Up Extra Practice with Exponential Growth and Decay

Read each problem carefully! Determine whether each problem should be modeled using the exponential growth, decay, or compounded growth formula.

1. You drink a beverage with 120 mg of caffeine. Each hour, the amount of caffeine in your system decreases by 12%. How much caffeine will be left in your system after 30 minutes? After 4 hours?

$t = .5$ $t = 4$

$y = 120(1 - .12)^{.5}$ $\boxed{112.6 \text{ mg}}$

$y = 120(1 - .12)^4$ $\boxed{72.0 \text{ mg}}$

2. You need to have $40,000 for a down payment on a home in 5 years. The bank will pay 5½% compounded daily. How much should you deposit in order to meet your goal?

$$\frac{40{,}000}{\left(1 + \frac{.055}{365}\right)^{365.5}} = \frac{a\left(1 + \frac{.055}{365}\right)^{365 \cdot 5}}{\left(1 + \frac{.055}{365}\right)^{365 \cdot 5}}$$

$30,383.51

3. A principle of $2000 is deposited into an account that pay 6¼% interest compounded monthly. How much will you have earned on the account in 2 years?

Two parts: subtract the principle

$y = 2000\left(1 + \frac{.0625}{12}\right)^{12 \cdot 2} \rightarrow y = 2265.56$
-2000

$265.56

4. Six students return from spring break with a viral contagion. The CDC estimates that the virus will spread at a rate of 12.8% each day. How many students will be infected after 3 days? After 3 weeks? How many days will it take for the entire school population of NCHS to be infected? (2700 students) ← use tables in calculator

$y = 6(1 + .128)^3$
$y \approx 8$ or 9

$y = 6(1 + .128)^{21}$
$y \approx 75$ students

≈ 51 days

5. A gas-guzzling SUV purchased for $42,000 in 2006 was sold in 2012 for $20,000. Estimate the rate of depreciation of the vehicle.

$20{,}000 = 42{,}000(1 - r)^6$

$\left(\frac{20{,}000}{42{,}000}\right)^{\frac{1}{6}} = \left((1 - r)^6\right)^{\frac{1}{6}}$

$0.8836\ldots = 1 - r$
$-0.1163\ldots = -r$
$r \approx 11.6\%$

6. Andy and Jeff have decided to raise rabbits to make fur coats. They pool their money and are able to purchase 10 rabbits for breeding. If the rabbit population increases at a rate of 40% each year, estimate the number of rabbits they will have after 1 month. After 6 months. After 2 years.

$y = 10(1 + .40)^{\frac{1}{12}}$
they still have only 10

$y = 10(1 + .40)^{.5}$
they will have 11.8

$y = 10(1 + .40)^{2}$
19.6

7. In 1962, my parents invested $250 into an account that paid 6% interest compounded quarterly. If they have not touched the account in all these years, how much money is in the account today? How much interest have they earned?

$y = 250\left(1 + \frac{.06}{4}\right)^{4 \cdot 55}$
$6614.08

balance − principle
$6614.08
−250.00

$6364.08

8. The population of a small town since 1960 can by modeled by the function $P = 65{,}000(.918)^t$, where t is the number of years since 1960.

A. What was the population in 1960? 65,000

B. Is the population increasing or decreasing? decreasing by $(1-.918)=.082$ or 8.2%

C. What was the population in 1976? use $t=16$ $y=65{,}000(.918)^{16} \approx 16{,}535$

D. Estimate the population of the town today. $y=65{,}000(.918)^{57} \approx 495$

9. The equation $y = 2500e^{-.0965t}$ models the population of a species of saltwater fish since the year 2015.

A. Is the population of this species increasing or decreasing? How do you know? By what percentage?

decreasing (because it's negative); by 9.65%

B. Approximate the current population of this species.

$y = 2500e^{-.0965 \cdot 2} \approx 2061$

10. You are investing your savings in an account with quarterly compounding. You estimate that your money will double in ten years. If this is true, what is the interest rate on the account?

Challenge

$2 = 1\left(1+\frac{r}{4}\right)^{4 \cdot 10}$

$2 = \left(1+\frac{r}{4}\right)^{40}$

$2^{\frac{1}{40}} = 1+\frac{r}{4}$

$1.017\ldots = 1+\frac{r}{4}$

$.017\ldots = \frac{r}{4}$

$r \approx .0699$ or 6.99%

11. Find the interest rate needed on an account that compounds semi-annually if you wish to quadruple your investment by the year 2050.

Challenge

$4 = 1\left(1+\frac{r}{2}\right)^{2 \cdot 33}$

$4 = \left(1+\frac{r}{2}\right)^{66}$

$(4)^{\frac{1}{66}} = 1+\frac{r}{2}$

$1.021\ldots = 1+\frac{r}{2}$

$.021 = \frac{r}{2}$

$r \approx .042$ or 4.2%

12. If Christopher Columbus invested a nickel with the Native American Bank when he first arrived, how much would his account be worth if he was given 6.09% compounded semi-annually?

$t = 2017-1492 = 525$

$y = .05\left(1+\frac{.0609}{2}\right)^{2 \cdot 525} \approx \$2.38 \times 10^{12}$

13. How much more will you earn on an account that pays 5⅞% compounded quarterly for 10 years than you will on an account that compounds semi-annually for the same period of time given an initial investment of \$25,000?

Two part

$y = 25{,}000\left(1+\frac{.05875}{4}\right)^{4 \cdot 10}$

\$ 44,795.30

$y = 25{,}000\left(1+\frac{.05875}{2}\right)^{2 \cdot 10}$

\$44,607.96

SUBTRACT!

$\boxed{\$187.34}$

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Ex 1: Finding the final amount

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EX 2: Finding the initial amount

You have a balance of $2983.76 in your bank account. If the bank has been paying you 5.85% annual interest since 2010, what did you deposit into the account?

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Compounded Growth $y = a\left(1+\frac{r}{n}\right)^{nt}$

y = the final amount
a = the starting amount
r = the growth/interest rate
n = # times interest is compounded per year
t = time

Read the problem carefully to determine the value of n:

annually	n = ______	*monthly*	n = ______
semi-annually	n = ______	*weekly*	n = ______
quarterly	n = ______	*daily*	n = ______

EX 1: Finding the final amount

Find the balance of an account that has been earning 6¼% interest compounded monthly for 5 years given the initial deposit was $2000.

EX 2: Finding the initial amount

Your account balance today is $5496.92. If you opened the account in 2009 and the account is paying 5.75% compounded quarterly, how much did you deposit into the account?

EX 3: Multi-step problems

Assume that you have $10,000 to invest for the next 20 years. How much more will you earn in an account that offers 5⅞% compounded semi-annually than one that compounds annually at the same rate?

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Things to know/when to use:

EX 1: Finding the final amount

Your grandparents opened a savings account in 1970 with $7500. The account has been earnings 5½% compounded continuously. What is the balance of the account today?

EX 2: Finding the initial amount

An account that has been earning 6.19% interest compounded continuously for 20 years has a current balance of $130,000. What was the initial deposit amount?

EX 3: Using x/y tables to estimate

In 1492, Christopher Columbus invested a nickel with the Native American Credit Union. If the account has been paying 4% interest compounded continuously since then, in what year did Columbus become a millionaire?

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B. Is the population increasing or decreasing?

C. What was the population in 1976?

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