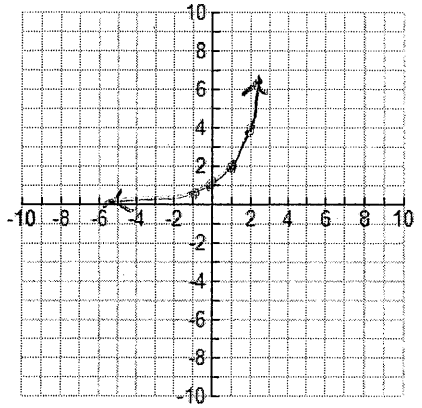
Graph and analyze the following exponential growth functions. Your graph should accurately show the y – intercept and the asymptote. Determine at least two additional points on the right side of the graph. USE A PENCIL!

$f(x) = (2)^x$

x	f(x)
-1	$\frac{1}{2}$
0	1
1	2
2	4



y-intercept (0, 1) asymptote y = 0

domain all ℝ range (0, +∞)

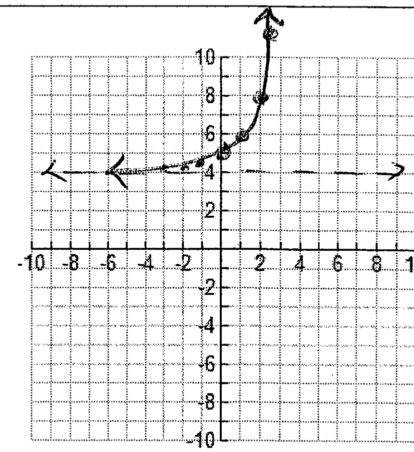
end behavior: as x → +∞, f(x) → +∞, and

as x → -∞, f(x) → 0

2. $f(x) = (2)^x + 4$

x	f(x)
0	5
1	6
2	8
3	12

-1 $4\frac{1}{2}$
-2 $4\frac{1}{4}$
-3 $4\frac{1}{8}$
ASYMPTOTE



y-intercept (0, 5) asymptote y = 4

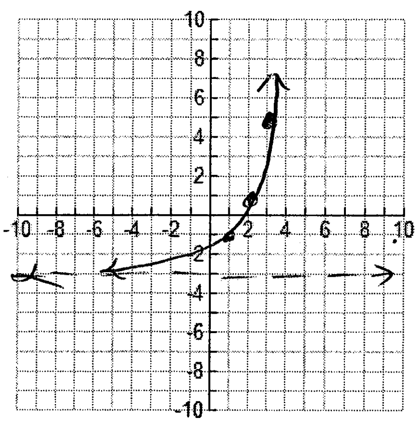
domain all ℝ range (4, +∞)

end behavior: as x → +∞, f(x) → +∞, and

as x → -∞, f(x) → 4

3. $f(x) = (2)^x - 3$

x	f(x)
0	-2
1	-1
2	1
3	5



y-intercept (0, -2) asymptote y = -3

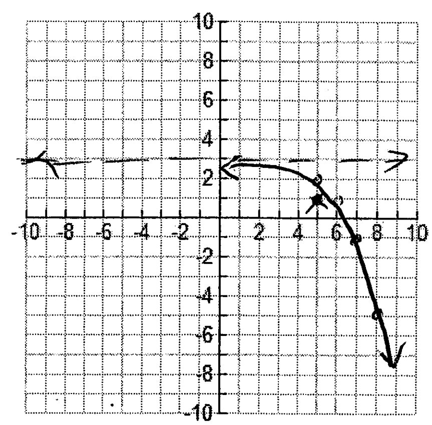
domain all ℝ range (-3, +∞)

end behavior: as x → +∞, f(x) → +∞, and

as x → -∞, f(x) → -3

4. $f(x) = -(2)^{x-5} + 3$

x	f(x)
5	2
6	1
7	-1
8	-5



y-intercept (0, 2.97) asymptote y = 3

domain all ℝ range (-∞, 3)

end behavior: as x → +∞, f(x) → -∞, and

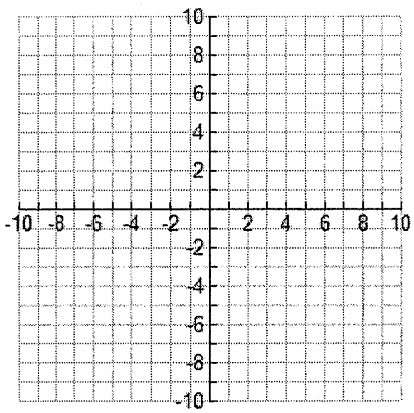
as x → -∞, f(x) → 3

Advanced Algebra Worksheet 4.1 More Transformations on Exponential Growth Functions

Graph and analyze the following exponential growth functions. Your graph should accurately show the y – intercept and the asymptote. Determine at least two additional points on the right side of the graph. USE A PENCIL!

1. $f(x) = (5)^{x-2}$

x	f(x)



y-intercept ________ asymptote ________

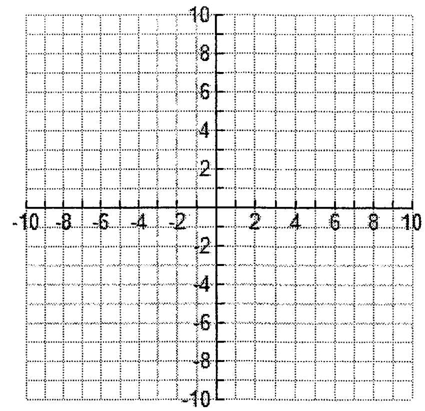
domain ________ range ________

end behavior: as x → +∞, f(x) → __________, and

as x → -∞, f(x) → __________

2. $f(x) = -(2)^x + 8$

x	f(x)



y-intercept ________ asymptote ________

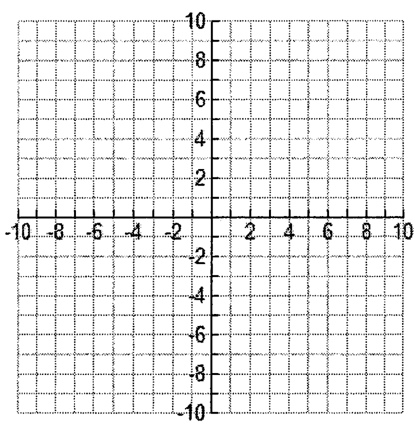
domain ________ range ________

end behavior: as x → +∞, f(x) → __________, and

as x → -∞, f(x) → __________

3. $f(x) = -3(2)^{x+2} + 9$

x	f(x)



y-intercept ________ asymptote ________

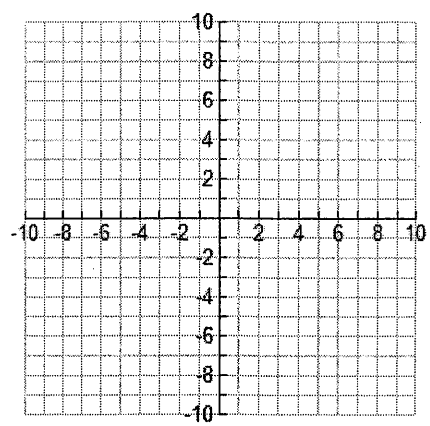
domain ________ range ________

end behavior: as x → +∞, f(x) → __________, and

as x → -∞, f(x) → __________

4. $f(x) = -2(4)^x + 5$

x	f(x)



y-intercept ________ asymptote ________

domain ________ range ________

end behavior: as x → +∞, f(x) → __________, and

as x → -∞, f(x) → __________

Modeling Exponential Growth Functions

Exponential growth can be modeled by the formula $y = a(1 + r)^t$, where

y = the final amount amount (also called balance)	
a = the initial amount amount (also called principle, deposit)	
r = the interest/growth rate (must be written as a decimal)	
t = the time (typically in years, but depends on the problem)	

Write a model and solve. Round answers appropriately.

EX 1: You deposit \$1500 into an account that pays 6.2% interest. What will the balance be after 5 years? After 10 years? $a = 1500$ $r = .062$

$y = 1500(1 + .062)^5 \approx$ \$2026.35

$y = \$1500(1 + .062)^{10}$ \$2737.39

EX 2: Eight students returned to school today with the flu virus. If the virus is expect to spread at a rate of 11% each day, how many students will be affected after one week? After 10 days?

$y = 8(1 + .11)^7$ ~ 16 students

$y = 8(1 + .11)^{10}$ ~ 22 or 23 students

EX 3: Your grandparents purchased an acre of land in 1960 which has appreciated at a rate of 4.5% each year. What is the land worth today? Assume purchase price \$2500

$y = 2500(1 + .045)^{57}$ \$30,730.42

EX 4: How much would you need to deposit into an account that pays 6¼% annual interest in order to have saved \$10,000 by the year 2020? FIND A!

$10{,}000 = a(1 + .0625)^3$

$\frac{10{,}000}{(1 + .0625)^3} = a$

$a \approx$ \$8337.06

EX 5: The equation $y = 6800(1.065)^t$ models the value of a piece of property since its purchase price in 1979.

a, $1 + r$

A. What was the purchase price? \$6800
B. What is the rate of appreciation? 6.5%
C. What is the value of the property today? $t = 2017 - 1979 = 38$

$y = 6800(1.065)^{38}$ \$74,437.88

EX 6: Today it was announced that 70 Emory University students were diagnosed with a serious intestinal virus. If the virus is spreading at a rate of 28% each day, approximately how many days would it take before the entire student population of 14,500 was sickened? (use the tables feature of your graphing calculator to estimate)

$y = 70(1 + .28)^x$

Between days 21 and 22 (we will learn how to solve exactly using logarithms in a few weeks")

Advanced Algebra Y   Name ___________________________

Exponential Growth Problems

Remember! The exponential growth model is $n = a(1+r)^t$ where a is the initial amount, r is the rate (change percent to decimal) and t is the time. If the rate is giving per year, t should be in years. If the rate is given per hour, t should be in hours and so forth.

Frogs

1) A population of 100 frogs increases at an annual rate of 22%. How many frogs will there be in 5 years? Write your model!

$y = 100(1+.22)^5$

$y \approx 270$ frogs

Using this same model for the exponential growth of the frogs, what will be the frog population in

a) 10 years $y = 100(1.22)^{10} \approx 730$ frogs

b) 25 years $y = 100(1.22)^{25} \approx 14{,}421$ frogs

2) A type of bacteria has a very high exponential growth rate at $\boxed{\text{80\% every hour}}$. If there are 10 bacteria, determine how many there will be in 5 hours, 1 day, and 1 week? Write your model!

a) 5 hours $y = 10(1+.8)^5 \approx 189$

b) 1 day $y = 10(1+.8)^{24} \approx 13382588$

c) 1 week $y = 10(1+.8)^{(24\times 7)} \approx 7.687 \times 10^{43}$

3) A species of extremely rare, deep water fish has an extremely rarely have children. If there are a 821 of this type of fish and their growth rate is $\boxed{\text{2\% each month}}$, how many will there be in half of a year, in 10 years and 100 years? Write your model!

a) Half a year (6 months) $y = 821(1+.02)^6 \approx 924$

b) 1 year (12 months) $y = 821(1+.02)^{12} \approx 1041$

c) 10 year $y = 821(1+.02)^{(12\times 10)} \approx 8838$

4) MULTIPLE CHOICE The population of Henderson City was 3,381,000 in 1994, and is growing at an annual rate of 1.8%. If this growth continues, what will the approximate population of Henderson City be in the year 2000.

$y = 3381000(1+.018)^6$

a) 3,696,000 b) 3,798, 000 c) 3,763,000 d) 3,831,000 3762979

5) MULTIPLE CHOICE A culture of bacteria contained 3,842,700 cells on one day and is growing at a daily rate of 6.8%. How many cells would be present 4 days later?

a) 4,999,442 b) 5,043,878 c)5,339,404 d) 15,370,800

$y = 3842700(1+.068)^4$

6. Find a bank account balance if the account starts with $100, has an annual rate of 4%, and the money left in the account for 12 years. Write your model!

$y = 100(1+.04)^{12}$

$160.10

7. In 1985, there were 285 cell phone subscribers in the small town of Centerville. The number of subscribers increased by 75% per year after 1985. How many cell phone subscribers were in Centerville in 1994? Write your model!

$y = 285(1+.75)^9$

43,871

8. The population of Winnemucca, Nevada, can be modeled by P=6191(1.04)t where t is the number of years since 1990. What was the population in 1990? By what percent did the population increase by each year? Write your model!

6191 ; the population grew by 4% each year

$y = 6191(1.04)^0$ # years at beginning

9. You have inherited land that was purchased for $30,000 in 1960. The value of the land increased by approximately 5% per year. What is the approximate value of the land in the year 2011? Write your model!

$y = 30,000(1+.05)^{51}$

$51,310

10. The equation $y = 2(1.162)^t$ models the growth of bacteria in a petri dish after t hours.

A. How many bacteria are in the dish at the beginning of the experiment? 2

B. What is the growth rate or the bacteria per hour? 16.2%

C. How many bacteria will be present after 12 hours? $y = 2(1.162)^{12}$ ≈12

D. How many bacteria will be present after 3 days? $y = 2(1.162)^{(24\times 3)}$

≈99,054

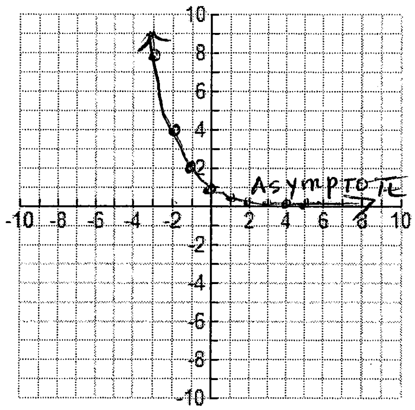
Exponential Decay Functions

Graph and analyze the following exponential growth functions. Your graph should accurately show the y – intercept and the asymptote. Determine at least two additional points on the right side of the graph. USE A PENCIL!

1. $f(x) = ½^x$

x	f(x)
0	1
1	1/2
2	1/4
3	1/8
4	1/16
-1	2
-2	4
-3	8

ASYMPTOTE



y-intercept (0,1) asymptote y=0

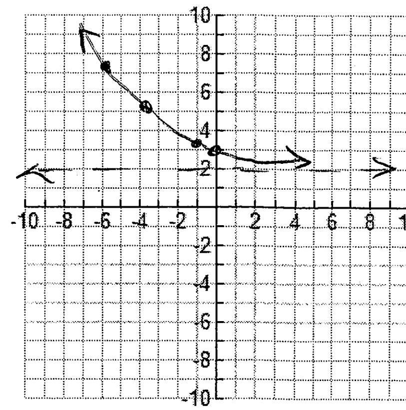
domain all ℝ range (0,+∞)

end behavior: as x → +∞, f(x) → 0,

and as x → -∞, f(x) → +∞

2. $f(x) = .75^x + 2$

x	f(x)
0	3
-1	3.3
-4	5.2
-6	7.6



y-intercept (0,3) asymptote y=2

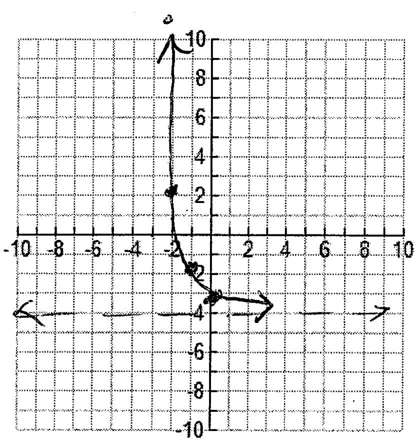
domain all ℝ range (2,+∞)

end behavior: as x → +∞, f(x) → 2, and

as x → -∞, f(x) → +∞

3. $f(x) = .4^x – 4$

x	f(x)
0	-3
-1	-1.5
-2	2.25
-3	11.625



y-intercept (0,-3) asymptote y=-4

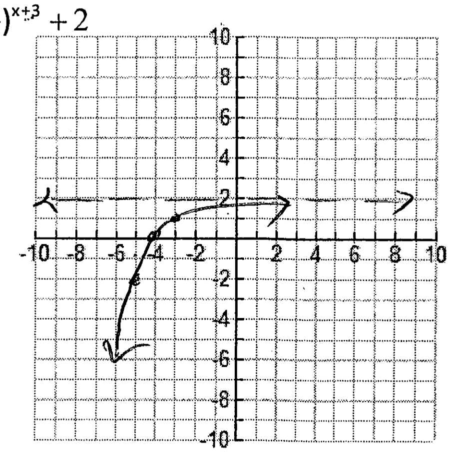
domain all ℝ range (-4,+∞)

end behavior: as x → +∞, f(x) → -4,

and as x → -∞, f(x) → +∞

4. f(x) = $f(x) = -(\frac{1}{2})^{x+3} + 2$

x	f(x)
-3	1
-4	0
-5	-2
0	1.9



y-intercept (0, asymptote (0,1.875)

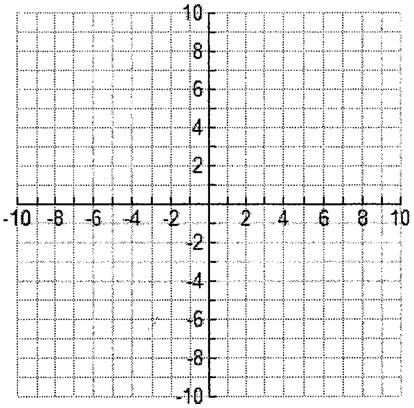
domain all ℝ range (-∞, 2)

end behavior: as x → +∞, f(x) → 2, and

as x → -∞, f(x) → -∞

1. $f(x) = .5^{x-1} - 4$

x	f(x)



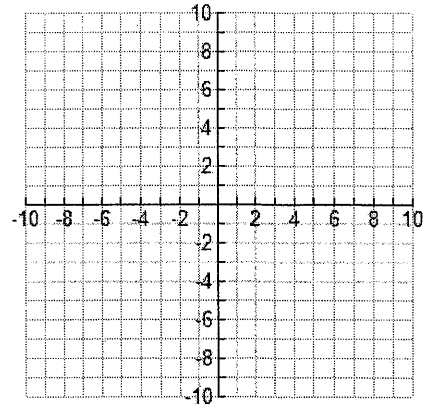
y-intercept ________ asymptote __________

domain ________ range __________

end behavior: as x → +∞, f(x) → __________,
and as x → -∞, f(x) → __________

2. $f(x) = -.6^x + 5$

x	f(x)



y-intercept ________ asymptote __________

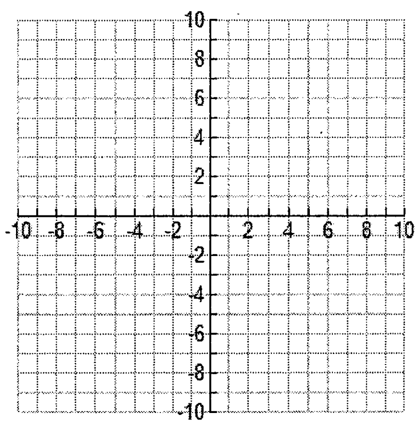
domain ________ range __________

end behavior: as x → +∞, f(x) → __________, and

as x → -∞, f(x) → __________

3. $f(x) = .25^x - 4$

x	f(x)



y-intercept ________ asymptote __________

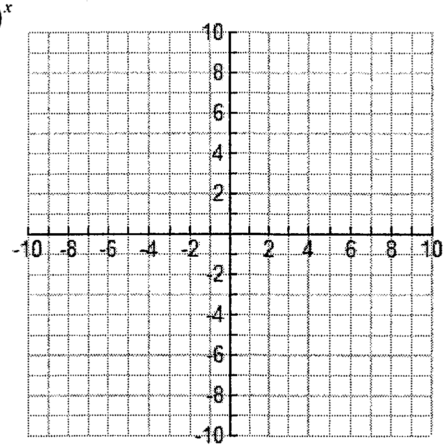
domain ________ range __________

end behavior: as x → +∞, f(x) → __________,

and as x → -∞, f(x) → __________

4. f(x) = $f(x) = -(\frac{1}{2})^x$

x	f(x)



y-intercept ________ asymptote __________

domain ________ range __________

end behavior: as x → +∞, f(x) → __________, and

as x → -∞, f(x) → __________

Modeling Exponential Decay Functions

NOTES

xponential growth can be modeled by the formula $y = a(1-r)^t$, where

y = the final amount amount (also called balance, savings)

a = the starting amount amount (also called principle, deposit)

r = the decay rate/depreciation rate rate (must be written as a decimal)

t = the time (typically in years, but depends on the problem)

Write a model and solve. Round answers appropriately.

Purchase price \$ 21,800

EX 1: You purchased a car in 2010 which is given to have an annual depreciation rate of 9.4%.

$a = 21{,}800 \quad t = 7 \quad r = .094$

a. Approximate the resale value of the car today.

$y = 21{,}800(1 - .094)^t$

$y \approx \$10{,}923.30$

b. Find the resale value in 2020.

$y = 21{,}800(1 - .094)^{10} \approx \$8123.41$

EX 2: A piece of construction equipment purchased in 2006 for \$230,000 is estimated to be worth \$100,000 today. Approximate the depreciation rate of the equipment.

t a y

$100{,}000 = 230000(1-r)^{11} \quad t = 11$

$\frac{100{,}000}{230{,}000} = (1-r)^{11}$ ISOLATE THE BASE FIRST

RAISE BOTH SIDES TO RECIPROCAL POWER

$\left(\frac{100{,}000}{230{,}000}\right)^{\frac{1}{11}} = 1 - r$

$.927\ldots = 1 - r$

$-.073\ldots = -r$

$r \approx 7.3\%$

EX 3: In 1980, the World Wildlife Organization modeled the Giant Panda Population using the model $P = 2100(.94)^t$, where t represents years.

a

a. According to the model, what was the Giant Panda Population in 1980? 2100

b. According to the model, was the Giant Panda population increasing or decreasing? How do you know? By what percentage?

decreasing; the base is .94 which is less than 1

$1 - .94 = .06$ or 6%

c. According to the model, what would the Giant Panda population be in the year 2000? In the year 2050?

$t = 20$

$y = 2100(.94)^{20}$

≈ 609 in 2000

$t = 70$

$y = 2100(.94)^{70}$

≈ 27 there would only be 27 pandas in 2050

Extra Practice with Exponential Growth and Decay Name ____________________

Determine whether each of the following is an application of growth or decay, then solve, rounding appropriately.

1. Your grandparents purchased an acre of ocean front property in Panama City in 1960 for $1800. If the value of the land has appreciated at an average rate of 11% each year, what is the value of the land today? Write a model for the function and find the value.

$y = 1800(1+.11)^{57}$

$y \approx \$689{,}735.05$

2. In 1972, your Dad purchase his first car, a canary yellow Camaro, for $2600. The car has depreciated at a rate of approximately 8.5% each year. What is the value of the car today? Write a model for the function and find the value.

$y = 2600(1-.085)^{45}$ $t = 45$ $r = .085$

$\$47.74$

3. A car valued at $14,900 today has depreciated at an average rate of 7.6% each year. If the car was purchased in 2001, estimate its purchase price.

$\frac{14{,}900}{(1-.076)^{16}} = \frac{a\cancel{(1-.076)^{16}}}{\cancel{(1-.076)^{16}}}$

$y \approx \$52{,}775.79$

4. In 1999, Edna invested $5000 into an account whose balance today is $9075.37. What is the interest rate she is earning on the account?

$9075.37 = 5000(1+r)^{18}$

$\frac{9075.37}{5000} = (1+r)^{18}$

$\left(\frac{9075.37}{5000}\right)^{\frac{1}{18}} = 1+r$

$1.033\ldots = 1+r$

$.034 \approx r$ or 3.4%

5. A construction company purchased a piece of heavy equipment in 2008. The equipment is now worth only half its purchase price. At what rate did the equipment diminish in value?

↑ use Ratios

$\frac{1}{2} = 1(1-r)^9$

$\left(\frac{1}{2}\right)^{\frac{1}{9}} = 1-r$

$.925\ldots = 1-r$

$-.074\ldots = -r$

oops; wrote in wrong space

use ratios

6. A car purchased in the year 2000 is now considered to be worth one third of its purchase price. What is the depreciation rate of the car? $t = 17$

$\frac{1}{3} = 1(1-r)^{17}$

$\left(\frac{1}{3}\right)^{\frac{1}{17}} = 1-r$

$.925\ldots = 1-r$

$-.074\ldots = -r$

$r \approx 7.4\%$

$\frac{1}{3} = 1(1-r)^{17}$

$\left(\frac{1}{3}\right)^{\frac{1}{17}} = 1-r$

$.937\ldots = 1-r$

$-.0628\ldots = -r$

$r \approx 6.3\%$

7. If you wish to grow your savings by 40% in 5 years, what interest rate will you need to earn on an account paying simple interest?

challenge

$1.4 = 1(1+r)^5$

$(1.4)^{\frac{1}{5}} = 1+r$

$1 \times .4 = .4$

↑100% ↑ + 40% more

$1.069\ldots = 1+r$

$.0696\ldots \approx r$

$r \approx 7.0\%$

8. A valuable painting was sold last week at auction for 20 times its purchase price in 1980. Estimate the appreciation rate of the painting?

$20 = 1(1+r)^{37}$

$(20)^{\frac{1}{37}} = 1+r$

$1.08\ldots = 1+r$

$r \approx 8.4\%$

9. A colony of bacteria doubles in 6 hours. What is its growth rate?

$2 = 1(1+r)^6$ $\qquad$ $1.122\ldots = 1+r$

$2^{\frac{1}{6}} = 1+r$ $\qquad$ $0.122 = r$

$r \approx 12.2\%$

10. What is the depreciation rate of a laptop computer if it is worth 25% ~~less than its purchase~~ price after 6 months?

$.75 = 1(1-r)^{.5}$

$.5625 = 1-r$

$-.4375 = -r$

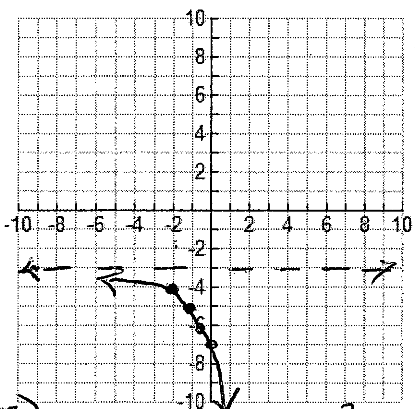
$r = 43.75\%$

this means the y value is .75 of original value

Do the following functions represent growth or decay? How do you know? Graph and analyze.

1. $f(x) = -(2)^{x+2} - 3$

x	f(x)
-2	-4
-1	-5
0	-7
1	-11



y-intercept $(0,-7)$ asymptote $y=-3$

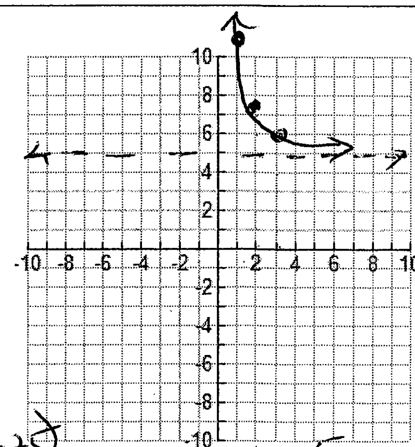
domain all $\mathbb{R}$ range $(-\infty,-3)$

end behavior: as $x \to +\infty$, $f(x) \to$ $-\infty$, and

as $x \to -\infty$, $f(x) \to$ -3

2. $f(x) = (.4)^{x-3} + 5$

x	f(x)
3	6
2	7.5
1	11.25
0	20.625



y-intercept $(0, 20.625)$ asymptote $y=5$

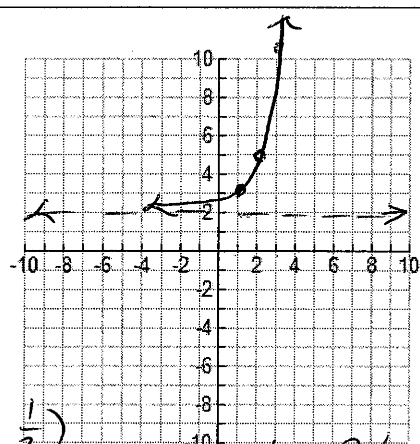
domain all $\mathbb{R}$ range $(5,+\infty)$

end behavior: as $x \to +\infty$, $f(x) \to$ 5, and

as $x \to -\infty$, $f(x) \to$ $+\infty$

1. $f(x) = (3)^{x-1} + 2$

x	f(x)
1	3
2	5
3	11
+0	$2\frac{1}{3}$



y-intercept $(0,2\frac{1}{3})$ asymptote $y=2$

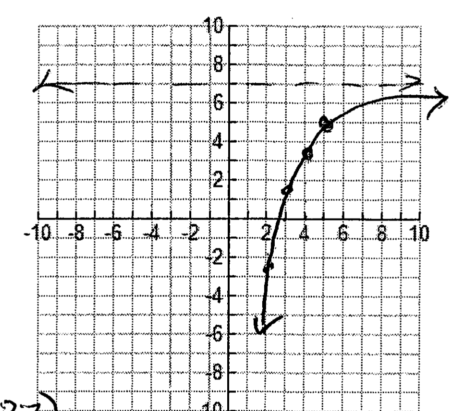
domain all $\mathbb{R}$ range $(2,+\infty)$

end behavior: as $x \to +\infty$, $f(x) \to$ $+\infty$, and

as $x \to -\infty$, $f(x) \to$ 2

2. $f(x) = -2(.6)^{x-5} + 7$

x	f(x)
5	5
4	3.7
3	1.4
2	-2.3



y-intercept $(0,-18.7)$ asymptote $y=7$

domain all $\mathbb{R}$ range $(-\infty, 7)$

end behavior: as $x \to +\infty$, $f(x) \to$ 7, and

as $x \to -\infty$, $f(x) \to$ $-\infty$