

Warm-up/Extra Practice with Expanding and Condensing Logarithms

Expand the following by applying the properties of logarithms.

1. $\log_7(4xy)^3$

2. $\log_7 4xy^3$

3. $\log_7 \frac{4x}{y^3}$

4. $\log_7 \frac{4}{xy^3}$

5. $\log_7 \sqrt[3]{4xy}$

6. $\log_7 \frac{\sqrt{4x}}{y^3}$

Condense the following expressions into a single logarithmic term. Evaluate, if possible.

7. $\log_2 32 - \log_2 128$

8. $\log 0.1 + \log 1 + \log 10$

9. $5\ln x + 4\ln y + 3\ln z$

10. $\frac{1}{2}\log_3 x - \frac{1}{2}\log_3 y$

11. $\log(x^2 - 4) - \log(x - 2)$

12. $5\log_2 x - \log_2 y + \frac{1}{4}\log_2 z$



To EXPAND

- write one term for each factor
- give each term the appropriate sign (+ or -)
- give each term a coefficient, if applicable

Justification

1. $\log_7 (4xy)^3$

Step 1

There are 3 factors = 4, x and y
therefore you must write
3 terms in your expansion

Step 1

$\log_7 4$

$\log_7 x$

$\log_7 y$

Step 2

$\log_7 4 + \log_7 x + \log_7 y$

Step 2

All 3 factors are in
the numerator so
put + signs in
between them

Step 3

$3\log_7 4 + 3\log_7 x + 3\log_7 y$

Step 3 All 3 factors are
in parentheses therefore
they all get the power of 3.
The property of logs
dictates that an exponent
becomes a coefficient
when expanding.

2. $\log_7 4xy^3$

Step 1

$\log_7 4$

$\log_7 x$

$\log_7 y$

Step 2

$\log_7 4 + \log_7 x + \log_7 y$

Step 3

$\log_7 4 + \log_7 x + 3\log_7 y$

← only the y is
raised to the 3rd
power so only
y gets the
coefficient 3

$$3. \log_7 \frac{4x}{y^3}$$

$$\text{Step 1} \quad \log_7 4 \quad \log_7 x \quad \log_7 y$$

$$\text{Step 2} \quad \log_7 4 + \log_7 x - \log_7 y$$

$$\text{Step 3} \quad \log_7 4 + \log_7 x - 3\log_7 y$$

The factor y
is in the
denominator
therefore it
gets a
subtraction
sign.

$$4. \log_7 \frac{4}{xy^3}$$

$$\text{Step 1} \quad \log_7 4 \quad \log_7 x \quad \log_7 y$$

$$\text{Step 2} \quad \log_7 4 - \log_7 x - \log_7 y$$

$$\text{Step 3} \quad \log_7 4 - \log_7 x - 3\log_7 y$$

Both x and y
are in the
denominator,
so both are
negative in the
expansion.

$$5. \log_7 \sqrt[3]{4xy} \rightarrow \text{same as } \log_7 (4xy)^{\frac{1}{3}}$$

$$\text{Step 1} \quad \log_7 4 \quad \log_7 x \quad \log_7 y$$

$$\text{Step 2} \quad \log_7 4 + \log_7 x + \log_7 y$$

$$\text{Step 3} \quad \frac{1}{3} \log_7 4 + \frac{1}{3} \log_7 x + \frac{1}{3} \log_7 y$$

$$6. \log_7 \frac{\sqrt{4x}}{y^3} \rightarrow \text{same as } \log_7 \frac{2\sqrt{x}}{y^3} \rightarrow \log_7 \frac{2x^{\frac{1}{2}}}{y^3}$$

Step 1 $\log_7 2 \quad \log_7 x \quad \log_7 y$

Step 2 $\log_7 2 + \log_7 x - \log_7 y$

Step 3 $\log_7 2 + \frac{1}{2} \log_7 x - 3 \log_7 y$

$$7. \log_2 32 - \log_2 128$$

$$\log_2 \frac{32}{128}$$

quotient prop.

EVALUATE $\log_2 \frac{1}{4}$ (By definition, this asks $2^x = \frac{1}{4}$)

-2

$$8. \log 0.1 + \log 1 + \log 10$$

$$\log (0.1)(1)(10)$$

product prop.

EVALUATE $\log 1$

because $\log_b 1 = 0$

$$9. 5 \ln x + 4 \ln y + 3 \ln z$$

$$\ln x^5 + \ln y^4 + \ln z^3$$

power prop.

product prop.

$\ln x^5 y^4 z^3$

$$10. \frac{1}{2} \log_3 x - \frac{1}{2} \log_3 y$$

$$\log_3 x^{\frac{1}{2}} - \log_3 y^{\frac{1}{2}}$$

(same as $\log_3 \sqrt{x} - \log_3 \sqrt{y}$)

$$\log_3 \frac{\sqrt{x}}{\sqrt{y}}$$

or $\log_3 \sqrt{\frac{x}{y}}$

$$11. \log(x^2-4) - \log(x-2)$$

$$\log \frac{(x^2-4)}{(x-2)}$$

quotient prop.

$$\log \frac{(x-2)(x+2)}{(x-2)}$$

difference of squares

$$\log(x+2)$$

simplify

factoring

$$12. 5 \log_2 X - \log_2 Y + \frac{1}{4} \log_2 Z$$

$$\log_2 X^5 - \log_2 Y + \log_2 Z^{\frac{1}{4}}$$

$$\log_2 \frac{X^5 \cdot Z^{\frac{1}{4}}}{Y}$$

product +

quotient prop

$$\log_2 \frac{X^5 \sqrt[4]{Z}}{Y}$$