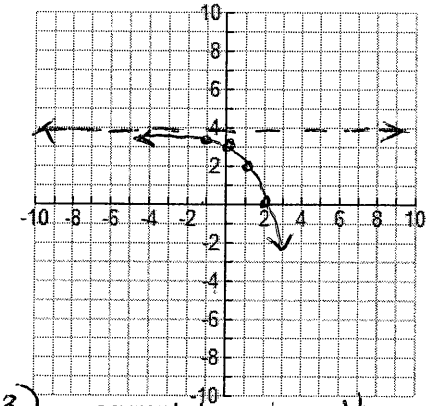
S Advanced Algebra Y Name _____

Review: Exponential Growth and Decay, Compounded Interest and Base e functions

Graph and analyze the following exponential growth functions. Your graph should show the y-intercept, the asymptote, and at least two additional points.

1. $f(x) = -2^x + 4$

x	f(x)
0	3
1	2
2	0
-1	$3\frac{1}{2}$



y-intercept (0,3) asymptote $y = 4$

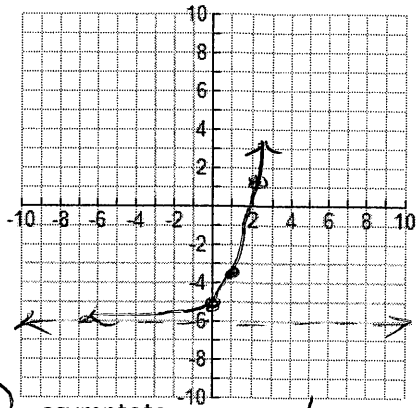
domain all R range $(-\infty, 3)$

end behavior: as $x \to +\infty$, $f(x) \to$ $-\infty$, and

as $x \to -\infty$, $f(x) \to$ 3

2. $f(x) = e^x - 6$

x	f(x)
0	-5
1	-3.3
2	1.4



y-intercept (0,-5) asymptote $y = -6$

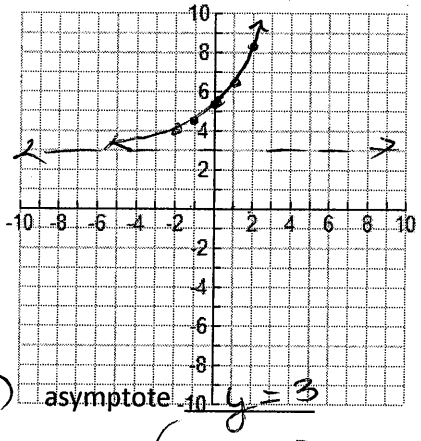
domain all R range $(-6, +\infty)$

end behavior: as $x \to +\infty$, $f(x) \to$ $+\infty$, and

as $x \to -\infty$, $f(x) \to$ -6

3. $f(x) = 1.5^{x+2} + 3$

x	f(x)
-2	4
-1	4.5
0	5.3
1	6.4
2	8.1



y-intercept (0,5.3) asymptote $y = 3$

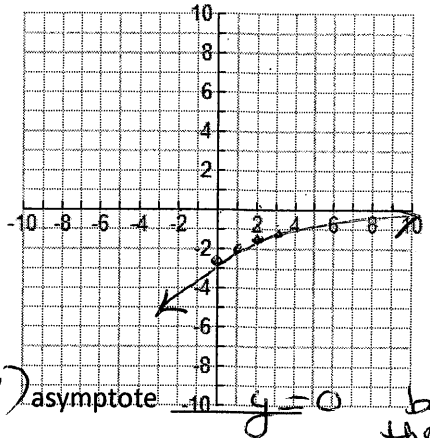
domain all R range $(3, +\infty)$

end behavior: as $x \to +\infty$, $f(x) \to$ $+\infty$, and

as $x \to -\infty$, $f(x) \to$ 3

4. $f(x) = -2(.75)^{x-1}$

x	f(x)
0	-2.7
1	-2
2	-1.5
3	-1.1



y-intercept (0,-2.7) asymptote $y = 0$ because there is no constant!

domain all R range $(-\infty, 0)$

end behavior: as $x \to +\infty$, $f(x) \to$ 0, and

as $x \to -\infty$, $f(x) \to$ $-\infty$

Read each of the following applications problems carefully. Determine whether the situation models growth or decay, compounded or continuously compounded interest. Write an equation to model the problem, then solve. Round appropriately

$y = a(1 + \frac{r}{n})^{nt}$ $y = Pe^{rt}$ $y = a(1+r)^t$ $y = a(1-r)^t$

↓ like shoes, this means 4!

5. So you want to breed rabbits? If you start with two pairs of bunnies and the rabbit population increases at an average rate of 23% each month, how many rabbits will you expect to have at the end of the year?

$y = 4(1+.23)^{12} \approx 48$ rabbies at end of year

↑ change to 12 months

6. The local credit union is advertising $4\frac{7}{8}$% interest compounded continuously. If you invest your $150,000 inheritance, how much money would you have at the end of 20 years?

$y = 150{,}000e^{(.04875)(20)}$

$y \approx 397{,}675.08$

7. Congratulations! You have finally saved the $50,000 you needed to put a down payment on a house. If your money has been in an account that paid 6% interest compounded semi-annually for the last 10 years, how much money did you invest?

$50{,}000 = a\left(1+\frac{.06}{2}\right)^{(2)(10)}$

$\frac{50{,}000}{\left(1+\frac{.06}{2}\right)^{20}} = a$

$a \approx \$27{,}683.79$

8. A $74,000 luxury SUV is estimated to lose its value at a rate of approximately 9.2% each year. What will its value be after 1 year? What will its value be after 10 years?

$y = 74{,}000(1-.092)^1$

$\$67{,}192$ after 1 year

$y = 74{,}000(1-.092)^{10}$

$y = \$28{,}189.68$ after 1 year

9. If you invest $2000 into an account that pays 5.15% interest compounded quarterly, how much will be in your account if you do not touch it until 2020?

$y = 2000\left(1+\frac{.0515}{4}\right)^{4\times 3}$

$y \approx \$2331.85$

10. A piece of beach front property in Destin, Florida was purchased for $4000 in 1972. If the land has appreciated at a rate of approximately 15% each year, what is the land worth today?

$y = 4000(1+.15)^{45}$

$y \approx \$2{,}155{,}077.08$

11. You would like to save $10,000 for college expenses. If you are planning on starting college six years from now, how much money should you deposit into an account that pays $5\frac{1}{4}$% annual interest?

$10{,}000 = a(1+.0525)^6$ (10,000 = y)

$\frac{10{,}000}{(1+.0525)^6} = a$

$y \approx \$7356.43$

12. A piece of heavy construction equipment purchased in 2001 is worth $23,900 today. What was its initial value if it has decreased in value at an approximate rate of 12% each year?

~~$23{,}900a(1-.12$~~

$23{,}900 = a(1-.12)^{16}$

$\frac{23{,}900}{(1-.12)^{16}} = a$

$a \approx \$184{,}788.59$

13. A family of six vampires has just moved into Legacy Park. If the vampire population grows at a rate of 25% each day, how many vampires will there be in one week? How many vampires will there be in one year?

$y = 6(1+.25)^7$

$y \approx 28$ or 29 after 1 week

$y = 6(1+.25)^{365}$

1.41×10^{36} after 1 year

14. Eight bacterium are placed in a petri dish at 8:00 a.m. If the bacteria population increases at a rate of approximately 42% each hour, how many bacteria will there be at midnight of the same day?

$y = 8(1+.42)^{16} \approx 2186$

15. What interest rate do you need on an account with monthly compounding if you wish to triple your investment in 20 years?

$3 = 1\left(1+\frac{r}{12}\right)^{12 \times 20}$

$3 = \left(1+\frac{r}{12}\right)^{240}$

RECIPROCAL POWER

$3^{\frac{1}{240}} = \left(\left(1+\frac{r}{12}\right)^{240}\right)^{\frac{1}{240}}$

$1.00\ldots = 1+\frac{r}{12}$

$.0045\ldots = \frac{r}{12}$

16. When the US auto industry tanked, so did the population of many towns fed by this industry. A Michigan town with a population of 345,000 in 1980 has approximately 200,000 inhabitants today. At what rate is the population decreasing?

$r \approx 5.5\%$

~~$200,000($~~ $200,000 = 345,000(1-r)^{37}$

$\frac{200,000}{345,000} = (1-r)^{37}$

$\left(\frac{200,000}{345,000}\right)^{\frac{1}{37}} = 1-r$

$.985\ldots = 1-r$

$-.014\ldots = -r$

$r \approx 1.46\%$

17. The equation $C = 5600(1.12)^t$ models the tuition at a private college since the year 1984.

Is the tuition increasing or decreasing? How do you know? increasing; the base > 1

What was the tuition in 1984? $ 5600

Estimate the tuition for the year 2000. $y = 5600(1.12)^{16}$

$\approx \$34,330$ per year

18. The equation $V = 34{,}950(.928)^t$ models the value of a car t years after its initial purchase.

Is the value of the car increasing or decreasing? How do you know? decreasing; the base < 1

What is the growth/decay rate of the car? to find, set $(1-r) = .928$

$-r = .928 - 1$

$-r = -.072$

$r = .072$

or 7.2%

Approximate the value of the car after 6 years.

$y = 34{,}950(.928)^6$

$y \approx \$22,322$

19. Johnny purchased a lime green Camaro in 1972 for $4600. The car has lost value at a rate of approximately 9.8% each year. What is the value of his car today?

$y = 4600(1-.098)^{45}$

$y \approx \$4.37$

20. The Giant Panda population is shrinking. In 1976, there were an estimated 6200 Pandas remaining in the world.

$t = 41$

A. If the population has decreased at a rate of approximately 12% each year, approximately how many Pandas remain in the world today?

$y = 6200(1 - .12)^{41} \approx 33$ pandas

B. Approximately how many Pandas can we expect to have in the year 2050?

$y = 6200(1 - .12)^{74} \approx 0.5$ (so, theoretically, there would be none left :)

21. On the day of your grandchild's birth, you deposited $25,000 in a trust fund that pays 5.75% interest, compounded continuously. Determine the balance in this account on your grandchild's 25th birthday.

$y = 25000e^{.0575 \times 25} \approx \$105,253.93$

22. Suppose you deposit $5000 in a trust fund that pays 7.5% interest, compounded continuously. In the trust fund, you specify that the balance will be given to the college from which you graduated after the money has earned interest for 50 years. How much will your college receive after 50 years?

$y = 5000e^{.075 \times 50} \approx \$212,605.41$

23. Let Q represent the mass of radium whose half-life is 1620 years. The quantity of radium present after t years is given by $Q = 25(.5)^{\left(\frac{t}{1620}\right)}$.

a. Determine the initial quantity (when t = 0) 25 grams (possibly mg?)

b. Determine the quantity present after 1000 years. $Q = 25(.5)^{\left(\frac{1000}{1620}\right)} \approx 16.3$ g

24. If the annual rate of inflation averages 5% over the next 10 years, then the approximate cost C of goods or services during any year in that decade will be given by $C = P(1.05)^t$, where t is the time in years and P is the present cost. If the price of an oil change for your car is presently $19.95, estimate the price 10 years from now.

$y = 19.95(1.05)^{10}$

In 10 years, it will cost $32.50

25. How much would you need to deposit into an account that pays 6¼% interest compounded quarterly in order to have saved $10,000 by the year 2020?

↑ this is y

$10,000 = a\left(1 + \frac{.0625}{4}\right)^{4 \times 3}$

$a = \frac{10,000}{\left(1 + \frac{.0625}{4}\right)^{12}}$

$a \approx \$8302.32$

Choose the best answer.

D 26. Suppose the amount of radioactive substance remaining in a 100 milligram sample after t years can be modeled by $A = 100e^{-0.01653t}$. How much is remaining after 24 years?

A. 0.67 mg B. 39.67 mg C. 148.69 mg D. 67.25 mg

C 27. What is the y-intercept of the equation $y = 2^{x+6} - 11$?

$y = 2^{0+6} - 11$

$y = 2^6 - 11$

$y = 64 - 11$

$y = 53$

A. -6 B. -11 C. 53 D. -9

C 28. Which of the following equations represents exponential decay?

A. $Y = 2^{x-1}$ B. $y = -2^x - 1$ C. $y = .2^x$ D. $y = 1.2^x$

↑ .2 is less than 1 base < 1

A 29. The domain of exponential functions is always "all real numbers".

A. True B. False

A 30. The constant of the exponential equation represents the asymptote.

A. True B. False

A 31. The range of an exponential function depends on the asymptote.

A. True B. False

C 32. A population of 390 animals decreases at an annual rate of 11%. Identify the exponential function that models this situation.

A. $Y = 390(0.11)^x$
B. $Y = 390^{0.11x}$
C. $Y = 390(0.89)^x$
D. $Y = (0.89)390^x$

$y = 390(1 - .11)^x$

$y = 390(.89)^x$

C 33.. Which of the following functions has a depreciation rate of 6%? →

$y = a(1 - .06)^t$

$y = a(.94)^t$

A. $y = 100(1.6)^t$ B. $y = 100(.4)^t$ C. $y = 100(1.06)^t$ C. $y = 100(.94)^t$