How can I factor a polynomial?

Thursday

Squares + cubes patterns

Always do this first!

FACTOR OUT THE GCF!
(greatest common factor)

Next step:

Count the number of terms

If it is a trinomial

If it is a binomial
(2 terms)

If it is a 4 term polynomial

How can I recognize a perfect square?

1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256, 289, 324, 361, 400

The exponents will be even

$x^2, x^4, x^6, x^8, x^{10}, \ldots$

Then look for one of these patterns

Do the terms contain only perfect squares or perfect cubes?

$x^3, x^6, x^9, x^{12}, x^{15}, \ldots$

How can I recognize a perfect cube?

1, 8, 27, 64, 125, 216, 343, 512, 729, 1000

The exponents will be multiples of 3

Squares

Cubes

$x^2 - y^2 = (x-y)(x+y)$

$x^3 + y^3 = (x+y)(x^2 - xy + y^2)$

OR

$x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

Don't forget to use S.O.A.P.!

SAME, OPPOSITE, ALWAYS POSITNE

DIFFERENCE OF SQUARES EXAMPLES

1. $x^2 - 1$
   $(x-1)(x+1)$

2. $x^2 - 4$
   $(x-2)(x+2)$

3. $9x^2 - 16$
   $(3x-4)(3x+4)$

4. $(25x^4 - 49)$
   $(5x^2-7)(5x^2+7)$

5. $-100x^2 + 121$
   (same as $121 - 100x^2$)
   $(11-10x)(11+10x)$

CUBES EXAMPLES

1. $x^3 + 1$
   $(x+1)(x^2 - x + 1)$

2. $27x^3 - 8$
   $(3x-2)(9x^2 + 6x + 4)$

3. $(1000x^3 + 343)$
   $(10x+7)(100x^2 - 70x + 49)$

Key

Factoring Monomials

To factor a polynomial means to write the polynomial as a product of prime polynomials.

1. Find the GCF first and factor it out. Make sure to write to include as part of your final answer.
2. Factor out the GCF from each of the terms
3. Look for one of the special patterns. If none applies, you are done.

1. $6a^3 + 15a$ GCF: $3a$

$2 \cdot 3 \cdot a \cdot a \cdot a$

$3 \cdot 5 \cdot a$

$3a(2a^2 + 5)$

2. $32b^2 + 12b$ GCF: $4b$

$4b(8b + 3)$

4. $12a^5b^2 + 16a^4b$ GCF: $4a^4b$

$4a^4b(3ab + 4)$

4. $9x^2 + 18y^4$ GCF: 9

$9(x^2 + 2y^4)$

5. $7x^2 - 15y$

$7 \cdot x \cdot x$

$-3 \cdot 5 \cdot y$

} there are no common factors ∴ this is "PRIME"

6. $y^4 - 3y^2 - 2y$ GCF: y

$y(y^3 - 3y - 2)$

Difference of Squares

1. Find the GCF first and factor it out. Make sure to write to include as part of your final answer.
2. Look for perfect square numbers and even powers.
3. Apply the pattern $\mathbf{A^2 - B^2 = (A - B)(A + B)}$

1. $5x^2 - 80$ GCF: 5

$5(x^2 - 16)$

$5(x - 4)(x + 4)$

2. $81x^2 - 4$

$(9x - 2)(9x + 2)$

3. $8x^5 - 72x^3$ GCF: $8x^3$

$8x^3(x^2 - 9)$

$8x^3(x - 3)(x + 3)$

4. $100a^2 - 225$ GCF: 25

$25(4a^2 - 9)$

$25(2a + 3)(2a - 3)$

5. $625 - 49y^2$

$(25 + 7y)(25 - 7y)$

6. $-121 + 64x^2$ same as $64x^2 - 121$

$(8x + 11)(8x - 11)$

7. $a^2b^2 - 25$

$(ab - 5)(ab + 5)$

8. $4 - 36x^2$ GCF: 4

$4(1 - 9x^2)$

$4(1 - 3x)(1 + 3x)$

9. $400x^2y^2 - 49a^2$

$(20xy - 7a)(20xy + 7a)$

10. $A^2 + B^2$

↑ the sum OF SQUARES

"primes" never factors

The Sum and Difference of Cubes

1. Find the GCF first and factor it out. Make sure to write to include as part of your final answer.
2. Look for numbers that are perfect cubes and variables that are multiples of three.
3. Factor using one of the patterns:

$A^3 + B^3 = (A + B)(A^2 - AB + B^2)$ or **$A^3 - B^3 = (A - B)(A^2 + AB + B^2)$**

1. $x^3 - 8$ $(x - 2)(x^2 + 2x + 4)$

2. ~~$x^3y^3 - 27$~~

$(xy - 3)(x^2y^2 + 3xy + 9)$

3. $8x^3 - 1000$ GCF: 8

$8(x^3 - 125)$

$8(x - 5)(x^2 + 5x + 25)$

4. $216x^3 + 125$

$(6x + 5)(36x^2 - 30x + 25)$

5. $4x^3 + 32y^3$ GCF: 4

$4(x^3 + 8y^3)$

$4(x + 2y)(x^2 - 2xy + 4y^2)$

6. ~~$x^3 + 8y^6$~~

$(x + 2y^2)(x^2 - 2xy^2 + 4y^4)$

7. $8y^3 + 729$

$(2y + 9)(4y^2 - 18y + 81)$

8. $250x^3 + 2$ GCF: 2

$2(125x^3 + 1)$

$2(5x + 1)(25x^2 - 5x + 1)$

9. $1000a^3b^6 + 27$

$(10ab^2 + 3)(100a^2b^4 - 30ab^2 + 9)$

10. $4 - 256p^6$ GCF: 4

$4(1 - 64p^6)$

$4(1 - 4p^2)(1 + 4p^2 + 16p^4)$