Remember the inverse property of logarithms!

$\log_b y = x$ if and only if $b^x = y$

Case I: Use the inverse property of logarithms (BOB) to solve equations which contain a Single logarithm or which can be Condensed to a single logarithm on one side of the equation.

EX 1: $\log_2(3x-1)=5$

Rewrite in exp. form

$2^5 = 3x-1$

$32 = 3x-1$

$33 = 3x$

$x = 11$

$3(11)-1$
$33-1$
$32>0$ ✓

EX 2: $4-\ln 7x = 2$

$-4 \quad -4$

$-\ln 7x = -2$

$\ln_e 7x = 2$

$e^2 = 7x$

$7.38\ldots = 7x$

$x \approx 1.06$

EX 3: $\log_{10}(x^2+36)=2$

$10^2 = x^2+36$

$100 = x^2+36$

$64 = x^2$

$x = \pm 8$

$y=8,\ x=-8$

The argument of a logarithmic expression must be greater than zero. For that reason, you must always check for extraneous solutions!

ANSWERS

EX 4: $\log_3 x + \log_3(x-6) = 3$

$x=9$

Condense into a single $\log_3$

$\log_3 x(x-6) = 3$ BOB

$3^3 = x(x-6)$ QUADRATIC

$27 = x^2-6x \rightarrow x^2-6x-27=0$

$(x-9)(x+3)=0$

$x=9,\ x=-3$ extraneous

BIG x

EX 5: $\log_2(7x-8)-\log_2 x = 3$

CONDENSE

$\log_2 \frac{7x-8}{x} = 3$ BOB

$2^3 = \frac{7x-8}{x}$

$\frac{8}{1} = \frac{7x-8}{x}$

$8x = 7x-8$

$x=-8$

NO SOLUTION

You try: $\log_2(x+1)+\log_2(x-5)=4$

CONDENSE

$\log_2 (x+1)(x-5) = 4$ BOB

$2^4 = (x+1)(x-5)$ FOIL

$16 = x^2-5x+x-5$

PUT IN STANDARD FORM

$0 = x^2-4x-21$

$0 = (x-7)(x+3)$

$x=7$, $\cancel{x=-3}$

The *One-to-One Property* of Logarithms

If $\log_b x = \log_b y$, then $x = y$

Case2: Use this property to solve logarithmic equations with a logarithmic equation on both sides of the equation.

EX 1: $\cancel{\log_5} 4x = \cancel{\log_5} 8$

$4x = 8$

$x = 2$

EX 2: $\underbrace{\log_7 36 - \log_7 x}_{\text{CONDENSE}} = \log_7 4$

CAREFUL: DO NOT WRITE $36 - x = 4$

$\log_7 \frac{36}{x} = \log_7 4$

$\frac{36}{x} = \frac{4}{1}$

$4x = 36$

$x = 9$

EX 3: $\log(3x+10) = \log x^2$

QUADRATIC

$3x + 10 = x^2$

$x^2 - 3x - 10 = 0$

$(x-5)(x+2) = 0$

$x = 5,\ x = -2$

- Condense each side of the equation (if needed) into a single logarithm
- Equate the arguments
- Solve
- Check for extraneous solutions

EX 4: $\log_3 x = \boxed{\frac{1}{2}}\log_3 25 - \boxed{5}\log_3 2$

Coefficients become exponents!

$\cancel{\log_3} x = \cancel{\log_3} \frac{25^{\frac{1}{2}}}{2^5}$

$x = \frac{5}{32}$

EX 5: $\ln(x+5) + \ln(x-2) = \ln 18$

$\cancel{\ln} (x+5)(x-2) = \cancel{\ln} 18$

QUADRATIC

$\underbrace{x^2 + 5x - 2x - 10}$ $= 18$

$x^2 + 3x - 18 = 0$

$(x+6)(x-3) = 0$

$x = -6$

$x = 3$

You try: $\log_5 24 - \log_5 x + \log_5 3 = 7\log_5 2$

$\log_5 \frac{24 \cdot 3}{x} = \log_5 2^7$

$\log_5 \frac{72}{x} = \log_5 128$

$\frac{72}{x} = \frac{128}{1}$

$72 = 128x$

$x = \frac{72}{128} = \frac{9}{16}$

You try: $\log 2x = \log(2x-8)$

$2x = 2x - 8$

$0 = -8$

NO SOLUTION

1. $\log_2 3 - \log_2 7 = \log_2 x$

$\log_2 \frac{3}{7} = \log_2 x$

$\frac{3}{7} = x$

2. $\log_3 14 + \log_3 y = \log_3 42$

$\log_3 14y = \log_3 42$

$14y = 42$

$y = 3$

3. $\log_9 x = \frac{1}{2}\log_9 144 - \frac{1}{3}\log_9 8$

$\log_9 x = \log_9 144^{\frac{1}{2}} - \log_9 8^{\frac{1}{3}}$

$\log_9 x = \log_9 12 - \log_9 2$

$\log_9 x = \log_9 \frac{12}{2}$

$x = 6$

4. $\log_3 56 - \log_3 8 = \log_3 x$

$\log_3 \frac{56}{8} = \log_3 x$

$7 = x$

5. $\log 7 + \log(n-2) = \log 6n$

$\log 7(n-2) = \cancel{6n}\ \log 6n$

$7(n-2) = 6n$

$7n - 14 = 6n$

$-14 = -n$

$n = 14$

6. $3\log x = \log 27$

$$\log x^3 = \log 27$$
$$x^3 = 27$$
$$x = 3$$

7. $\ln(m+3) - \ln m = \ln 4$

$$\ln \frac{(m+3)}{m} = \ln 4$$
$$\frac{m+3}{m} = \frac{4}{1}$$
$$m+3 = 4m$$
$$3 = 3m$$
$$m = 1$$

8. $3\log_5 x - \log_5 4 = \log_5 16$

$$\log_5 \frac{x^3}{4} = \log_5 16$$
$$\frac{x^3}{4} = \frac{16}{1}$$
$$x^3 = 64$$
$$x = 4$$

9. $\log_2 15 + \log_2 14 - \log_2 105 = \log_2 x$

$$\log_2 \frac{(15 \cdot 14)}{105} = \log_2 x$$
$$\frac{210}{105} = x$$
$$2 = x$$

10. $\log_4(x+3) + \log_4(x-3) = 2$ BOB

$\log_4 (x+3)(x-3) = 2$

$4^2 = (x+3)(x-3)$

$16 = x^2 - 9$

$25 = x^2$

$x = 5$, $\cancel{x = -5}$

11. $\log_8(n+1) - \log_8 n = \log_8 4$

$\log_8 \frac{(n+1)}{n} = \log_8 4$

$\frac{n+1}{n} = \frac{4}{1}$

$n + 1 = 4n$

$1 = 3n$

$n = \frac{1}{3}$

12. $\log_2(y+2) - 1 = \log_2(y-2)$

$\log_2(y+2) - \log_2(y-2) = 1$

$\log_2 \frac{(y+2)}{(y-2)} = 1$ BOB

$2 = \frac{y+2}{y-2}$

$2(y-2) = y+2$

$2y - 4 = y + 2$

$y - 4 = 2$

$y = 6$

ooh.. I forgot to mention. If there is a constant, isolate it!

13. $\log_3(4x+5) - \log_3(3-2x) = 2$

$\log_3 \frac{(4x+5)}{(3-2x)} = 2$

$3^2 = \frac{4x+5}{3-2x}$

$\frac{9}{1} = \frac{4x+5}{3-2x}$

$9(3-2x) = 4x+5$

$27 - 18x = 4x+5$

$27 = 22x+5$

$22 = 22x$

$x = 1$

14. $\ln(x+2) + \ln 5 = 4$

$\ln(x+2)(5) = 4$

BOB

$e^4 = 5(x+2)$

$e^4 = 5x+10$

$54.598.. = 5x+10$

$44.598.. = 5x$

$x \approx 8.92$

GSE Algebra 2Y Name ________________________________

Quiz: Properties of Logarithms, Solving log equations, Graphing logs as inverses of exponential functions

Rewrite in exponential form.

1. $\log_3 243 = 5$ ______________ 2. $\log \frac{1}{1000} = -3$ ______________

Rewrite in logarithmic form.

3. $2^9 = 512$ ______________ 4. $64^{\frac{1}{2}} = 8$ ______________

Solve the equation, showing all steps. Round to two decimal places, if necessary.

5. $\log_2(x+4) = 5$ ______________ 6. $7 + \log(2x-8) = 10$ ______________

7. $2\ln(3x) = 4.6$ ______________ 8. $\log_5(2x-1) = 1$ ______________