Complete the table by evaluating the logarithm without using a calculator. NO DECIMAL ANSWERS.

$\log_{10} .01 = -2$ $10^{-2} = .01$ $\ln e^{6x} = 6x$ $e^{6x} = e^{6x}$

1. Rewrite the logarithmic equation in exponential form.

$\log_3 81 = 4$ $3^4 = 81$	$\log_{\frac{1}{2}} 8 = -3$ $\left(\frac{1}{2}\right)^{-3} = 8$	$\log_{64} 16 = \frac{2}{3}$ $64^{\frac{2}{3}} = 16$

2. Rewrite the exponential equation in logarithmic form.

$2^7 = 128$ $\log_2 128 = 7$ base → subscript	$(\frac{3}{5})^{-2} = \frac{25}{9}$ $\log_{\frac{3}{5}} \frac{25}{9} = -2$	$16^{\frac{1}{4}} = 2$ $\log_{16} 2 = \frac{1}{4}$

3. Complete the table by evaluating the logarithm ~~without using a calculator.~~ NO DECIMAL ANSWERS.

$\frac{\log(81^{-2})}{\log 3}$

$log_4 256$ 4	$log_2 \frac{1}{16}$ → $\frac{\log\left(\frac{1}{16}\right)}{\log 2}$ → -4	$log_{49} 7$ $\frac{1}{2}$
$log_9 9^7$ 7	$log_3 \boxed{81}^{-2}$ $\log_3 \boxed{3^4}^{-2}$ → $\log_3 3^{-8}$ → -8	$log_{\frac{2}{5}} \frac{125}{8}$ -3
$5^{\log_5 39}$ inverse 39	$\ln e^{6x}$ $6x$	$\log_{10} 10^{\sqrt{x+2}}$ $\sqrt{x+2}$

6. Expand the logarithmic expression. Your final answer should include as many terms as there are factors. Factors from the numerator will be positive; factors from the numerator will be negative; exponents must be rewritten as coefficients.

$log_4 \frac{5x^3}{3y}$ Factors: 5, x, 3, y 4 is not a factor! $\log_4 5 + 3\log_4 x - \log_4 3 - \log_4 y$	$ln 7\sqrt[6]{z}$ index $\ln 7z^{\frac{1}{6}}$ factors: 7, z $\ln 7 + \frac{1}{6}\ln z$
$\log \frac{5}{ab^2}$ $\log 5 - \log a - 2\log b$	$\log_3 \sqrt[4]{mn}$ $\frac{1}{4}\log_3 m + \frac{1}{4}\log_3 n$
$\log_4 3x^5\sqrt{yz}$ $\log_4 3 + 5\log_4 x + \frac{1}{2}\log_4 y + \frac{1}{2}\log_4 z$	$\log \frac{1}{ab}$ $\log 1 - \log a - \log b$ $0 - \log a - \log b$ $-\log a - \log b$

7. Condense the logarithmic expression. There should be only one log term in your final answer. Change coefficients into exponents first. Terms that are positive multiply together in the numerator. Terms that are negative multiply together in the denominator.

$3\log_6 x - \log_6 y - 5\log_6 z$ $\log_6 \frac{x^3}{yz^5}$	$8\log_3 x - 4\log_3 y$ $\log_3 \frac{x^8}{y^4}$
$\frac{1}{2}\ln x + \frac{1}{2}\ln y + \frac{1}{2}\ln z - \ln 6$ $\frac{1}{2}$ power $= \sqrt{\ }$ fraction? $\ln \frac{\sqrt[2]{xyz}}{6}$	$\frac{1}{3}\log_2 8 + 3\log_2 4$ $\sqrt[3]{8}$ $\log_2 8^{\frac{1}{3}} \cdot 4^3$ $\log_2 2 \cdot 64$ $\log_2 128$ 7

8. Solve the following exponential equations. If the bases are like or can be made like, apply the property of equality to the exponents in order to solve. If the bases are not like and cannot be made like, take a logarithm with the same base as the exponent on each side. Round to three decimal places, where necessary.

$4^{x-1} = 4^{3x}$ log ? $x - 1 = 3x$ $-1 = 2x$ $x = -\frac{1}{2}$	$\left(\frac{1}{4}\right)^{x-6} = 32^x$ $(2^{-2})^{x-6} = (2^5)^x$ $2^{-2x+12} = 2^{5x}$ $-2x + 12 = 5x$ $12 = 7x$ $x = \frac{12}{7}$
$25^{x+2} = 625^{3x}$ $(5^2)^{x+2} = (5^4)^{3x}$ $5^{2x+4} = 5^{12x}$ $2x + 4 = 12x$ $4 = 10x$ $x = 4/10 = 2/5$	$10^{2x} + 3 = 94$ $10^{2x} = 91$ $\log 10^{2x} = \log 91$ $2x = 1.959...$ $x \approx .980$
$e^{3x} = 347$ take ln $\ln e^{3x} = \ln 347$ $3x = 5.84...$ $x \approx 1.950$	$5^x = 2061$ $\log_5 5^x = \log_5 2061$ $x = \frac{\log 2061}{\log 5}$ $x \approx 4.741$
$\frac{5}{4}(.79)^{-x} = 14$ $(.79)^{-x} = 14\left(\frac{4}{5}\right)$ $(.79)^{-x} = 11.2$ $\log_{.79} .79^{-x} = \log_{.79} 11.2$ $-x = \frac{\log 11.2}{\log .79}$ $-x \approx -10.249$ $x \approx 10.249$	$216^{3x} = 36^{4x-2}$ $(6^3)^{3x} = (6^2)^{4x-2}$ $6^{9x} = 6^{8x-4}$ $9x = 8x - 4$ $x = -4$
$7^{6x} = 11^{2x-1}$ omit $\log_7 7^{6x} = \log_7 11^{2x-1}$ $6x = (2x-1)\log_7 11$	$11 - 3^{-2x} = 4$

$6x = (2x-1)\frac{\log 11}{\log 7}$

$6x = (1.232)(2x-1)$

$6x = 1.464x - 1.232$

$4.536x = -1.232$

$x \approx -.272$

$11 - 3^{-2x} = 4$

$-3^{-2x} = -7$

$3^{-2x} = 7$

$\log_3 3^{-2x} = \log_3 7$

$-2x = \frac{\log 7}{\log 3}$

$-2x = 1.77...$

$x \approx -.886$

BOB ! NOT BOB?

14. Solve the following logarithmic equations. Check for extraneous solutions.

$log_2(x+6)=7$ BOB $2^7=x+6$ $128=x+6$ $x=122$	$log_3(4x-3)=log_3(2x+11)$ NO $4x-3=2x+11$ $2x-3=11$ $2x=14$ $x=7$
$2lnx+13=11$ BOB $2\ln x=-2$ $\ln x=-1$ $e^{-1}=x$ $x\approx 0.368$	$ln5x=6$ BOB $e^6=5x$ $403.428..=5x$ $x\approx 80.686$
$\log_4(x+2)+\log_4(x-4)=2$ BOB $\log_4(x+2)(x-4)=2$ $4^2=(x+2)(x-4)$ $16=x^2-2x-8$ $0=x^2-2x-24$ $0=(x-6)(x+4)$ $x=6, \cancel{x=-4}$	$2\log_2 x-\log_2(x+3)=2$ BOB $\log_2 \frac{x^2}{x+3}=2$ $2^2=\frac{x^2}{x+3}$ $4(x+3)=x^2$ $4x+12=x^2$ $x^2-4x-12=0$ $(x-6)(x+2)=0$ $x=6, \cancel{x=-2}$
$\log(x^2+x)=\log 12$ NO $x^2+x=12$ $x^2+x-12=0$ $(x+4)(x-3)=0$ $x=-4, x=3$	$\log_{\sqrt{7}} x=4$ BOB $\sqrt{7}^4=x$ $x=49$

ENRICHMENT ONLY NOT ON TEST

Applications of Logarithms (Problems #15 and #16 are multi-part problems, not multiple choice)

15. The wind speed, s (in miles per hour) near the center of a tornado can be modeled by *s = 93 log d + 65*, where d is the distance (in miles) that the tornado travels.

Find the distance (to the nearest tenth of a mile that the tornado has traveled given the following wind speeds.

a. 90 mph

$90=93\log d+65$
$25=93\log d$
$\frac{25}{93}=\log_{10} d$
$10^{\frac{25}{93}}=d$
$d\approx 1.9$ miles

b. 150 mph

$150=93\log d+65$
$85=93\log d$
$\frac{85}{93}=\log_{10} d$
$10^{\frac{85}{93}}=d$
$d\approx 8.2$ miles

c. 220 mph

$220=93\log d+65$
$155=93\log d$
$\frac{155}{93}=\log_{10} d$
$d=10^{\frac{155}{93}}$
$d\approx 46.4$ miles

16. The management at a factory has determined that a worker can produce a maximum of 45 units per day. The model $y = 45 - 42e^{-0.05t}$ indicates the number of units, y, that a new employee can produce per day after t days on the job. To the nearest tenth, estimate the number of days it will take before a worker can produce

5 units

b. 25 units

c. 42 units

$5=45-42e^{-0.05t}$
$-40=-42e^{-.05t}$
$\frac{40}{42}=e^{-.05t}$
$\ln\left(\frac{40}{42}\right)=-.05t$
$-.048...=-.05t$
$t\approx 0.98$
$t<1$ day on the job

17. If you deposit $10,000 into an account that pays 5.4% compounded quarterly, how long will it take for the balance to reach $25,000? Round to the nearest tenth.

$25{,}000 = 10{,}000\left(1 + \frac{.054}{4}\right)^{4t}$

$2.5 = \left(1 + \frac{.054}{4}\right)^{4t}$

$2.5 = (1.0135)^{4t}$

$\log_{1.0135} 2.5 = \log_{1.0135} 1.0135^{4t}$

$\frac{\log 2.5}{\log 1.0135} = 4t$

$68.33\ldots = 4t$

$t \approx 17.1$ years

18. Twenty years ago, there were approximately 4900 of a rare species of bird left in the world. If there are approximately 3000 of the species left today, at what rate is the population decreasing? (use $A = Pe^{rt}$). Round to three significant digits.

$3000 = 4900e^{r \cdot 20}$

$\frac{3000}{4900} = e^{20r}$

$\ln\left(\frac{3000}{4900}\right) = \ln e^{20r}$

$-0.490\ldots = 20r$

$r \approx -2.5\%$

For natural base models, negative rates show decay

20. Forty years ago, Zippy bought a mood ring for $9.95. The mood ring, now considered a collector's item, is today valued at $280. At what rate did the ring appreciate?

$280 = 9.95(1+r)^{40}$

$\frac{280}{9.95} = (1+r)^{40}$

$\left(\frac{280}{9.95}\right)^{\frac{1}{40}} = 1+r$

$1.087\ldots = 1+r$

$.087\ldots = r$

$r \approx 8.7\%$

21. The half-life of Comodium -16 is 12,500 years. How long will it take for 15 grams of Comodium-16 to decompose to 14 grams? Round to one decimal place.

Part I : Find the decay rate (half-life)

Part II : Use the rate to answer the actual question

$\frac{1}{2} = 1e^{r \cdot 12,500}$

$\ln\left(\frac{1}{2}\right) = \ln e^{12,500r}$

$-0.693\ldots = 12{,}500r$

$r = -.000055452$

$14 = 15e^{-.000055452 \cdot t}$

$\left(\frac{14}{15}\right) = e^{-.000055452t}$

$\ln\left(\frac{14}{15}\right) = -.000055452t$

$-.0689\ldots = -.000055452t$

$t \approx 1244.2$ years

22. In a controlled lab setting, 100 bacteria increase to 1600 count in 12 hours. How long will it take for 2 bacterium to increase to 1000? Round to one decimal place.

Part I : Find the growth rate of the bacteria

$1600 = 100e^{r \cdot 12}$

$16 = e^{12r}$

$\ln 16 = \ln e^{12r}$

$2.77\ldots = 12r$

$r \approx 0.231$

Part II : use the growth rate to answer the question

$1000 = 2e^{.231t}$

$500 = e^{.231t}$

$\ln 500 = \ln e^{.231t}$

$6.214\ldots = .231t$

$t \approx 26.9$ hours ~~years~~

- Make the bases like

Name:

Date:

Algebra 2

Solving Exponential Equations

Find all values of x that solve the following equations. For many problems, you may want to write both sides with the same base.

1. $2^x = 8$ $x = 3$

Keep base

$\boxed{\cancel{2}^x = \cancel{2}^3}$

2. $3 \cdot 2^x = 48$

divide by 3 first!

$2^x = 16$

$2^x = 2^4$

$x = 4$

3. $3^x = \frac{1}{9}$

use a negative exponent!

$3^x = 3^{-2}$

$x = -2$

4. $4^x + 7 = 71$

$4^x = 64$

$4^x = 4^3$

$x = 3$

5. $2^x \cdot 2^{x-2} = \sqrt{2}$

multiply powers of same base = ADDITION

$\cancel{2}^{x+x-2} = \cancel{2}^{\frac{1}{2}}$

$2x - 2 = \frac{1}{2}$

$\frac{2x}{1} = \frac{5}{2}$

$4x = 5$

$x = \frac{5}{4}$

6. $2^x = \frac{1}{32}$

$2^x = 2^{-5}$

$x = -5$

7. $2^{x+2} = 8$

$2^{x+2} = 2^3$

$x + 2 = 3$

$x = 1$

8. $\sqrt[4]{3} = 9^x$

$3^{\frac{1}{4}} = (3^2)^x$ power to a power = MULTIPLY

$\frac{1}{4} = 2x$

$x = \frac{1}{8}$

9. $2^x = \frac{4}{\sqrt{2}}$

$2^x = \frac{4}{2^{1/2}}$

$2^x = \frac{2^2}{2^{1/2}}$

$2^x = 2^{\frac{3}{2}}$

$x = \frac{3}{2}$

10. $3 \cdot 2^x = 24$

$2^x = 8$

$2^x = 2^3$

$x = 3$

11. $2 \cdot 3^{x+3} + 1 = 19$

$2 \cdot 3^{x+3} = 18$

$3^{x+3} = 9$

$\log_3 3^{x+3} = \log_3 9$

$x + 3 = 2$

$x = -1$

12. $2^x = \frac{1}{\sqrt{8}}$

$2^x = \frac{1}{\sqrt{8}}$

$2^x = \frac{1}{\sqrt{2^3}}$

$\frac{2^x}{1} = \frac{1}{2^{3/2}}$

$2^x = 2^{-\frac{3}{2}}$

$x = -\frac{3}{2}$

13. $2 \cdot 3^{2x-1} + 7 = 61$

$2 \cdot 3^{2x-1} = 54$

$3^{2x-1} = 27$

$3^{2x-1} = 3^3$

$2x-1 = 3$

$2x = 4$

$x = 2$

14. $3^x = 9^7$

$3^x = (3^2)^7$

$3^x = 3^{14}$

$x = 14$

15. $2^x = 2\sqrt{2}$

$2^x = 2 \cdot 2^{\frac{1}{2}}$

$2^x = 2^1 \cdot 2^{\frac{1}{2}}$

$2^x = 2^{1\frac{1}{2}}$

$x = \frac{3}{2}$

16. $5^x = \frac{1}{\sqrt[3]{5}}$

$5^x = \frac{1}{5^{\frac{1}{3}}}$

$5^x = 5^{-\frac{1}{3}}$

$x = -\frac{1}{3}$

17. $2 \cdot 2^x = 8$

$2^1 \cdot 2^x = 2^3$

$2^{1+x} = 2^3$

$1 + x = 3$

$x = 2$

18. $\left(\frac{1}{5}\right)^x = 25$

$(5^{-1})^x = 5^2$

$5^{-x} = 5^2$

$-x = 2$

$x = -2$

19. $2^{2x+1} \cdot 2^x = 16$

$2^{2x+1+x} = 2^4$

$2x + 1 + x = 4$

$3x + 1 = 4$

$3x = 3$

$x = 1$

20. $6^{-x} = \frac{6}{\sqrt[5]{6}}$

$6^{-x} = \frac{6^1}{6^{1/5}}$

$6^{-x} = \frac{6^{5/5}}{6^{1/5}}$

$6^{-x} = 6^{4/5}$

$-x = 4/5$

$x = -\frac{4}{5}$

21. $(\sqrt{2})^x = 8$

$(2^{\frac{1}{2}})^x = 8$

$2^{\frac{1}{2}x} = 2^3$

$\frac{1}{2}x = 3$

$x = 6$

22. $3^x = \left(\frac{1}{9}\right)^{4-x}$

$3^x = (3^{-2})^{4-x}$

$x = -8 + 2x$

$-x = -8$

$x = 8$

23. $(2^{x+1})^2 = \frac{1}{4}$

$(2^{x+1})^2 = 2^{-2}$

$2^{2x+2} = 2^{-2}$

$2x + 2 = -2$

$2x = -4$

$x = -2$

24. $4^{2x} = 8^{x-3}$

$(2^2)^{2x} = (2^3)^{x-3}$

$2^{4x} = 2^{3x-9}$

$4x = 3x - 9$

$x = -9$

Solving Exponential Functions

Recall solving exponential equations where bases were like or could be made like:

Ex 1: $3^x = 3^{2x+4}$

$x = 2x + 4$

$0 = x + 4$

$-4 = x$

Ex 2: $25^{x+3} = 125^{4x}$

power to a power = MULTIPLY

$(5^2)^{x+3} = (5^3)^{4x}$

$5^{2x+6} = 5^{12x}$

$2x + 6 = 12x$

$6 = 10x$

$x = \frac{6}{10} = \frac{3}{5}$

What happens when the bases cannot be made the same?

Change-of-Base Formula:

$\log_a x = \frac{\log x}{\log a}$ $\quad \frac{\log 7}{\log 3}$

$3^{1.771} \approx 7$

Ex.

$\log_3 7 = 1.771$ $\quad 3^? = 7$

Steps to Solve: $3^1 \quad 3^2$

1. Identify the base of the exponent.
2. Take a logarithm on both sides of the equation. The base of the log must be the same as the base of the exponent.
3. CHECK YOUR ANSWER!

Remember, if the base is 10 or e, you do not need to use the change of base formula☺

1. $8^x = 23$ — the base is 8

$(\log_8 8)^x = \log_8 23$

INVERSE PROPERTY

$x = \frac{\log 23}{\log 8}$ use change of base formula

≈ 1.508

2. $11^x = 72$ the base is 11

$(\log_{11} 11)^x = \log_{11} 72$

$x = \frac{\log 72}{\log 11} \approx 1.784$

3. $3^{5x} = 42$

$\log_3 3^{5x} = \log_3 42$

$5x = \frac{\log 42}{\log 3}$

$5x \approx 3.402\ldots$ $x \approx .680$

4. $4^{x+3} = 11$

$\log_4 4^{x+3} = \log_4 11$

$x + 3 = \frac{\log 11}{\log 4}$

~~$x+3 = 1.729\ldots$~~

$x \approx -1.270$

4. $3e^x + 2 = 5$

↑ use ln on calculator

– isolate base e

$3e^x = 3$

$e^x = 1$ take ln on both sides

$\ln_e e^x = \ln 1$

$x \approx 0$

5. $10^{x-9} + 3 = 21$

$10^{x-9} = 18$ take log on both sides

$\log_{10} 10^{x-9} = \log_{10} 18$

$x - 9 = 1.255...$

$x \approx 10.255$

6. $-7(5^{x+2}) - 1 = -15$

$-7(5^{x+2}) = -14$

$5^{x+2} = 2$

$\log_5 5^{x+2} = \log_5 2$

$x + 2 = \frac{\log 2}{\log 5}$

$x + 2 = .430...$

$x \approx -1.569$

7. $4e^{6x+7} - 9 = 23$

$4e^{6x+7} = 32$

$e^{6x+7} = 8$

$\ln e^{6x+7} = \ln 8$

$6x + 7 = 2.079...$

$6x = -4.92...$

$x \approx -.820$

8. $25^{x+3} = 5^{2x+7}$

$(5^2)^{x+3} = 5^{2x+7}$

$2x + 6 = 2x + 7$

$6 = 7$

no solution

9. $3 \cdot 10^{2x} - 7 = 11$

$3 \cdot 10^{2x} = 18$

$10^{2x} = 6$

$\log 10^{2x} = \log 6$

$2x = .778...$

$x \approx .389$

CCGPS Adv. Alg. **Logs - Homework**

Name ______________________________ **Date** ____________________

1. Condense:
$\ln 3 + \ln x - \ln y - 2\ln z$

$\ln \frac{3x}{yz^2}$

2. Condense:
$\ln 5 + \frac{1}{2}\ln x + 2\ln z$

$\ln 5z^2\sqrt{x}$

3. Condense:
$\log_5 9 + \frac{1}{2}\log_5 k + 2\log_5 p$

$\log_5 9p^2\sqrt{k}$

4. Condense:
$\ln 4 + 3\ln x - 5\ln y$

$\ln \frac{4x^3}{y^5}$

5. Condense:
$\log_3 a - 2\log_3 b - 3\log_3 c$

$\log_3 \frac{a}{b^2c^3}$

6. Condense:
$\log x + 2\log y + 3\log z - \log 5$

$\log \frac{xy^2z^3}{5}$

7. Solve:
$4 - \log(3x + 7) = 1$

$-\log(3x+7) = -3$

$\log_{10}(3x+7) = 3$

$10^3 = 3x + 7$

$1000 = 3x + 7$

$993 = 3x$

$x = 331$

8. Condense:
$\log 5 + \log x - \log y$

$\log \frac{5x}{y}$

9. Condense:
$3\log_3 x - \log_3 7 - 4\log_3 y$

$\log_3 \frac{x^3}{7y^4}$

10. Condense:
$3\log_5 3 + 2\log_5 a - \log_5 b - 2\log_5 c$

$\log_5 \frac{3^3 \cdot a^2}{bc^2}$

or $\log_5 \frac{27a^2}{bc^2}$

Name ______________________ Date ______________

11. Solve

$\ln 3x + 2 = 3.7$

$\ln 3x = 1.7$

$e^{1.7} = 3x$

$5.47\ldots = 3x$

$x \approx 1.825$

12. Solve

$\log_4(5x-4) = 2$

$4^2 = 5x-4$

$16 = 5x - 4$

$20 = 5x$

$x = 4$

13. Solve

$25\log_2 8x + 11 = 261$

$25\log_2 8x = 250$

$\log_2 8x = 10$

$2^{10} = 8x$

$1024 = 8x$

$x = 128$

14. Solve

$-15\log_2(3x+2) = -45$

$\log_2(3x+2) = 3$

$2^3 = 3x+2$

$8 = 3x+2$

$6 = 3x$

$x = 2$

15. Expand

$\ln 2a^2b^3$

$\ln 2 + 2\ln a + 3\ln b$

16. Expand

$\log_2 \frac{3\sqrt{b}}{a^4c^2}$

$\log_2 3 + \frac{1}{2}\log_2 b - 4\log_2 a - 2\log_2 c$

17. Expand

$\log \frac{3x^4}{y\sqrt[3]{z}}$

$\log 3 + 4\log x - \log y - \frac{1}{3}\log z$

18. Expand

$\log_2 w^3k^5\sqrt[4]{n}$

$3\log_2 w + 5\log_2 4 + \frac{1}{4}\log_2 n$