

1. Complete the table by evaluating the logarithm without using a calculator. NO DECIMAL ANSWERS.

$\log_4 256$ $4^? = 256$ ANSWER: 4	$\log_2 \frac{1}{16}$ $2^? = \frac{1}{16}$ ANS: -4	$\log_{49} 7$ $49^? = 7$ ANS: $\frac{1}{2}$
$\log_9 9$ $9^? = 9$ ANS: 1	$\log_3 81$ $3^? = 81$ ANS: 4	$\log_{\frac{2}{5}} \frac{125}{8}$ $(\frac{2}{5})^? = \frac{125}{8}$ ANS: -3
$\log_6 1$ $6^? = 1$ ANS: 0	$\log_4 64$ $4^? = 64$ ANS: 3	$\log_{\frac{1}{2}} 32$ $(\frac{1}{2})^? = 32$ ANS: -5
$\log_6 216$ $6^? = 216$ ANS: 3	$\log_7 2401$ $7^? = 2401$ ANS: 4	$\log_{64} 4$ $64^? = 4$ ANS: $\frac{1}{3}$
$\ln e^3$ $e^? = e^3$ ANS: 3	$\log_{14} 14$ $14^? = 14$ ANS: 1	$\log_9 0$ $9^? = 0$ UNDEFINED
$\log_2 1$ $2^? = 1$ ANS: 0	$\log_{27} 3$ $27^? = 3$ ANS: $\frac{1}{3}$	$\log_{25} (-4)$ $25^? = -4$ UNDEFINED

Rewrite the equation in exponential form.

2. $\log_2 32 = 5$ $2^5 = 32$

3. $\log_{10} 100 = 2$ $10^2 = 100$

4. $\log_{25} 16 = -4$ $25^{-4} = 16$

5. $\log_{625} 5 = \frac{1}{4}$ $625^{\frac{1}{4}} = 5$

6. $\log_6 36 = 2$ $6^2 = 36$

6. $\log_{49} 7 = \frac{1}{2}$ $49^{\frac{1}{2}} = 7$

Rewrite the equation in logarithmic form.

7. $5^3 = 125$ $\log_5 125 = 3$

8. $10^{-2} = 0.01$ $\log 0.01 = -2$

9. $64^{\frac{1}{3}} = 4$ $\log_{64} 4 = \frac{1}{3}$

10. $81^{\frac{1}{4}} = 3$ $\log_{81} 3 = \frac{1}{4}$

11. $(\frac{25}{49})^{\frac{1}{2}} = \frac{7}{5}$ $\log_{\frac{25}{49}} \frac{7}{5} = -\frac{1}{2}$

12. $36^{-\frac{3}{2}} = \frac{1}{216}$ $\log_{36} \frac{1}{216} = -\frac{3}{2}$

Rewrite the equation in exponential form. Then solve.

13. $\log_{343} x = \frac{1}{3}$ $x = 7$

$343^{\frac{1}{3}} = x$

14. $\log_x 8 = \frac{1}{2}$ $x = 64$

$x^{\frac{1}{2}} = 8$ Mental math

15. $\log_2 128 = x$ $x = 7$

$2^x = 128$

16. $\log_4(2x+6) = 3$ $x = 29$

$4^3 = 2x+6$

$58 = 2x+6$

$64 = 2x+6$

$x = 29$

17. $\log_7(x-1) = 5$ $16,808$

$7^5 = x-1$

$16,807 = x-1$

18. $\log_{\frac{1}{3}}(2x) = -4$ $x = 40.5$

$\left(\frac{1}{3}\right)^{-4} = 2x$

$81 = 2x$

19. $\log_2(x-3) = 6$ $67 = x$

$2^6 = x-3$

$64 = x-3$

20. $\log(2x+4) = 2$ $x = 48$

$10^2 = (2x+4)$

$96 = 2x$

$100 = 2x+4$

21. $\log_8(7-3x) = 3$ $x = -\frac{505}{3}$

$8^3 = 7-3x$

$512 = 7-3x$

$505 = -3x$

22. $\ln(x-6) = -2$ $x \approx 6.135$

$e^{-2} = x-6$

23. $\log_4(6x-13) = 5$ $x = \frac{1037}{6}$

$4^5 = 6x-13$

$1024 = 6x-13$

$1037 = 6x$

25. $\log_{\frac{1}{4}}(2x) = -4$ $x = 128$

$\left(\frac{1}{4}\right)^{-4} = 2x$

$256 = 2x$

27. $7 = \ln(1-3x)$ $x \approx -365.211$

$e^7 = 1-3x$

$1096.63... = 1-3x$

29. $\log_{\frac{1}{3}} x = -4$ $x = 81$

$\left(\frac{1}{3}\right)^{-4} = x$

31. $\ln(4-x) = 5$ $x \approx -144.413$

$e^5 = 4-x$

$148.41... = 4-x$

$144.41... = -x$

$x \approx -144.413$

24. $9 = \log_{32}(3x+7)$ $x \approx 2.333$

$32^9 = (3x+7)$

26. $\log(x^2-5) = 2$ $x = \pm\sqrt{105}$

$10^2 = x^2-5$

$x^2 = 105$

$100 = x^2-5$

28. $\log_2(x^2+4x) = 5$ $x = -8, x = 4$

$2^5 = x^2+4x$

$0 = x^2+4x-32$

$32 = x^2+4x$

$0 = (x+8)(x-4)$

30. $\log x^{\frac{3}{2}} = 3$ $x = 100$

$10^3 = \left(x^{\frac{3}{2}}\right)^{\frac{2}{3}}$

32. $6 = \log_3(3x+1)$ $x = \frac{728}{3}$

$3^6 = 3x+1$

$729 = 3x+1$

$728 = 3x$

Algebra 2
Section 7-4 Day 1
Properties of Logarithms

Properties of Logarithms

- PRODUCT PROPERTY : $\log_b mn = \log_b m + \log_b n$
- QUOTIENT PROPERTY : $\log_b \frac{m}{n} = \log_b m - \log_b n$
- POWER PROPERTY : $\log_b m^n = n \log_b m$

Condensing Logarithms

Examples: Condense each expression into a single logarithm. Simplify when possible.

1. $\log 7 + \log 2$

$\log(7 \cdot 2)$
 $\log 14$

2. $\log 3 - \log 8$

$\log \frac{3}{8}$

3. $5 \log 3$

$\log 3^5$
 $\log 243$

4. $\log_2 9 - \log_2 3$

$\log_2 \frac{9}{3}$
 $\log_2 3$

5. $5 \log 3 + \log 4$

$\log 3^5 + \log 4$
 $\log(3^5 \cdot 4)$
 $\log(243 \cdot 4)$
 $\log 972$

6. $\log 8 - 2 \log 6 + \log 3$

$\log 8 - \log 6^2 + \log 3$
 $\log \left(\frac{8 \cdot 3}{6^2} \right)$

7. $4 \log m - \log n$

$\log m^4 - \log n$
 $\log \frac{m^4}{n}$

8. $\log_6 5 + \log_6 x$

$\log_6 5x$

$\log \frac{24}{36}$
 $\log \frac{2}{3}$

9. $\log_4 32 - \log_4 2$

$\log_4 \frac{32}{2}$
 $\log_4 16$
 2

EVALUATE using mental math

10. $6 \log_2 x + 5 \log_2 y$

$\log_2 x^6 + \log_2 y^5$
 $\log_2 x^6 y^5$

11. $\log_4 5x + \log_4 3x$

$\log_4 (5x \cdot 3x)$
 $\log_4 (15x^2)$

12. $2 \log_4 6 - \log_4 9$

$\log_4 6^2 - \log_4 9$
 $\log_4 \frac{6^2}{9}$

$\log_4 \frac{36}{9} \rightarrow \log_4 4 \rightarrow 1$

MENTAL MATH ☺

Expanding Logarithms

- Take a _____ and write it as a _____ or a _____ of two or more logarithms.
- Use your Properties of Logarithms to help you expand logarithms

13. $\log \frac{4x}{y}$ $\log 4x - \log y$
 $\log 4 + \log x - \log y$

14. $\log_9 \frac{x^4}{729}$ $\log x^4 - \log 729$
 $4 \log x - \log 729$

15. $\log_3 \frac{250}{37}$
 $\log_3 250 - \log_3 37$

16. $\log_3 9x^5$ $\log_3 9 + \log_3 x^5$
 $2 + 5 \log_3 x$

17. $\log x^3 y^5$
 $\log x^3 + \log y^5$
 $3 \log x + 5 \log y$

18. $\log_7 49xyz$
 $\log_7 49 + \log_7 x + \log_7 y + \log_7 z$
 $2 + \log_7 x + \log_7 y + \log_7 z$

19. $\log_3 (2x)^2$
 $2 \log_3 (2x)$
 $2 (\log_3 2 + \log_3 x)$

20. $\log \sqrt[4]{\frac{xy}{z}}$ Same as $\left(\frac{xy}{z}\right)^{\frac{1}{4}}$
 $\frac{1}{4} \log \left(\frac{xy}{z}\right)$
 $\frac{1}{4} [\log(xy) - \log z]$

Simplifying Logarithms (Try using mental math!)

21. $\log_4 32 - \log_4 2$ $\log_4 \frac{32}{2}$
 $\log_4 16$
 2

22. $2 \log_4 6 - \log_4 9$ $\log_4 6^2 - \log_4 9$
 $\log_4 \frac{36}{9} \rightarrow \log_4 4 \rightarrow 1$

23. $\log_2 4 - \log_2 16$
 $\log_2 \frac{4}{16} \rightarrow \log_2 \frac{1}{4} \rightarrow -2$

24. $\log_2 96 - \log_2 3$
 $\log_2 \frac{96}{3} \rightarrow \log_2 32 \rightarrow 5$

25. $\log_3 27 - 2 \log_3 3$
 $\log_3 27 - \log_3 3^2$
 $\log_3 \frac{27}{3^2} \rightarrow \log_3 \frac{27}{9} \rightarrow \log_3 3 \rightarrow 1$

26. $\log_6 12 + \log_6 3$ $\log_6 (12 \cdot 3)$
 $\log_6 36$
 2

Properties of Logarithms

Expand each logarithm.

1) $\log(6 \cdot 11)$

$$\log 6 + \log 11$$

2) $\log(5 \cdot 3)$

$$\log 5 + \log 3$$

3) $\log\left(\frac{6}{11}\right)^5$

$$5 \log\left(\frac{6}{11}\right)$$

$$5 [\log 6 - \log 11] \text{ OR}$$

$$5 \log 6 - 5 \log 11$$

5) $\log \frac{2^4}{5}$

$$\log 2^4 - \log 5$$

$$4 \log 2 - \log 5$$

4) $\log(3 \cdot 2^3)$

$$\log 3 + \log 2^3$$

$$\log 3 + 3 \log 2$$

6) $\log\left(\frac{6}{5}\right)^6$

$$6 \log\left(\frac{6}{5}\right)$$

$$6 [\log 6 - \log 5]$$

$$6 \log 6 - 6 \log 5$$

7) $\log \frac{x}{y^6}$

$$\log x - \log y^6$$

$$\log x - 6 \log y$$

Start:

$$\log x - \log y^6$$

8) $\log(a \cdot b)^2$

$$2 \log(a \cdot b)$$

$$2 [\log a + \log b]$$

$$2 \log a + 2 \log b$$

9) $\log \frac{u^4}{v}$

$$\log u^4 - \log v$$

$$4 \log u - \log v$$

10) $\log \frac{x}{y^5}$

$$\log x - \log y^5$$

$$\log x - 5 \log y$$

11) $\log \sqrt[3]{x \cdot y \cdot z}$

$$\log (xyz)^{\frac{1}{3}}$$

$$\log x^{\frac{1}{3}} y^{\frac{1}{3}} z^{\frac{1}{3}}$$

$$\frac{1}{3} \log x + \frac{1}{3} \log y + \frac{1}{3} \log z$$

12) $\log(x \cdot y \cdot z^2)$

$$\log x + \log y + 2 \log z$$

Condense each expression to a single logarithm.

13) $\log 3 - \log 8$

$\log \frac{3}{8}$

14) $\frac{\log 6}{3}$ same as $\frac{1}{3}(\log 6)$
so $\log \sqrt[3]{6}$

15) $4\log 3 - 4\log 8$

$\log \frac{3^4}{8^4}$ OR $\log \frac{81}{4096}$

16) $\log 2 + \log 11 + \log 7$

$\log(2 \cdot 11 \cdot 7)$ OR $\log(154)$

17) $\log 7 - 2\log 12$

$\log 7 - \log 12^2$
 $\log \frac{7}{144}$

18) $\frac{2\log 7}{3}$ $\frac{1}{3} \log 7^2$

$\log \sqrt[3]{49}$

19) $6\log_3 u + 6\log_3 v$

$\log_3 u^6 + \log_3 v^6$
 $\log_3 u^6 v^6$

20) $\ln x - 4\ln y$

$\ln \frac{x}{y^4}$

21) $\log_4 u - 6\log_4 v$

$\log_4 \frac{u}{v^6}$

22) $\log_3 u - 5\log_3 v$

$\log_3 \frac{u}{v^5}$

23) $20\log_6 u + 5\log_6 v$

$\log_6 u^{20} + \log_6 v^5$
 $\log_6 u^{20} v^5$

24) $4\log_3 u - 20\log_3 v$

$\log_3 \frac{u^4}{v^{20}}$

Critical thinking questions:

25) $2(\log 2x - \log y) - (\log 3 + 2\log 5)$

$\log \left(\frac{2x}{y} \right)^2 - \log(3 \cdot 5^2)$

$\log \left(\frac{4x^2}{75y^2} \right) \rightarrow \log \left(\frac{4x^2}{75y^2} \right)$

26) $\log x \cdot \log 2$

$\log y$