Matrix Multiplication

If $A = m \times n$ and $B = n \times p$, then $A \cdot B$ is defined. Its dimensions will be $m \times p$.

Use only the rows from the left matrix

$$\begin{array}{c} R1 \\ R2 \end{array}\begin{bmatrix} 2 & -3 & 5 \\ -1 & -4 & 12 \end{bmatrix} \square \begin{array}{c} \begin{array}{cc} C1 & C2 \end{array} \\ \begin{bmatrix} -5 & 7 \\ 3 & 0 \\ -8 & 10 \end{bmatrix} \end{array}$$

Use only the columns from the right matrix

Begin with row 1 of left matrix Multiply column 1 of right matrix

This answer goes into R1 C1 of product matrix

Next take row 1 of left matrix Multiply column 2 of right matrix

Place this answer into R1 C2 of product matrix

Next take Row 2 of left matrix. Multiply by column 1 of RIGHT MATRIX

Then take Row 2 of left matrix. Multiply by column 2 of RIGHT MATRIX

$$\begin{array}{c} R1 \\ R2 \end{array}\begin{bmatrix} 2 & -3 & 5 \\ -1 & -4 & 12 \end{bmatrix} \square \begin{array}{c} \begin{array}{cc} C1 & C2 \end{array} \\ \begin{bmatrix} -5 & 7 \\ 3 & 0 \\ -8 & 10 \end{bmatrix} \end{array} = \begin{array}{c} \begin{array}{cc} C1 & C2 \end{array} \\ \begin{bmatrix} \underline{-59} & \underline{64} \\ \underline{-103} & \underline{113} \end{bmatrix} \end{array} \begin{array}{c} R1 \\ R2 \end{array}$$

R2·C2

$[-1 \quad -4 \quad 12]$
$[7 \quad 0 \quad 10]$
$-7 + 0 + 120$
$= 113$

R2·C1 =

$[-1 \quad -4 \quad 12]$
$[-5 \quad 3 \quad -8]$
$5 - 12 - 96$
$= -103$

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R1·C1 = $[2 \quad -3 \quad 5]$
$[-5 \quad 3 \quad -8]$ ↓ MULTIPLY

ADD → $-10 \quad -9 \quad -40 = -59$

R1·C2

$[2 \quad -3 \quad 5]$
$[7 \quad 0 \quad 10]$ ↓

$14 + 0 + 50 = 64$

MATRIX MULTIPLICATION

Remember. . . We only use the rows of the left-hand matrix and the columns of the right-hand matrix!

Step 1: Verify that the product is defined.

Step 2: Determine the dimensions of the product matrix.

Step 3: Fill in the entries of the product matrix by multiplying each row of the left matrix with each column of the right matrix.

EX 1 Multiply $\begin{bmatrix} -2 & 6 & 3 \\ 4 & 1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 & -5 \\ 6 & 2 & 4 \\ 1 & -1 & 3 \end{bmatrix} = \begin{bmatrix} _____ & _____ & _____ \\ _____ & _____ & _____ \end{bmatrix}$

$= \begin{bmatrix} __ & __ & __ \\ __ & __ & __ \end{bmatrix}$

EX 2 Multiply $\begin{bmatrix} 7 & 2 & -3 & 4 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ 1 & -6 \\ 9 & 2 \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} _____ & _____ \end{bmatrix}$

$= \begin{bmatrix} __ & __ \end{bmatrix}$

EX 3 Multiply $\begin{bmatrix} 8 & -1 \\ 7 & 5 \end{bmatrix}\begin{bmatrix} 4 & 4 & 3 & 2 \\ -4 & 5 & 3 & 1 \end{bmatrix} = \begin{bmatrix} _____ & _____ & _____ & _____ \\ _____ & _____ & _____ & _____ \end{bmatrix}$

$= = \begin{bmatrix} __ & __ & __ & __ \\ __ & __ & __ & __ \end{bmatrix}$

Math 3 WS 1.9 Matrix Multiplication

Let $A = \begin{bmatrix} -1 & 3 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 5 & -4 \\ 2 & -3 \end{bmatrix}$, and $C = \begin{bmatrix} 10 & -6 \\ -1 & 7 \end{bmatrix}$

1. Find AB, then find BA. Are the products the same? NO

$AB = \begin{bmatrix} -1 & 3 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 5 & -4 \\ 2 & -3 \end{bmatrix} = \begin{bmatrix} 1 & -5 \\ 12 & -11 \end{bmatrix}$ $\quad BA = \begin{bmatrix} 5 & -4 \\ 2 & -3 \end{bmatrix}\begin{bmatrix} -1 & 3 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} -13 & 11 \\ -8 & 3 \end{bmatrix}$

2. Find BC, then find CB. Are the products the same? NO

$BC = \begin{bmatrix} 5 & -4 \\ 2 & -3 \end{bmatrix}\begin{bmatrix} 10 & -6 \\ -1 & 7 \end{bmatrix} = \begin{bmatrix} 54 & -72 \\ 23 & -33 \end{bmatrix}$ $\quad CB = \begin{bmatrix} 10 & -6 \\ -1 & 7 \end{bmatrix}\begin{bmatrix} 5 & -4 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 38 & -58 \\ 9 & 25 \end{bmatrix}$

3. Based on your findings, is matrix multiplication Commutative? Explain.

No; the product depends on the order

Determine whether the product of the matrices is defined. If so, find the product. If not, write "undefined".

4. $\underset{1\times 3}{\begin{bmatrix} 2 & 3 & 4 \end{bmatrix}}\underset{3\times 2}{\begin{bmatrix} -1 & 4 \\ 0 & 1 \\ 5 & 2 \end{bmatrix}} = (1 \times 2)$

$\begin{bmatrix} 18 & 19 \end{bmatrix}$

5. $\underset{3\times 1}{\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}}\underset{1\times 2}{\begin{bmatrix} 1 & -3 \end{bmatrix}} = (3 \times 2)$

$\begin{bmatrix} -1 & 3 \\ 0 & 0 \\ 1 & -3 \end{bmatrix}$

6. $\underset{3\times 2}{\begin{bmatrix} 0 & 1 \\ 4 & 3 \\ 5 & -1 \end{bmatrix}}\underset{2\times 3}{\begin{bmatrix} 2 & -1 & 0 \\ 3 & 4 & 1 \end{bmatrix}} = (3 \times 3)$

$\begin{bmatrix} 3 & 4 & 1 \\ 17 & 8 & 3 \\ 7 & -9 & -1 \end{bmatrix}$

7. $\underset{1\times 3}{\begin{bmatrix} 3 & 1 & 1 \end{bmatrix}}\underset{3\times 1}{\begin{bmatrix} 2 \\ -6 \\ 0 \end{bmatrix}} = (1 \times 1)$

$= \begin{bmatrix} 0 \end{bmatrix}$

Use matrix multiplication to solve for each variable.

8. $\begin{bmatrix} 1 & 1 & 3 \\ 2 & -1 & 4 \\ 3 & 0 & 2 \end{bmatrix}\begin{bmatrix} 2 \\ -1 \\ x \end{bmatrix} = \begin{bmatrix} -5 \\ y \\ 2 \end{bmatrix}$

$\begin{bmatrix} 2-1+3x \\ 4+1+4x \\ 6+0+2x \end{bmatrix} = \begin{bmatrix} -5 \\ y \\ 2 \end{bmatrix}$

$x = -2,\ y = -3$

Simplify LEFT MATRIX

$\begin{bmatrix} 1+3x \\ 4x+5 \\ 2x+6 \end{bmatrix} = \begin{bmatrix} -5 \\ y \\ 2 \end{bmatrix}$

$1+3x = -5$
$3x = -6$
$x = -2$

$2x+6 = 2$
$2x = -4$
$x = -2$
(confirms previous)

$4x+5 = y$
$4(-2)+5 = y$
$-8+5 = y$
$y = -3$

9. $\begin{bmatrix} 2 & 3 & 1 \\ -1 & 4 & 5 \end{bmatrix}\begin{bmatrix} 1 & x \\ 3 & -1 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 15 & 9 \\ 31 & y \end{bmatrix}$

$\begin{bmatrix} 2+9+4 & 2x-3+2 \\ -1+12+20 & -x-4+10 \end{bmatrix} = \begin{bmatrix} 15 & 9 \\ 31 & y \end{bmatrix}$

$x = 5,\ y = 1$

$\begin{bmatrix} 15 & 2x-1 \\ 31 & -x+6 \end{bmatrix} = \begin{bmatrix} 15 & 9 \\ 31 & y \end{bmatrix}$

$15 = 15$ ✓

$31 = 31$ ✓

$2x-1 = 9$
$2x = 10$
$x = 5$

$-x+6 = y$
$-5+6 = y$
$y = 1$

10. $\begin{bmatrix} -2 & 1 & 2 \\ 3 & 2 & 4 \\ 0 & -2 & 4 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ x & 2 \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} 6 & -6 \\ 19 & 9 \\ 8 & y \end{bmatrix}$

$\begin{bmatrix} -2+x+6 & -6+2-2 \\ 3+2x+12 & 9+4-4 \\ 0-2x+12 & 0-4-4 \end{bmatrix} = \begin{bmatrix} 6 & -6 \\ 19 & 9 \\ 8 & y \end{bmatrix}$

$x = 2,\ y = -8$

$\begin{bmatrix} x+4 & -6 \\ 2x+15 & 9 \\ -2x+12 & -8 \end{bmatrix} = \begin{bmatrix} 6 & -6 \\ 19 & 9 \\ 8 & y \end{bmatrix}$

$x+4 = 6$
$x = 2$

$2x+15 = 19$
$2x = 4$
$x = 2$ ✓

$-8 = y$

11. $\begin{bmatrix} 4 & 1 & 3 \\ -2 & x & 1 \end{bmatrix}\begin{bmatrix} 9 & -2 \\ 2 & 1 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} y & 5 \\ -13 & 11 \end{bmatrix}$

$y = 35,\ x = 3$

$\begin{bmatrix} 36+2-3 & -8+1+12 \\ -18+2x-1 & 4+x+4 \end{bmatrix} = \begin{bmatrix} y & 5 \\ -13 & 11 \end{bmatrix}$

$\begin{bmatrix} 35 & 5 \\ 2x-19 & x+8 \end{bmatrix} = \begin{bmatrix} y & 5 \\ -13 & 11 \end{bmatrix}$

$2x-19 = -13$
$2x = 6$
$x = 3$