Perfect Square Trinomials

Observe:

$(x+1)^2$	$\rightarrow x^2+2x+1$
$(x-1)^2$	x^2-2x+1
$(x+2)^2$	x^2+4x+4
$(x-2)^2$	x^2-4x+4
$(x+3)^2$	x^2+6x+9
$(x-3)^2$	x^2-6x+9
$(x+4)^2$	$x^2+8x+16$
$(x-4)^2$	$x^2-8x+16$
$(x+5)^2$	$x^2+10x+25$
$(x-5)^2$	$x^2-10x+25$
$(x+6)^2$	$x^2+12x+36$
$(x-6)^2$	$x^2-12x+36$

What patterns do you notice?

- all products are quadratic trinomials
- the constant term is a perfect square
- the middle coefficient is twice the value of the number in the binomial
- the sign of the middle term is the same as the sign of the binomial

These are all called "perfect square trinomials". They are derived from squaring a binomial.

Determine the value of "c" that makes a perfect square trinomial, then rewrite in binomial form.

Ex $x^2-10x+\_\_\_$

$\frac{-10}{2} \rightarrow -5 \rightarrow (-5)^2 \rightarrow 25$

In quadratic form (ax^2+bx+c), $b=-10$. To find "c", divide the b value by 2 and then square it.

$x^2-10x+25$

$(x-5)^2$

$\frac{b}{2} \rightarrow$

Ex $x^2 + 14x + \underline{49}$ $\quad \frac{b}{2} \to \frac{14}{2} \to 7 \to 7^2 \to 49$

$(x+7)^2$
$\uparrow \frac{b}{2}$

Ex $x^2 - 20x + \underline{100}$
$(x-10)^2$

SOLVE by COMPLETING THE SQUARE

Ex $x^2 + 6x + 8 = 0$
$x^2 + 6x + \_ = -8 + \_$ — add the value to complete the square
$x^2 + 6x + 9 = -8 + 9$
$(x+3)^2 = 1$
Rewrite in binomial form — $x + 3 = \pm 1$ — Square root both sides

$x + 3 = 1$ $\qquad$ $x + 3 = -1$
$x = -2$ $\qquad$ $x = -4$

Ex $x^2 + 4x - 20 = 0$
$x^2 + 4x + \_ = 20 + \_$ — Move constant
$x^2 + 4x + 4 = 20 + 4$ — Add c
$(x+2)^2 = 24$ — Rewrite binomial
$x + 2 = \pm\sqrt{24}$ — Square root
$x = -2 \pm \sqrt{4 \cdot 6}$ — solve for x
$x = -2 \pm 2\sqrt{6}$ — simplify

Ex $3x^2 - 6x + 12 = 0$ — divide all by 3
$x^2 - 2x + 4 = 0$ — move constant
$x^2 - 2x + \_ = -4 + \_$ — complete square
$x^2 - 2x + 1 = -4 + 1$
$(x-1)^2 = -3$
$(x-1) = \pm\sqrt{-3}$ $\to$ $x = 1 \pm i\sqrt{3}$