

# How Can I Sketch the Graph of a Polynomial?

It's now time to put everything we have learned about polynomials together in order to sketch the graph.

EX 1: Graphing a polynomial from its factored form

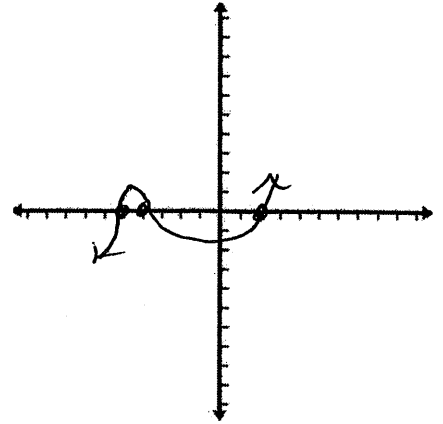
$$f(x) = (x-2)(x+4)(x+5)$$

3rd degree,  
positive LC

**Step 1:** Look at the degree and leading coefficient of your equation. What will the end behavior of your graph be?  $\swarrow \nearrow$ . Place arrows in the appropriate quadrants.

**Step 2:** Plot the x and y-intercepts. You may need to adjust the scale of your graph!

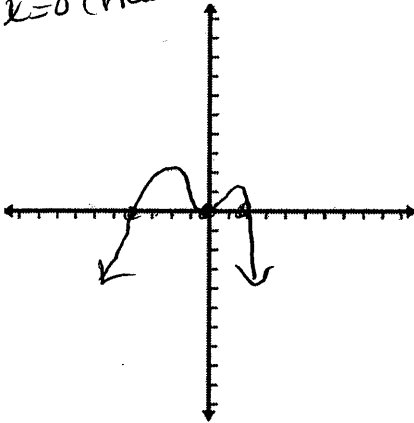
**Step 3:** Sketch the curve by connecting the points in left to right order. Remember to "bounce" where there is a repeated zero.



EX 2:  $f(x) = -x^2(x+4)(x-2)$

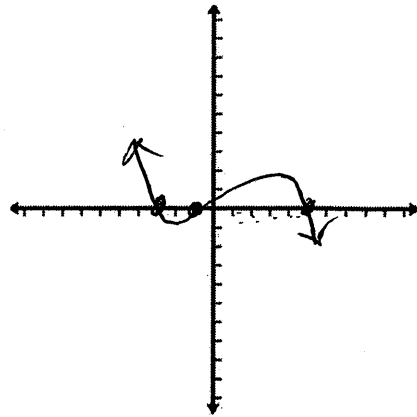
$x=0$  (mult 2)

$x = -4$  even,  
 $x = 2$  neg.

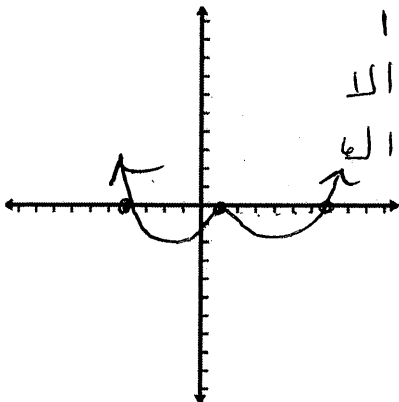


EX 3:  $f(x) = -2x^3 + 2x^2 + 34x + 30$ ; zeros are -1, -3 and 5

ODD, negative  
 $\swarrow \searrow$



EX 4:  $f(x) = x^4 - 4x^3 - 19x^2 + 46x - 24$ ;  $(x-1)$  and  $(x-6)$  are factors



1	-4	-19	46	-24
1	1	-3	-22	24
1	-3	-22	24	0
1	6	18	-24	0
1	3	-4	0	0

$$x^2 + 3x - 4 = 0$$

$$(x+4)(x-1) = 0$$

$$x = -4, x = 1$$

$\therefore$  Solutions are  $x = 1$  (mult 2)

$$x = 6$$

$$x = -4$$

EX 5:  $x^5 - 4x^3 = 0$

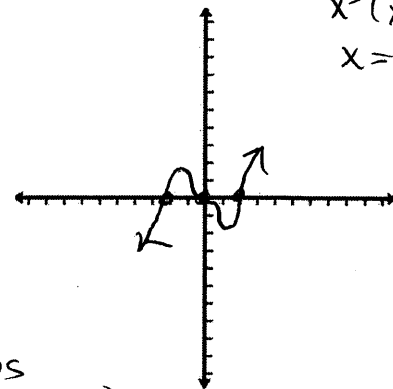
$$x^3(x^2 - 4) = 0$$

$$x^3(x-2)(x+2) = 0$$

$$x = 0 \text{ (mult 3)}$$

$$x = 2,$$

$$x = -2$$



# How can I solve a Polynomial Inequality?

## Method 1: Algebraically

- Step 1: Find all the zeros of the function  
 Step 2: Plot the zeros on a number line. Remember to use a closed circle for  $\leq$  and  $\geq$  and an open circle for  $<$  and  $>$ .  
 Step 3: Pick a test point in each interval  
 Step 4: Emphasize the intervals of the solution set on the number line by overshading.

Example 1: Solve  $4x^2 + 17x < 42$  UNDERNEATH  
 $4x^2 + 17x - 42 < 0$

*STRICTLY UNDERNEATH*

$[-6, \frac{7}{4}]$  OR  $(-6, \frac{7}{4})$ ?

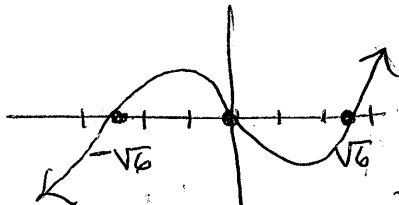
## Method 2: Graphically

- Step 1: Find all the zeros of the function  
 Step 2: Sketch a graph of the function by plotting the x-intercepts and using the appropriate end behavior.  
 Step 3: Examine the graph and note the intervals where the curve is above and below the line.  
 Step 4: Write the solution set in interval notation.

$+4x^2 + 17x < 42$   $\nearrow \nearrow$   
 SOLVE:  $4x^2 + 17x = 42$   
 $-42$   
 $\frac{24}{4} \quad \frac{-7}{4}$   
 $6 \quad 17 \quad 4$   
 $x = -6, x = \frac{7}{4}$

### Example 2: $x^3 - 6x \geq 0$

ODD,  
POS

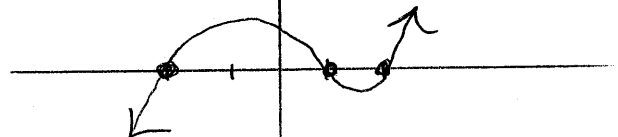


$[-\sqrt{6}, 0] \cup [\sqrt{6}, +\infty)$

$x^3 - 6x \geq 0$  ABOVE OR ON THE X AXIS  
 $x(x^2 - 6) \geq 0$  USE BRACKETS!  
 $x = 0$   
 $x^2 - 6 = 0$   
 $\sqrt{x^2} = \sqrt{6}$   
 $x = \pm 2.4$

### Example 3: $x^3 - x^2 - 4x + 4 > 0$ ABOVE → parentheses

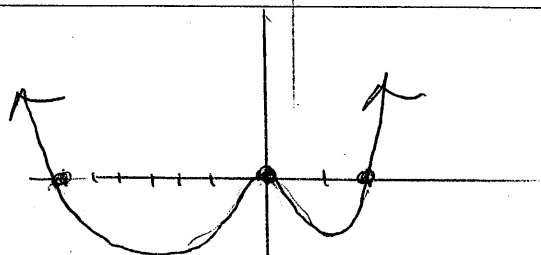
$x^2(x-1) - 4(x-1) > 0$  (STRICTLY)  
 $(x-1)(x^2 - 4) > 0$   
 $(x-1)(x+2)(x-2) > 0$   
 $x = 1, x = -2, x = 2$



$(-2, 1) \cup (2, +\infty)$

### Example 4: $x^4 + 4x^3 \leq 12x^2$ UNDERNEATH OR ON → BRACKETS

$x^4 + 4x^3 - 12x^2 \leq 0$   
 $x^2(x^2 + 4x - 12) \leq 0$   
 $x^2(x+6)(x-2) \leq 0$   
 $x = 0$  (mult 2)  $x = -6$   $x = 2$



$[-6, 2]$

Advanced Algebra Worksheet: Solving Polynomial Inequalities

Step 1: Find all the zeros of the polynomial, and then plot the solutions on the x-axis.

Step 2: Use appropriate end behavior and knowledge of repeated zeros to sketch the graph

Step 3: Write the solution set of the inequality using interval notation.

1.  $x^3 - 3x^2 > 10x$

$$x^3 - 3x^2 - 10x > 0$$

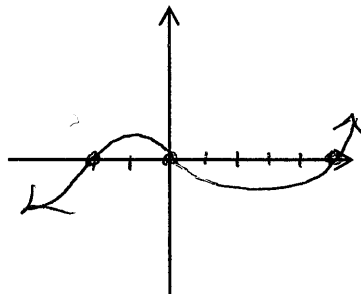
$$x(x^2 - 3x - 10) > 0$$

$$x(x-5)(x+2) > 0$$

$$x = 0$$

$$x = 5$$

$$x = -2$$



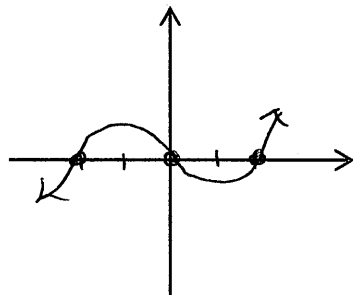
$$\underline{(-2, 0) \cup (5, +\infty)}$$

2.  $x^3 - 4x \geq 0$

$$x(x^2 - 4) \geq 0$$

$$x(x-2)(x+2) \geq 0$$

$$x = 0, x = 2, x = -2$$



$$\underline{[-2, 0] \cup [2, +\infty)}$$

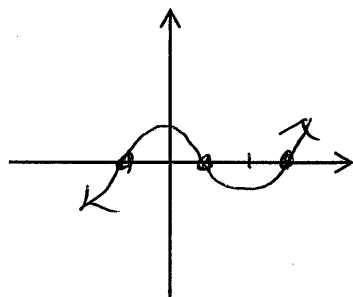
3.  $x^3 - 3x^2 - x + 3 < 0$

$$x^2(x-3) - 1(x-3) < 0$$

$$(x-3)(x^2-1) < 0$$

$$(x-3)(x-1)(x+1) < 0$$

$$x = 3, x = 1, x = -1$$



$$\underline{(-\infty, -1) \cup (1, 3)}$$

4.  $2x^3 - 3x^2 - 32x > -48$

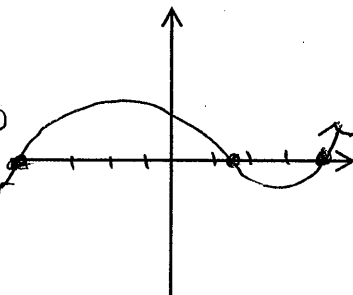
$$2x^3 - 3x^2 - 32x + 48 > 0$$

$$x^2(2x-3) - 16(2x-3) > 0$$

$$(2x-3)(x^2-16) > 0$$

$$(2x-3)(x-4)(x+4) > 0$$

$$2x-3=0 \rightarrow x=3/2, x=4, x=-4$$



$$\underline{(-4, \frac{3}{2}) \cup (4, +\infty)}$$

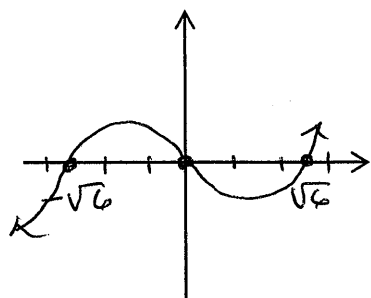
5.  $x^3 - 6x \leq 0$  ← BELOW OR ON

$x(x^2 - 6) \leq 0$

$x = 0 \quad x^2 - 6 = 0$

$x^2 = 6$

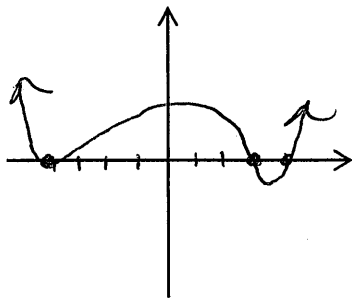
$x = \pm 2.4$



$(-\infty, -\sqrt{6}] \cup [0, \sqrt{6}]$

6.  $(x+4)^2(x-4)(x-3) < 0$

↑  
STRICTLY BELOW



$(3, 4)$

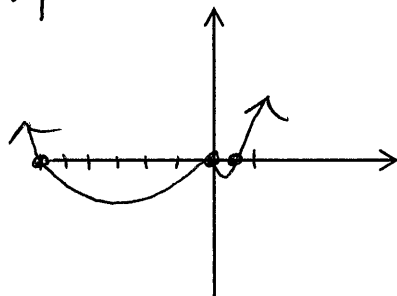
7.  $x^2(2x-1)(x+6) > 0$  ← STRICTLY ABOVE

↓  
 $2x - 1 = 0$

$2x = 1$

$x = \frac{1}{2}$

$x = -6$



$(-\infty, -6) \cup (\frac{1}{2}, \infty)$

$x = 0$  (mult 2)

8.  $x^5 - 8x^3 + 12x > 0$  ← STRICTLY ABOVE

$x(x^4 - 8x^2 + 12) > 0$

$x(x^2 - 6)(x^2 - 2) > 0$

$x = 0 \quad x^2 - 6 = 0$

$x^2 = 6$

$x = \pm\sqrt{6}$

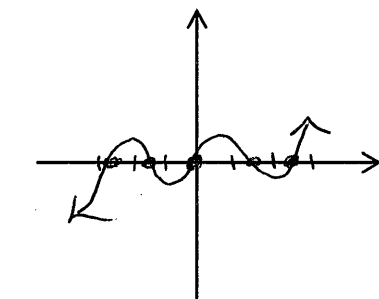
$x^2 - 2 = 0$

$x^2 = 2$

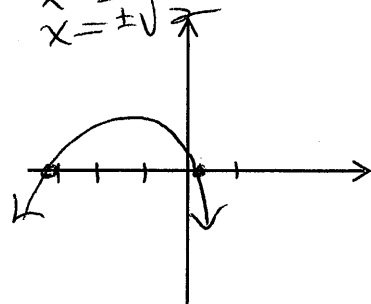
$x = \pm\sqrt{2}$

9.  $-6x^2 - 17x + 3 \leq 0$

$-(6x^2 + 17x - 3) \leq 0$



$(-\sqrt{6}, -\sqrt{2}) \cup (0, \sqrt{2})$   
 $\cup (\sqrt{6}, \infty)$



$(-\infty, -3] \cup [\frac{1}{6}, \infty)$

$\frac{18}{6} \times \frac{-1}{17} = \frac{-18}{17}$

$x = -3$   
 $x = \frac{1}{6}$

## Unit 2, Activity 7, Specific Assessment Graphing Polynomials

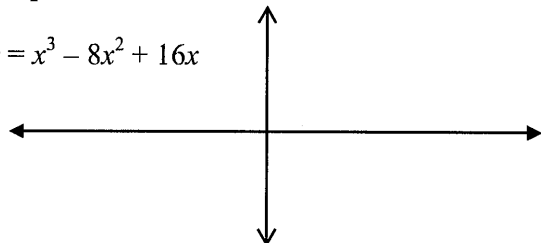
Name \_\_\_\_\_

Date \_\_\_\_\_

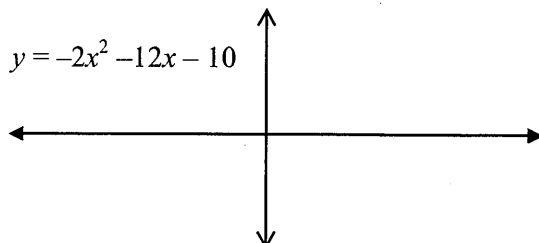
### Graphs of Polynomials

Find the zeros and use the rules developed in the Graphing Polynomials Discovery Worksheet to sketch the following graphs without a calculator. Label accurately the zeros, end-behavior, and  $y$ -intercepts. Do not be concerned with minimum and maximum values between zeros.

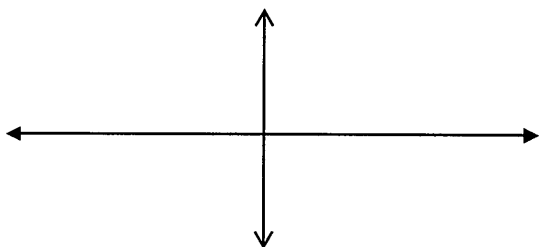
a.  $y = x^3 - 8x^2 + 16x$



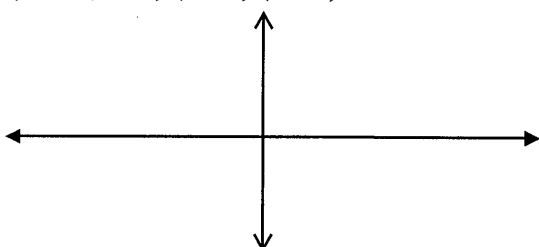
b.  $y = -2x^2 - 12x - 10$



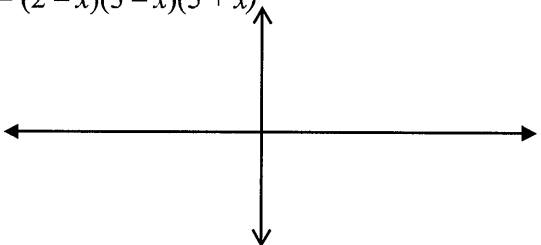
c.  $y = (x - 4)(x + 3)(x + 1)$



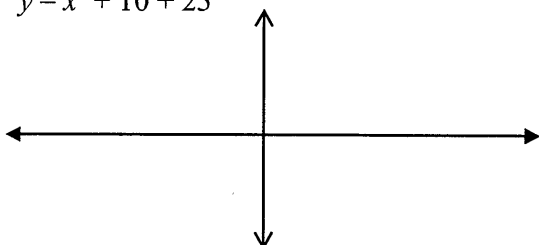
d.  $y = -(x + 2)(x - 7)(x + 5)$



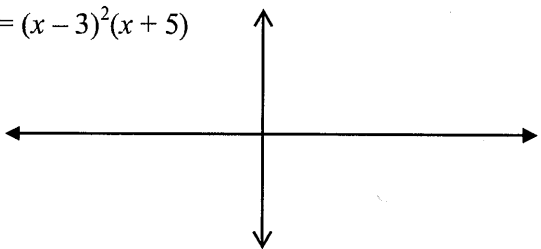
e.  $y = (2 - x)(3 - x)(5 + x)$



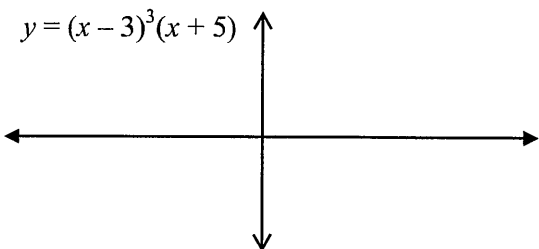
f.  $y = x^2 + 10x + 25$



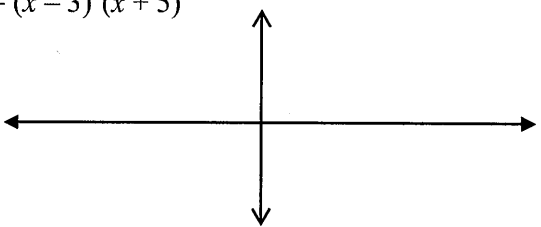
g.  $y = (x - 3)^2(x + 5)$



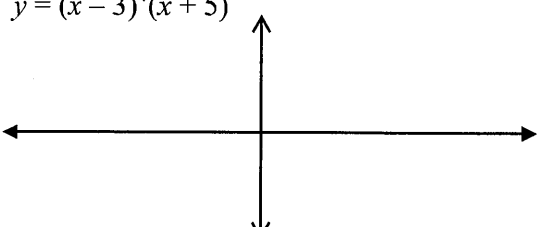
h.  $y = (x - 3)^3(x + 5)$

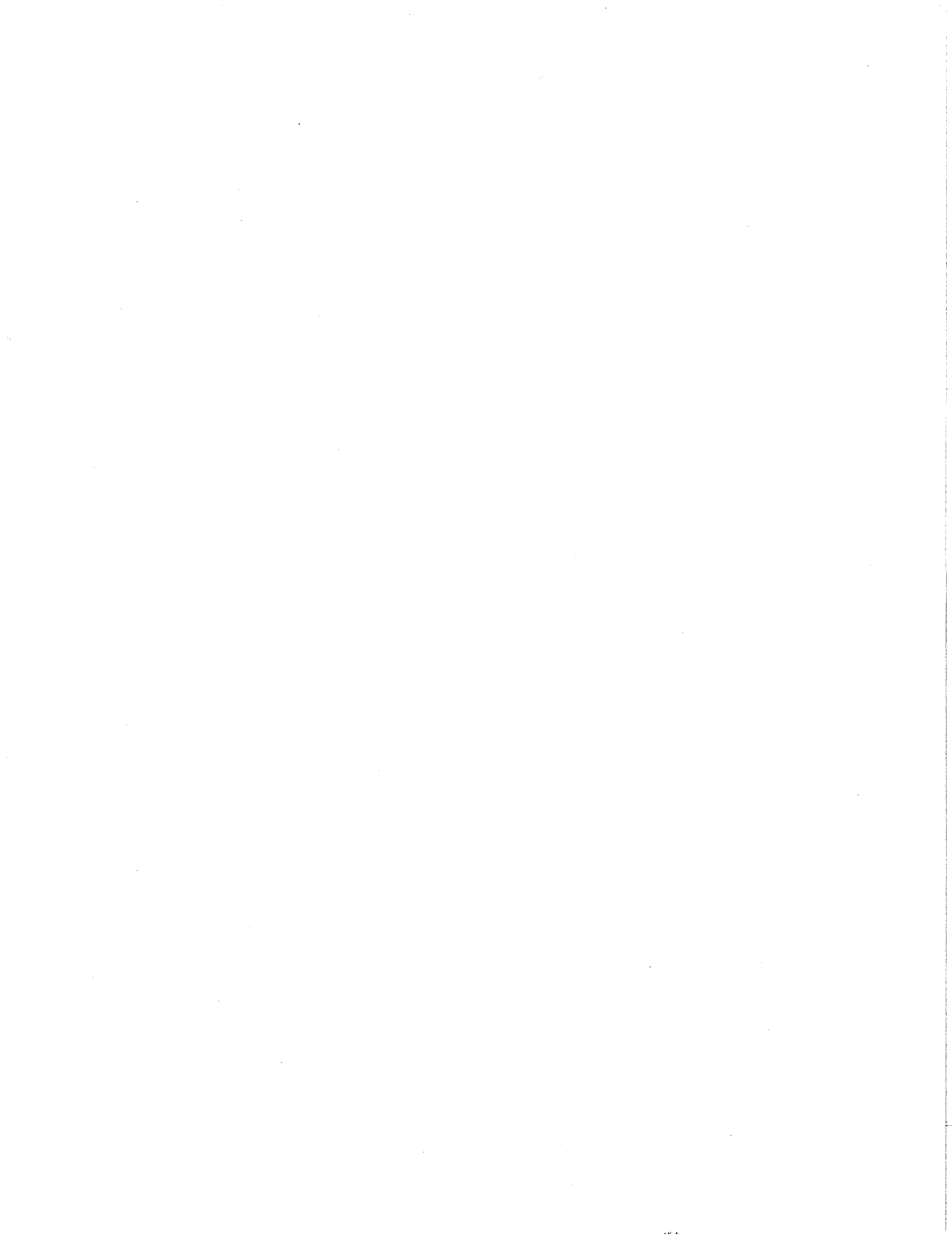


i.  $y = (x - 3)^3(x + 5)^2$



j.  $y = (x - 3)^4(x + 5)$





# Unit 2, Activity 7, Specific Assessment Graphing Polynomials

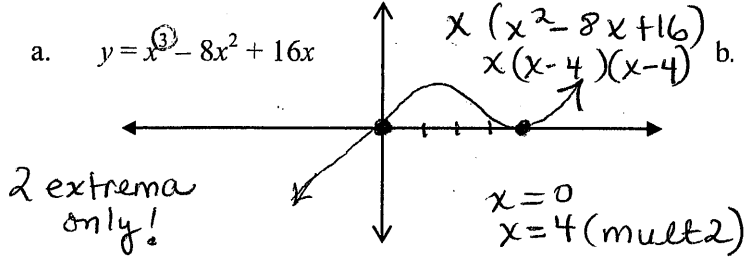
Name \_\_\_\_\_

Date \_\_\_\_\_

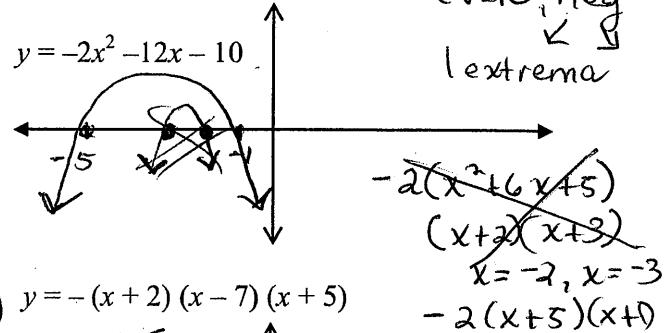
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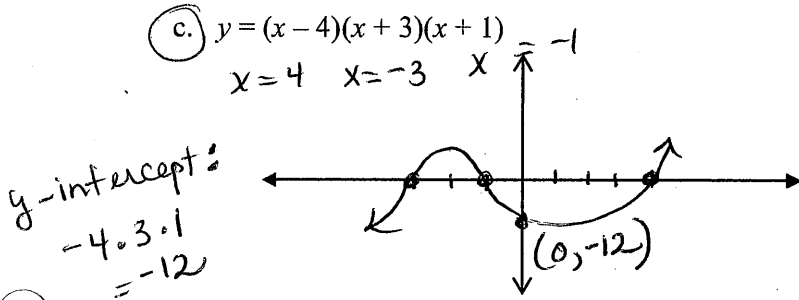
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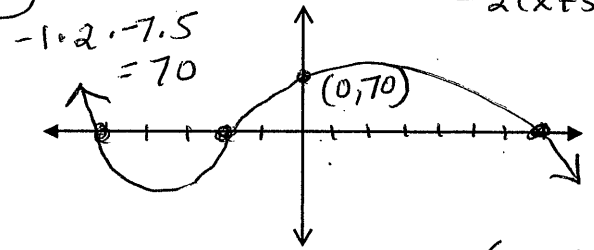
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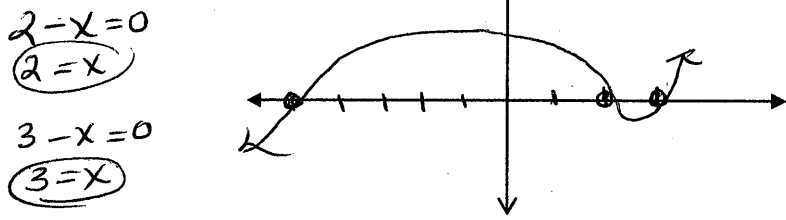
c.  $y = (x-4)(x+3)(x+1)$



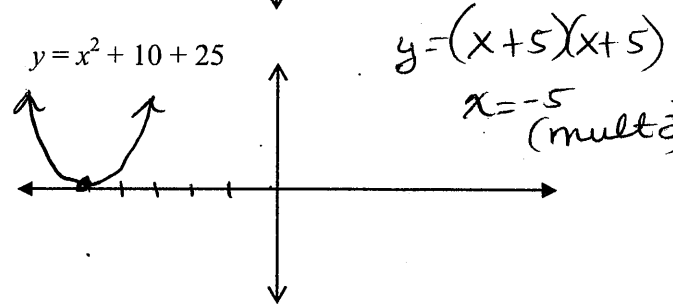
d.  $y = -(x+2)(x-7)(x+5)$



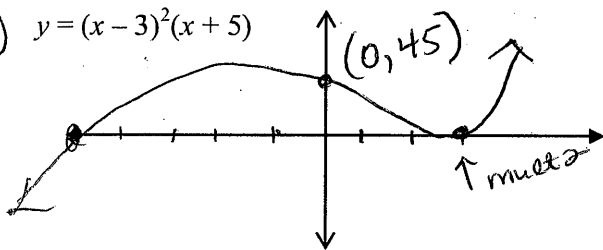
e.  $y = (2-x)(3-x)(5+x)$



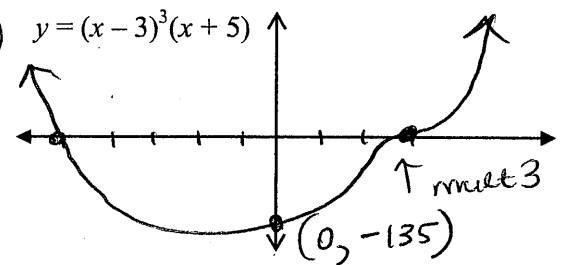
f.  $y = x^2 + 10x + 25$



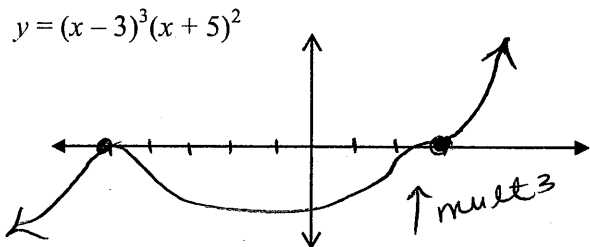
g.  $y = (x-3)^2(x+5)$



h.  $y = (x-3)^3(x+5)$



i.  $y = (x-3)^3(x+5)^2$



j.  $y = (x-3)^4(x+5)$

