WS: Solving Radical Equations   Name ______________________

Solve the radical equations. Check for extraneous solutions.

1. $\sqrt{4-x} = x-4$ — **$x=4$**

$4-x=(x-4)^2$

$4-x=(x-4)(x-4)$

$4-x=x^2-8x+16$

$0=x^2-7x+12$

$0=(x-3)(x-4)$

~~$x=3$~~, $x=4$

2. $+3x^{\frac{3}{4}}=+192$ — **$x=256$**

$x^{\frac{3}{4}}=64$

$x=(64)^{\frac{4}{3}}$

3. $\sqrt{x-2}=\sqrt{3x}$ — **NO SOLUTION**

$x-2=3x$

$-2=2x$

$x=-1$

NO SOLUTION

4. $\sqrt[3]{2x-2}=2\sqrt[3]{-4-x}$ — **$x=-3$**

$2x-2=8(-4-x)$

$2x-2=-32-8x$

$10x-2=-32$

$10x=-30$

$x=-3$

5. $(x-5)^3-1=98$ — **$x\approx 9.63$**

$(x-5)^3=99$

$x-5=99^{\frac{1}{3}}$

$x-5\approx 4.62\ldots$

$x\approx 9.63$

6. $\frac{2}{3}x^5=5184$ — **$x=6$**

$x^5=5184\left(\frac{3}{2}\right)$

$x^5=7776$

$x=(7776)^{\frac{1}{5}}$

7. $\sqrt{x+2}=-4$ — **NO SOLUTION**

$x+2=16$

$x=14$ EXTRANEOUS

8. $x=\sqrt{x+7}+5$ — **$x=9$**

$x-5=\sqrt{x+7}$

$(x-5)^2=x+7$

$(x-5)(x-5)=x+7$

$x^2-10x+25=x+7$

$x^2-11x+18=0$

$(x-2)(x-9)=0$

~~$x=2$~~, $x=9$

9. $(2x-5)^{\frac{4}{3}}=81$ — **$x=16$**

$2x-5=(81)^{\frac{3}{4}}$

$2x-5=27$

$2x=32$

$x=16$

10. $-2x^5=341$ — **$x\approx -2.79$**

$x^5=-170.5$

$x=(-170.5)^{\frac{1}{5}}$

11. $-4\sqrt{6x}=-24\sqrt{2x-4}$ — **$x=24/11$**

$\sqrt{6x}=6\sqrt{2x-4}$

$6x=36(2x-4)$

$6x=72x-144$

$-66x=-144$

$x=24/11$

12. $x-5=\sqrt{18-2x}$ — **$x=7$**

$(x-5)^2=18-2x$

$(x-5)(x-5)=18-2x$

$x^2-10x+25=18-2x$

$x^2-8x+7=0$

$(x-7)(x-1)=0$

$x=7$, ~~$x=1$~~

13. The period of a pendulum is the time T (in seconds) it takes for a pendulum of length L (in feet) to go through one cycle. The period is given by $T = 2\pi\sqrt{\frac{L}{32}}$.

A. Given the period of the pendulum is 1.5 seconds, find its length.

B. Given the period of the pendulum is 3 seconds, find its length.

C. Given the length of the pendulum is 4 feet, finds its period.

A. $1.5 = 2\pi\sqrt{\frac{L}{32}}$

$\frac{1.5}{2\pi} = \sqrt{\frac{L}{32}}$

$\left(\frac{1.5}{2\pi}\right)^2 = \frac{L}{32}$

$32\cdot\left(\frac{1.5}{2\pi}\right)^2 = L$

$L \approx 1.8$ ft

B. $32\cdot\left(\frac{3}{2\pi}\right)^2$

$L \approx 7.3$ ft

C. $T = 2\pi\sqrt{\frac{4}{32}}$

$T \approx 2.2$ seconds

14. SHOT PUT. The *shot* (a metal sphere) used in men's shot put has a volume of about 905 cubic centimeters. Find the radius of the shot given the formula for the volume of a sphere is $V = \frac{4}{3}\pi r^3$.

$905 = \frac{4}{3}\pi r^3$

$\frac{905\left(\frac{3}{4}\right)}{\pi} = r^3$

$216.05\ldots = r^3$

$r = (216.05\ldots)^{\frac{1}{3}}$

$r \approx 6.0$ cm

15. Philip Darlington discovered a rule of thumb that relates an island's land area A(in square miles) to the number of reptile and amphibian species the island can support by the model $A = 0.0779s^3$. The island of Puerto Rico is roughly 4000 square miles. About how many reptile and amphibian species can it support?

$4000 = .0779\,s^3$

$\frac{4000}{.0779} = s^3$

$51347.8819 = s^3$

$S = (51347.8819)^{\frac{1}{3}} \approx 37$ species

16. R.A. Moyer of Iowa State College found in comprehensive tests carried out on wet pavements that the braking distance d (in feet) for a particular automobile traveling at v miles per hour was given approximately by the model $d = 0.0212v^{\frac{7}{3}}$.

A. Approximate the braking distance to the nearest foot for a car traveling on wet pavement at 70 miles per hour.

$d = .0212\,(70)^{\frac{7}{3}}$

$d \approx 428.1$ ft

B. Approximate to the nearest whole number the speed of the car requiring 100 feet of braking distance on wet pavement.

$100 = .0212\,V^{\frac{7}{3}}$

$\frac{100}{.0212} = V^{\frac{7}{3}}$

$\left(\frac{100}{.0212}\right)^{\frac{3}{7}} = V$

$V \approx 38$ mph

Additional Notes and Examples on Solving Radical and Rational Exponent Equations

Example 1: Solving an equation with coefficients on both sides

$$\left(2\sqrt{x+8}\right)^2 = \left(3\sqrt{x-2}\right)^2$$

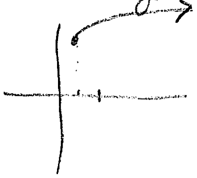
$4(x+8) = 9(x-2)$

$4x + 32 = 9x - 18$

$32 = 5x - 18$

$50 = 5x \rightarrow x = 10$

Example 2: Solving an equation with an extraneous solution. What does this look like on a graph?



$y = \sqrt{2x-1}+5 = 2$ Isolate the radical first

$\sqrt{2x-1} = -3$ Square both sides

$2x - 1 = 9$

$2x = 10$ $x = 5$ ✓ EXTRANEOUS SOLUTION

Check:

$\sqrt{2(5)-1} + 5 = 2$

$\sqrt{9} + 5 = 2$

$3 + 5 = 2$

$8 \neq 2$

$4 = \sqrt{16}$

Example : 3 Solving an equation with a binomial on one side

QUADRATIC SET EQUAL TO 0; BIG X

A. $(x-1)^2 = \left(\sqrt{5x-9}\right)^2$

$(x-1)(x-1) = 5x - 9$

$x^2 - x - x + 1 = 5x - 9$

$x^2 - 2x + 1 = 5x - 9$

$x^2 - 7x + 10 = 0$

$(x-5)(x-2) = 0$

$x = 5, x = 2$

(10: −5, −2; −7)

B. $x - 3 = \sqrt{30 - 2x}$

$7 - 3 = \sqrt{30-14}$

$-6 = \sqrt{36}$

$(x-3)^2 = 30 - 2x$

$(x-3)(x-3) = 30 - 2x$

$x^2 - 3x - 3x + 9 = 30 - 2x$

$x^2 - 6x + 9 = 30 - 2x$

$x^2 - 4x - 21 = 0$

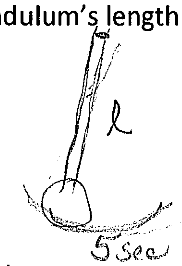
(−21: −7, +3; −4)

$(x-7)(x+3)$

$x = 7$ ~~$x = -3$~~

Real-Life Application Examples

Ex1: The period T (in seconds) of a pendulum can be modeled by $T = 1.11\sqrt{l}$, where l is the pendulum's length (in feet). How long is a pendulum with a period of 5 seconds?



$\frac{5}{1.11} = \frac{1.11\sqrt{l}}{1.11}$

$\left(\frac{5}{1.11}\right)^2 = \left(\sqrt{l}\right)^2$

$l = 20.29$ ft

EX 2: The velocity of a free-falling object is given by $V = \sqrt{19.62h}$, where V is the velocity (in meters per second) and h is the distance (in meters) the object has fallen. If an object hit the ground with a velocity of 55 meters per second, from what height was it dropped?

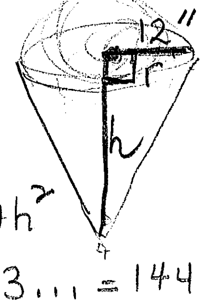
$(55)^2 = \left(\sqrt{19.62h}\right)^2$

$3025 = 19.62h$

$h \approx 154.18$ meters

EX 3: The lateral surface area of a cone is given by $S = \pi r\sqrt{r^2 + h^2}$ where r is the radius and h is the height of the cone. A cone has a radius of 12 inches and a lateral surface area of 912 square inches. What is the height of the cone?

FIND h



$912 = \pi(12)\sqrt{12^2 + h^2}$

$\left(\frac{912}{12\pi}\right)^2 = \frac{12\pi}{12\pi}\left(\sqrt{144 + h^2}\right)^2$

$\left(\frac{912}{12\pi}\right)^2 = 144 + h^2$

$585.23\ldots = 144 + h^2$

$441.23\ldots = h^2 \rightarrow \approx 21$ inches