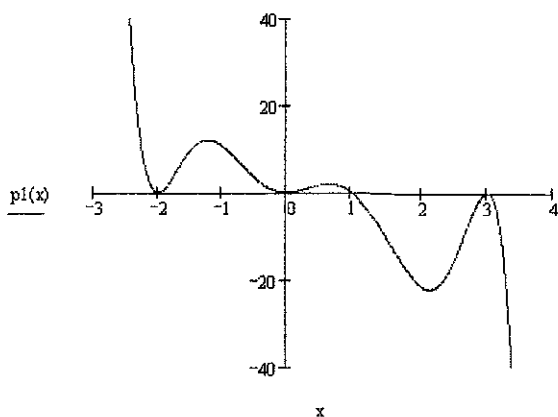


1. The function has a multiplicity of 2 at $x = 1$

2. What is a possible equation, in factored form, for the function shown?

$(x-1)^2(x-2)(x-3)$

3. The *least possible degree* of the function is 4



4. The function has a multiplicity of 2 at $x = -2$, $x = 0$, and $x = 3$.

5. What is a possible equation, in factored form, for the function shown?

$-(x+2)^2 x^2 (x-1)(x-3)^2$
 better written: $-x^2(x+2)^2(x-1)(x-3)^2$

6. The *least possible degree* of the function is 7

7. Given the factored form of the polynomial function $f(x) = (x-2)(x+3)(x+10)$, what are the zeros? Use Z.P.P.

$x-2=0 \rightarrow x=2$
 $x+3=0 \rightarrow x=-3$
 $x+10=0 \rightarrow x=-10$

8. Given the factored form of the function $f(x) = (x+8)^4(x-7)$, there is a multiplicity of 4 at $x = -8$

means the same as $(x+8)(x+8)(x+8)(x+8)(x-7)$

9. What are all the solutions of the equation: $x^3 - 5x^2 - 36x = 0$?

$x(x^2 - 5x - 36) = 0$
 $x(x-9)(x+4) = 0$

$x = 0$
 $x = 9$
 $x = -4$

10. What are the zeros of the function: $f(x) = x^4 - 6x^3 - 44x^2 + 184x + 96$?

- there are 4 solutions $x = -6, x = 4, x = 4 + 2\sqrt{5}, x = 4 - 2\sqrt{5}$

(WORK ON SEPARATE PAPER)

- using calculator, I see $(-6, 0)$ and $(4, 0)$ are solutions \therefore do synthetic division with these values

11. Make a list of all the *possible rational zeros* of the function: $f(x) = 7x^3 - 2x^2 + 9x - 12$ (do not solve!)

$\frac{p}{q} = \pm \frac{1, 2, 3, 4, 6, 12}{1, 7} \rightarrow \pm \left\{ 1, \frac{1}{7}, 2, \frac{2}{7}, 3, \frac{3}{7}, 4, \frac{4}{7}, 6, \frac{6}{7}, 12, \frac{12}{7} \right\}$

12. Use synthetic division to find the quotient and remainder for $(3x^5 - 2x^4 + x^2 - 4x + 12) \div (x-2)$.

Write your answer in the appropriate polynomial format.

$$\begin{array}{r} 3 \quad -2 \quad 0 \quad 1 \quad -4 \quad 12 \\ \quad \quad 6 \quad 8 \quad 16 \quad 34 \quad 60 \\ 2 \overline{) 3 \quad 4 \quad 8 \quad 17 \quad 30 \quad 72} \\ \underline{3 \quad 4 \quad 8 \quad 17 \quad 30 \quad 72} \\ 0 \end{array}$$

$3x^4 + 4x^3 + 8x^2 + 17x + 30$ remainder 72

Use synthetic division and appropriate quadratic methods to complete the following:

13. Factor the polynomial: $f(x) = 4x^3 - 8x^2 - 15x + 9$ completely given that $f(3) = 0$.

$$(x-3)(2x+3)(2x-1)$$

14. Given that $(x+1)$ is a factor of $f(x) = 2x^3 + x^2 - 7x - 6$, find all the zeros.

$$x = -1, x = 2, x = -\frac{3}{2}$$

15. Factor the polynomial $f(x) = 2x^3 + x^2 - 16x - 15$, completely given that $f(3) = 0$

$$(x-3)(2x+5)(x+1)$$

16. Find all the roots of the equation $2x^4 + 7x^3 - 4x^2 - 27x - 18 = 0$ given that $(x-2)$ and $(x+3)$ are factors.

$$x = 2, x = 3, x = -1, x = -\frac{3}{2}$$

For the following problems, make a list of possible rational zeros. Use the graphing calculator to shorten the list, then find all the zeros. Your solutions are only estimates until proven algebraically using any appropriate combination of synthetic division and quadratic methods.

17. Find all the zeros of the function $y = 6x^3 - 53x^2 + 111x + 20$

$$x = 4, x = 5, x = -\frac{1}{6}$$

18. Find all the roots of the equation $y = x^4 + 3x^3 + 11x^2 + 63x - 210$

$$x = -5, x = 2, \\ x = i\sqrt{21}, x = -i\sqrt{21}$$

19. Find all the roots of the function $f(x) = x^3 + 5x^2 + x - 5$

$$x = -5, x = i, x = -i$$

20. Write the standard form equation of the polynomial with zeros at 6, 8, and -7

$$y = x^3 - 7x^2 - 50x + 336$$

21. Write the standard form equation of the polynomial with solutions $\sqrt{7}$ and $-6i$

$$y = x^4 + 29x^2 - 252$$

10. $x^4 - 6x^3 - 44x^2 + 184x + 96$

$$\begin{array}{r} 1 \quad -6 \quad -44 \quad 184 \quad 96 \\ -6 \mid 1 \quad -6 \quad 72 \quad -168 \quad 96 \\ \hline \quad \quad -12 \quad 28 \quad -16 \quad 0 \end{array}$$

$$\begin{array}{r} 1 \quad -12 \quad 28 \quad 16 \\ 4 \mid 1 \quad -4 \quad -32 \quad -16 \\ \hline \quad \quad -8 \quad -4 \quad 0 \end{array}$$

$$x^2 - 8x - 4 = 0$$

$$x^2 - 8x + 16 = 4 + 16$$

$$(x - 4)^2 = 20$$

$$x - 4 = \pm \sqrt{20}$$

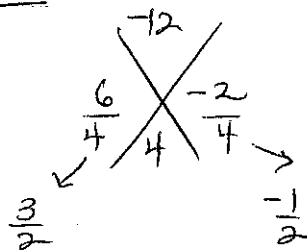
$$x = 4 \pm 2\sqrt{5}$$

13. $4x^3 - 8x^2 - 15x + 9$; $f(3) = 0 \rightarrow (x-3)$ is a factor

$$\begin{array}{r} 4 \quad -8 \quad -15 \quad 9 \\ 3 \mid 4 \quad -8 \quad -15 \quad 9 \\ \hline \quad \quad 4 \quad -3 \quad 0 \end{array}$$

$$4x^2 + 4x - 3$$

$$(2x+3)(2x-1)$$

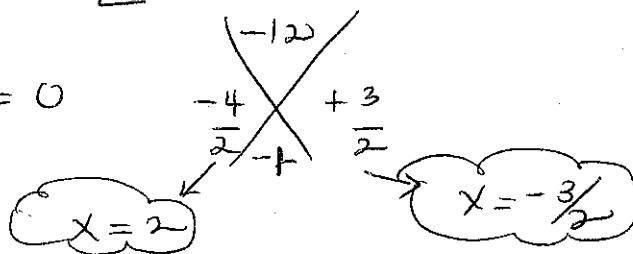


14. $2x^3 + x^2 - 7x - 6$; $(x+1)$ is a factor

$\rightarrow x = -1$ is a zero

$$\begin{array}{r} 2 \quad 1 \quad -7 \quad -6 \\ -1 \mid 2 \quad -1 \quad -6 \quad -6 \\ \hline \quad \quad -1 \quad -6 \quad 0 \end{array}$$

$$2x^2 - x - 6 = 0$$



15. $2x^3 + x^2 - 16x - 15$; $f(3) = 0 \rightarrow (x-3)$ is a factor

$$\begin{array}{r} 2 \quad 1 \quad -16 \quad -15 \\ 3 \overline{) 2 \quad 7 \quad 5 \quad 0} \\ \underline{6 \quad 21 \quad 15} \\ 0 \end{array}$$

$$2x^2 + 7x + 5 = 0$$

$$\begin{array}{r} 10 \\ \frac{5}{2} \quad \frac{2}{2} \\ \times \quad \frac{7}{2} \\ \hline \end{array}$$

$$(2x+5)(x+1)(x-3)$$

16. $2x^4 + 7x^3 - 4x^2 - 27x - 18 = 0$; $(x-2)$ and $(x+3)$ are factors

$$\begin{array}{r} 2 \quad 7 \quad -4 \quad -27 \quad -18 \\ 2 \overline{) 2 \quad 11 \quad 18 \quad 9 \quad 0} \\ \underline{4 \quad 22 \quad 36 \quad 18} \\ 0 \end{array}$$

$$\begin{array}{r} 2 \quad 11 \quad 18 \quad 9 \\ -3 \overline{) 2 \quad 5 \quad 3 \quad 0} \\ \underline{-6 \quad -15 \quad -9} \\ 0 \end{array}$$

$x=2$ and $x=3$ are solutions

$$2x^2 + 5x + 3 = 0$$

$$\begin{array}{r} 6 \\ \frac{2}{2} \quad \frac{3}{2} \\ \times \quad 5 \\ \hline \end{array}$$

$x = -1$ $x = -\frac{3}{2}$

$$17. \quad y = 6x^3 - 53x^2 + 111x + 20$$

$x=4$ } using
and $x=5$ } calculator

$$\begin{array}{r} 6 \quad -53 \quad 111 \quad 20 \\ \quad \quad 24 \quad -116 \quad -20 \\ \hline 4 \overline{) 6} \quad -29 \quad -5 \quad \underline{0} \end{array}$$

$$\begin{array}{r} 6 \quad -29 \quad -5 \\ \quad \quad 30 \quad 5 \\ \hline 5 \overline{) 6} \quad 1 \quad \underline{0} \end{array}$$

$$\begin{aligned} 6x + 1 &= 0 \\ 6x &= -1 \\ x &= -\frac{1}{6} \end{aligned}$$

$$18. \quad y = x^4 + 3x^3 + 11x^2 + 63x - 210$$

$x = -5$
 $x = 2$

$$\begin{array}{r} 1 \quad 3 \quad 11 \quad 63 \quad -210 \\ -5 \overline{) 1} \quad -5 \quad 10 \quad -105 \quad 210 \\ \quad \quad -2 \quad 21 \quad -42 \quad \underline{0} \end{array}$$

$$\begin{array}{r} 1 \quad -2 \quad 21 \quad -42 \\ 2 \overline{) 1} \quad 2 \quad 0 \quad 42 \\ \quad \quad 0 \quad 21 \quad \underline{0} \end{array}$$

$$\begin{aligned} x^2 + 21 &= 0 \\ x^2 &= -21 \\ x &= \pm \sqrt{-21} \\ x &= \pm i\sqrt{21} \end{aligned}$$

$$19. \quad y = \underbrace{x^3 + 5x^2} + \underbrace{x + 5}$$

$$x^2(x+5) + 1(x+5)$$

$$(x+5)(x^2+1)$$

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$$x+5=0$$

$$x^2+1=0$$

$$x = -5$$

$$x^2 = -1$$

$$x = \pm\sqrt{-1}$$

$$x = \pm i$$

$$20. \quad \underbrace{(x-6)(x-8)}(x+7)$$

$$x^2 - 8x - 6x + 48$$

$$(x^2 - 14x + 48)(x+7)$$

$$x^3 + 7x^2 - 14x^2 - 98x + 48x + 336$$

$$x^3 - 7x^2 - 50x + 336$$

$$21. \quad \sqrt{7} \quad \text{and} \quad -6i$$

$$\rightarrow -\sqrt{7} \quad \text{and} \quad +6i$$

are also solutions!

$$(x+\sqrt{7})(x-\sqrt{7})$$

$$x^2 - \sqrt{7}x - \sqrt{7}x - 7$$

$$(x^2 - 7)$$

$$(x+6i)(x-6i)$$

$$x^2 - 6ix + 6ix - 36i^2$$

$$(x^2 + 36)$$

Now MULTIPLY THESE!

$$x^4 + 36x^2 - 7x^2 - 252$$

$$x^4 + 29x^2 - 252$$