How Can I Write the Equation of a Polynomial?

Remember the **Factor Theorem** that states if (x – c) is a factor of a polynomial, then c is a root of the polynomial. Let's apply this theorem to writing the equations of polynomials.

EXAMPLE 1: Write the polynomial function given its roots are 3, -5, and ¾.

We can easily determine that two of the factors are $(x-3)$ and $(x+5)$. Follow these simple steps to determine the 3rd factor:

$x = \frac{3}{4}$

$4x = 3 \rightarrow (4x-3)=0$

Now apply the distributive property (FOIL) to write the standard form equation of the polynomial. (2)

$(x-3)(x+5)(4x-3)$ → distribute the product to remaining factor

① Pick two to distribute

$x^2+5x-3x-15$

$(x^2+2x-15)$ → $(x^2+2x-15)(4x-3)$

$4x^3-3x^2+8x^2-6x-60x+45$

$f(x)=4x^3+5x^2-66x+45$ ③ write in standard form

EXAMPLE 2: Find a polynomial function of least degree with roots 4 and 5i.

Please recall from Math 2 that complex numbers (*i*) always come in conjugate pairs. For example, if +3i is a root of the equation, then $-3i$ is a root of the equation. If 6 – 4i is a root of the equation, then $6+4i$ is a root of the equation. If – 2 + i is a root of the equation, then $-2-i$ is a root of the equation.

For this example, we may conclude that our roots are 4, $5i$, and $-5i$. We can now write the factorization of the polynomial:

$(x-4)(x-5i)(x+5i)$

There is a shortcut but it comes after observation and practice!

Apply the distributive property to write the standard form equation of the polynomial. It is important to remember that $i^2 =$ -1. Use this fact to simplify completely.

$(x-4)\boxed{(x-5i)(x+5i)}$ ← always distribute conjugate pairs first!

$x^2+\underbrace{5ix-5ix}_{\text{Zero pair}}-25i^2$

$x^2-25(-1)$ ← replace i^2 with -1

x^2+25 Now distribute to remaining factor

$(x^2+25)(x-4)$

$f(x)=x^3-4x^2+25x-100$

EXAMPLE 3: Find a polynomial function with zeros at $\boxed{\text{3 with a multiplicity of 2}}$, 0, and $\sqrt{7}$

this means $(x-3)(x-3)$ ↑ x is a monomial factor

Just as with complex solutions, irrational solutions ($\sqrt{\ }$) always come in conjugate pairs. For this problem, we have $\sqrt{7}$ is a solution and so we know that $-\sqrt{7}$ is also a solution. Let's write the factors:

$x(x-3)(x-3)(x-\sqrt{7})(x+\sqrt{7})$

Distribute all factors (be patient) to write the standard form equation.

Multiply conjugate pair first!

$(x-3)(x-3)$

$x^2-3x-3x+9$

$x\ (x^2-6x+9)$

$(x-\sqrt{7})(x+\sqrt{7})$

$x^2+\underbrace{\sqrt{7}x-\sqrt{7}x}_{\text{Zero pair}}-\sqrt{49})$

(x^2-49)

NOW MULTIPLY to remaining factor(s)

Altogether: $x(x^2-6x+9)(x^2-49)$

distribute last

$x(x^4-49x^2-6x^3+294x+9x^2-441)$

$x(x^4-6x^3-40x^2+294x-441)$

$x^5-6x^4-40x^3+294x^2-441x$

Advanced Algebra Worksheet

Writing Equations of Polynomials

Name ______________________________

Find the equation of the polynomial with the given roots. Show all work!

1. 4 and 7

$(x-4)(x-7)$

$x^2-7x-4x+28$

$x^2-11x+28$

2. $\frac{1}{2}$, -1 and 0

$x=\frac{1}{2}$

$x-\frac{1}{2}=0$

$x\,(2x-1)(x+1)$

$x(2x^2+2x-x-1)$

$x(2x^2+x-1)$

$2x^3+x^2-x$

3. 2i and -7

AND $x=-2i$

$(x+2i)(x-2i)(x+7)$

$x^2+2ix-2ix-4i^2$

$(x^2+4)(x+7)$

$x^3+7x^2+4x+28$

4. $\sqrt{11}$ and 4 with a multiplicity of 2

AND $x=-\sqrt{11}$

$(x+\sqrt{11})(x-\sqrt{11})(x-4)(x-4)$

$x^2-\sqrt{11}x+\sqrt{11}x-11$

(x^2-11)

$x^2-4x-4x+16$

$(x^2-8x+16)$

$x^4-8x^3+16x^2-11x^2+88x-176$

$x^4-8x^3+5x^2+88x-176$

5. $\frac{7}{5}$, -2 and 0 with a multiplicity of 3

$x^3(5x-7)(x+2)$

$x^3(5x^2+10x-7x-14)$

$x^3(5x^2+3x-14)$

$5x^5+3x^4-14x^3$

6. $\sqrt{2}$ and 6

AND $-\sqrt{2}$

$(x-\sqrt{2})(x+\sqrt{2})(x-6)$

$x^2+\sqrt{2}x-\sqrt{2}x-2$

$(x^2-2)(x-6)$

$x^3-6x^2-2x+12$

7. i and 8i

AND $-i$ and $-8i$

$(x+i)(x-i)\quad(x+8i)(x-8i)$

$x^2+ix-ix-i^2$ ↑ Replace i^2 with -1

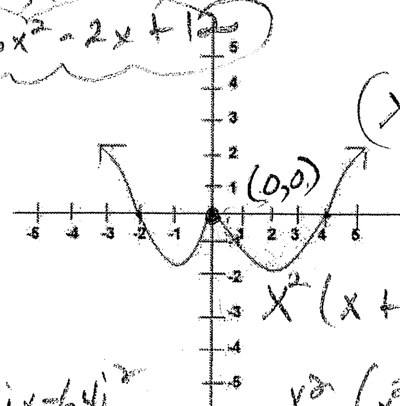
$x^2+8ix-8ix-64i^2$

$(x^2+1)\quad(x^2+64)$

$x^4+64x^2+x^2+64$

x^4+65x^2+64

8.



$(x^2$

$x^2(x+2)(x-4)$

$x^2(x^2-4x+2x-8)$

$x^2(x^2-2x-8)$

$x^4-2x^3-8x^2$