Standard Form

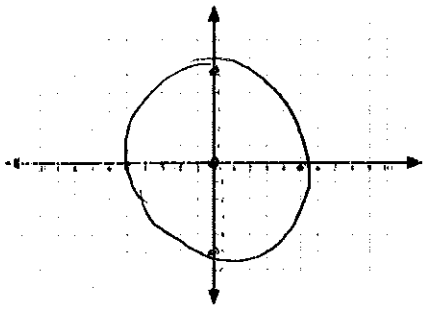
The standard form equation of a circle is $x^2+y^2=r^2$; the center is (0,0) and r is the radius (length).

Examples:

1. $x^2 + y^2 = 25$ has a center at (0,0) and a radius 5 units long.
2. $x^2 + y^2 = 19$ has a center at (0,0) and a radius $\sqrt{19} \approx 4.4$ units long.
3. $x^2 + y^2 = 1$ has a center at (0,0) and a radius 1 units long.

Sketch the circles:

1.



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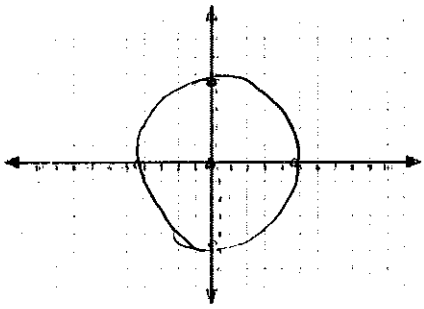

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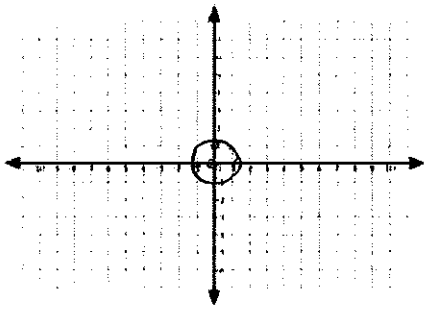


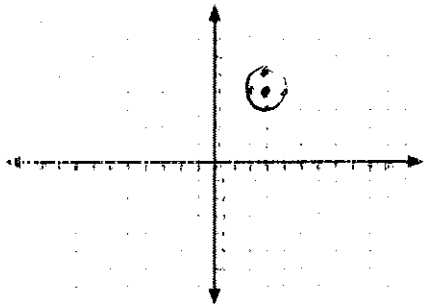
Transformations on Circles

We can apply transformations to circle of the form $(x-h)^2+(y-k)^2=r^2$ where *h* is the horizontal translation and *k* is the vertical translation.

Examples:

1. $(x - 3)^2 + (y - 4)^2 = 1$ has center (3, 4) and a radius 1 units long.
2. $(x + 5)^2 + y^2 = 8$ has center (−5, 0) and a radius $\sqrt{8} \approx 2.8$ units long.
3. $(x + 1)^2 + (y + 3)^2 = 16$ has center (−1, −3) and a radius 4 units long.

1.



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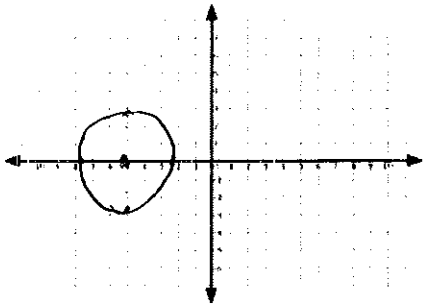

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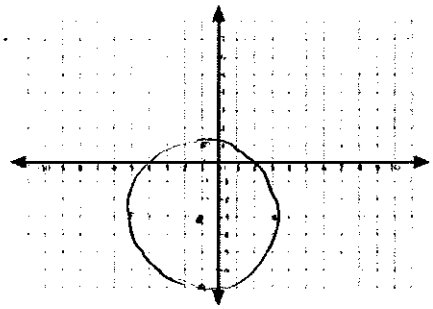


Δx = distance between x coordinates
Δy = distance between y-coordinates

$(\Delta x)^2 + (\Delta y)^2 = r^2$

Given a center and a point on the circle: Use the given points and distance formula in order to find the length of the radius, then substitute into the equation

1. center (0, -2); point (3, 8) $\Delta y = 10$

$\Delta x = 3$

$3^2 + 10^2 = r^2$
$9 + 100 = r^2$
$109 = r^2$

$\boxed{x^2 + (y+2)^2 = 109}$

2. Center (5, -3); point (0, 9) $\Delta y = 12$

$\Delta x = 5$

$5^2 + 12^2 = r^2$
$25 + 144 = r^2$
$169 = r^2$

$\boxed{(x-5)^2 + (y+3)^2 = 169}$

3. center (1, 1); points (5, 7) $\Delta y = 6$

$\Delta x = 4$

$4^2 + 6^2 = r^2$
$16 + 36 = r^2$
$52 = r^2$

$\boxed{(x-1)^2 + (y-1)^2 = 52}$

4. Center (3, 0); point (11, -4) $\Delta y = 4$

$\Delta x = 8$

$8^2 + 4^2 = r^2$
$64 + 16 = r^2$
$80 = r^2$

$\boxed{(x-3)^2 + y^2 = 80}$

Given the endpoints of a diameter: Use the midpoint formula to find the center; use the center and an endpoint of the diameter to find the length of the radius

1. endpoints of a diameter are at (8, -1) and (4, 3)

Step 1: find center (midpoint)

$\left(\frac{8+4}{2}, \frac{-1+3}{2}\right)$

$(6, 1)$

Step 2: pick either point and use the center

$(6,1), (8,-1)$

Step 3: Find $\Delta x, \Delta y$

$\Delta x = 2, \Delta y = 2$

$2^2 + 2^2 = r^2$
$4 + 4 = r^2$
$8 = r^2$

$\boxed{(x-7)^2 + (y+1)^2 = 2}$

$(x-6)^2 + (y-1)^2 = 8$

2. endpoints of a diameter are at (6, 0) and (8, -2)

Center $(7, -1)$ $(6, 0)$

$\Delta x = 1 \quad \Delta y = 1$

$1^2 + 1^2 = r^2$
$1 + 1 = r^2 \rightarrow r^2 = 2$

3. endpoints of a diameter are at (1, 4) and (9, 14)

center $(5, 9)$, $(1, 4)$

$\Delta x = 4 \quad \Delta y = 5$

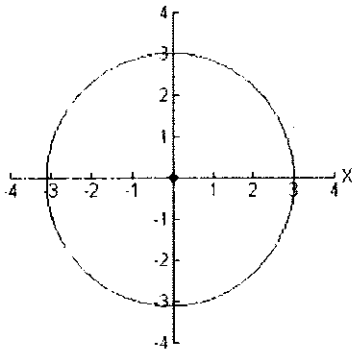
$4^2 + 5^2 = r^2$
$16 + 25 = r^2$
$41 = r^2$

$\boxed{(x-5)^2 + (y-9)^2 = 41}$

Writing Equations of Circles

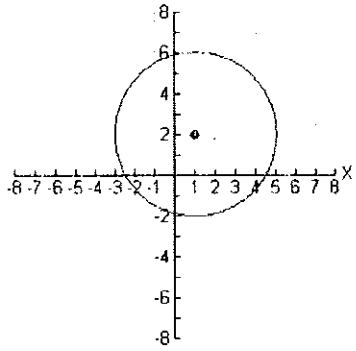
Given a graph: Identify the center and radius then substitute values into the equation

1.



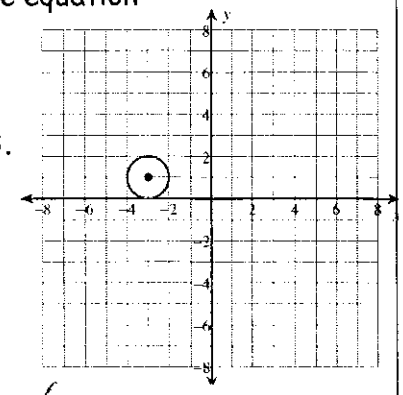
$x^2+y^2=9$

2.



$(x-1)^2+(y-2)^2=16$

3.



$(x+3)^2+(y-1)^2=1$

Given a center and a radius: Substitute values into the equation

1. center (0,-8) and r = 11

$x^2+(y+8)^2=121$

2. Center (4, 3) and r = 2.5

$(x-4)^2+(y-3)^2=6.25$

3. center (0, 0) and r = 4

$x^2+y^2=16$

4. Center (-2, 5) and r = √6

$(x+2)^2+(y-5)^2=6$

Before we go any further, let's stop and review how to use the distance and midpoint formulas.

Distance Formula $D=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ **Midpoint Formula** $M=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$

Find the distance and midpoint between the following pairs of points.

1. (1, 6) and (-2, 9)

$m=\left(-\frac{1}{2},\frac{15}{2}\right)$

FOR DISTANCE, USE Pythagorean Thm

$\Delta x=3$ (change in x)

$\Delta y=3$ (change in y)

$(\Delta x)^2+(\Delta y)^2=r^2$

$3^2+3^2=r^2$

$9+9=r^2$

$18=r^2$ so $r=\sqrt{18}$

2. (0, -3), and (-1, 4)

$m:\left(-\frac{1}{2},\frac{1}{2}\right)$

$\Delta x=1$

$\Delta y=7$

$1^2+7^2=r^2$

$1+49=r^2$

$50=r^2$

$r=\sqrt{50}$