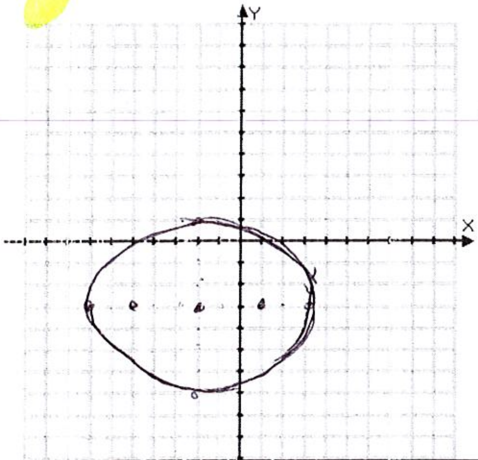
...c Sections Review

...rections: Graph the following conic sections and list the characteristics that are specific to that conic section. You will NOT use ALL of the blanks for any one conic section.

1. $\dfrac{(x+2)^2}{25}+\dfrac{(y+3)^2}{16}=1$ C E H P



$c^2 = 25-16$

$c^2 = 9$

$c = 3$

foci

$(-2,-3)$ +3, −3

Characteristics:

Center/Vertex: $(-2,-3)$ +5 +4, −5 −4

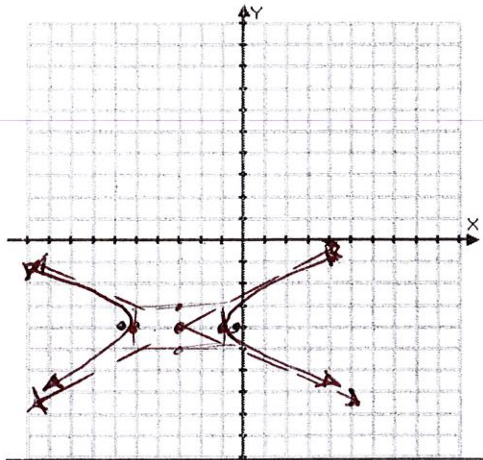
Vertices: $(3,-3)$ $(-7,-3)$

Co-Vertices: $(-2,1)$ $(-2,-7)$

Foci/Focus: $(1,-3)$ $(-5,-3)$

Radius or Directrix: ________

2. $\dfrac{(x+3)^2}{4}-(y+4)^2=1$ C E H P



$c^2 = a^2+b^2$

$c^2 = 4+1$

$c^2 = 5$

$c = \sqrt{5}$

Characteristics:

Center/Vertex: $(-3,-4)$

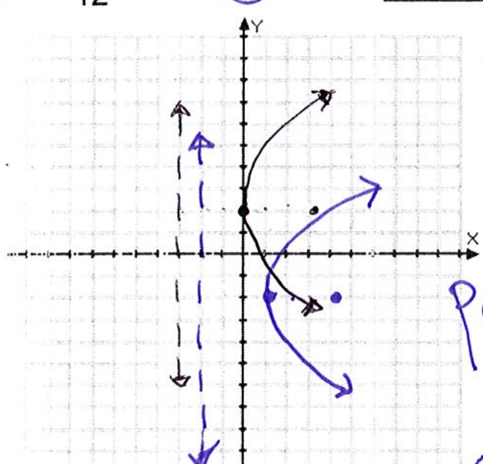
Vertices: $(-3,-4)$ +2 −2 $(-1,-4)$ $(-5,-4)$

Co-Vertices: $(-3,-4)$ +1 −1 $(-3,-3)$, $(-3,-5)$

Foci/Focus: $(-3\pm\sqrt{5},-4)$

Radius or Directrix: ________

3. $x=\dfrac{1}{12}(y-2)^2+1$ oops.. C E H P



$4p=12$

$p=3$

Purple graph is correct

Characteristics:

Center/Vertex: $(\cancel{0,2})$ +3 $(1,-2)$

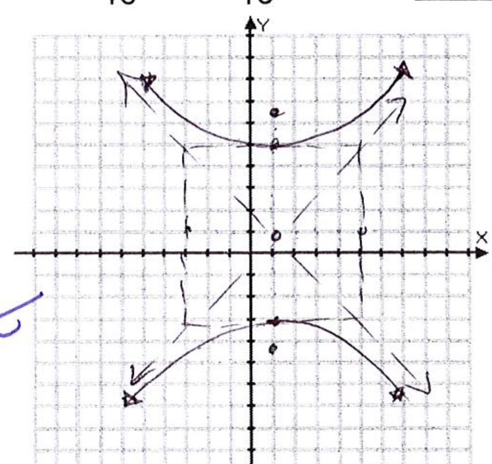
Vertices ________ ________

Co-Vertices: ________ ________

Foci/Focus: $\cancel{(3,2)}$ $(4,-2)$

Radius or Directrix: $x=\cancel{3}$ $x=-2$

4. $\dfrac{(y-1)^2}{16}-\dfrac{(x-1)^2}{16}=1$ C E H P



$c^2 = 16+16$

$c^2 = 32$

$c = 4\sqrt{2}$

Characteristics:

Center/Vertex: $(1,1)$

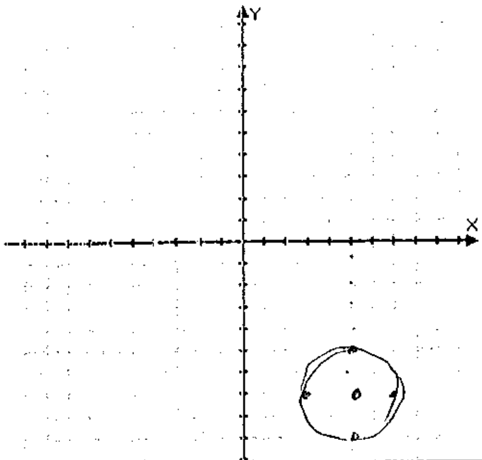
Vertices: $(1,5)$ $(1,-3)$

Co-Vertices: $(5,1)$ $(-3,1)$

Foci/Focus: $(1, 1\pm4\sqrt{2})$

Radius or Directrix: ________

5. $(x-5)^2+(y+7)^2=4$ **C** E H P



Characteristics:

Center/Vertex: $(5,-7)$

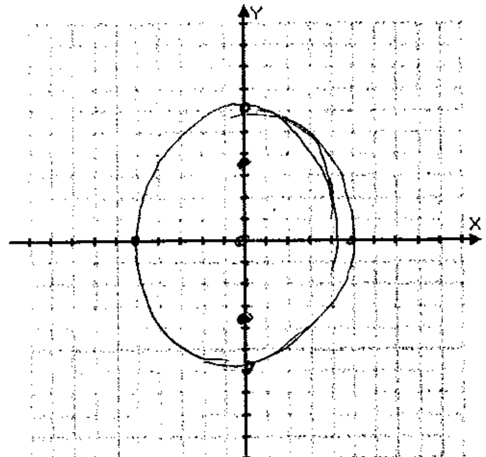
Vertices: ______ ______

Co-Vertices: ______ ______

Foci/Focus: ______ ______

Radius or Directrix: 2

6. $\dfrac{x^2}{25}+\dfrac{y^2}{36}=1$ C **E** H P



$c^2=36-25$

$c^2=11$

$c=\sqrt{11}$

Characteristics:

Center/Vertex: $(0,0)$

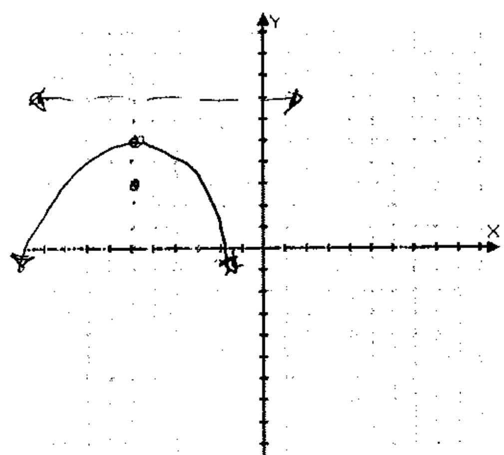
Vertices: $(0,6)$ $(0,-6)$

Co-Vertices: $(5,0)$ $(-5,0)$

Foci/Focus: $(0,\pm\sqrt{11})$

Radius or Directrix: ______

7. $y=-\dfrac{1}{8}(x+6)^2+5$ C E H **P**



Characteristics:

Center/Vertex: $(-6,5)$ +2 / −2

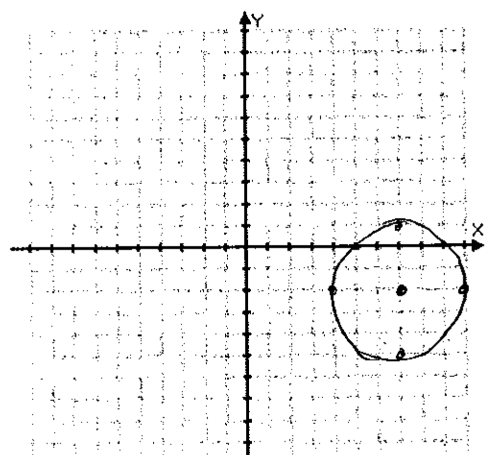
Vertices: ______ ______

Co-Vertices: $(-6,3)$

Foci/Focus: ______ ______

Radius or Directrix: $y=7$

8. $\dfrac{(x-7)^2}{9}+\dfrac{(y+2)^2}{9}=1$ **C** E H P



Characteristics:

Center/Vertex: $(7,-2)$

Vertices: ______ ______

Co-Vertices: ______ ______

Foci/Focus: ______ ______

Radius or Directrix: $r=3$

Identify the conic section, then rewrite in standard form.

CLASSIFY

$x^2 + y^2 - 6x - 2y + 1 = 0$ CIRCLE

$$x^2 - 6x \quad + y^2 - 2y \quad = -1$$
$$x^2 - 6x + 9 + y^2 - 2y + 1 = -1 + 9 + 1$$
$$(x-3)^2 + (y-1)^2 = 9$$

10. $y^2 - 12x - 72 = 0$ PARABOLA

$$-12x = -y^2 + 72$$
$$x = \frac{1}{12}y^2 - 6$$

11. $9x^2 + 4y^2 + 54x - 16y + 61 = 0$ ELLIPSE

$$9x^2 + 54x \quad + 4y^2 - 16y \quad = -61$$
$$9(x^2 + 6x + 9) + 4(y^2 - 4y + 4) = -61 + 81 + 16$$
$$9(x+3)^2 + 4(y-2)^2 = 36$$
$$\frac{(x+3)^2}{4} + \frac{(y-2)^2}{9} = 1$$

12. $-25x^2 + 16y^2 - 150x - 96y - 481 = 0$ HYPERBOLA

$$16y^2 - 96y \quad -25x^2 - 150x \quad = 481$$
$$16(y^2 - 6y + 9) - 25(x^2 + 6x + 9) = 481 + 144 - 225$$
$$16(y-3)^2 - 25(x+3)^2 = 400$$
$$\frac{(y-3)^2}{25} - \frac{(x+3)^2}{16} = 1$$

13. $4x^2 - y^2 + 8x - 12 = 0$ HYPERBOLA

$$4x^2 + 8x \quad -y^2 = 12$$
$$4(x^2 + 2x + 1) - y^2 = 12$$
$$4(x+1)^2 - y^2 = 12$$
$$\frac{(x+1)^2}{3} - \frac{y^2}{12} = 1$$

14. $2x^2 + 2y^2 - 8x + 4y - 2 = 0$ CIRCLE

$$x^2 + y^2 - 4x + 2y - 1 = 0$$
$$x^2 - 4x \quad + y^2 + 2y \quad = 1$$
$$x^2 - 4x + 4 + y^2 + 2y + 1 = 1 + 4 + 1$$
$$(x-2)^2 + (y+1)^2 = 6$$

15. $16x^2 - y^2 + 96x + 8y + 112 = 0$ HYPERBOLA

$$16x^2 + 96x \quad -y^2 + 8y \quad = -112$$
$$16(x^2 + 6x + 9) - (y^2 - 8y + 16) = -112 + 144 - 16$$
$$16(x+3)^2 - (y-4)^2 = 16$$
$$(x+3)^2 - \frac{(y-4)^2}{16} = 1$$

16. $x^2 + 2x - 4y + 9 = 0$ PARABOLA

$$-4y = -x^2 - 2x - 9$$
$$4y = x^2 + 2x + 9$$
$$4y = x^2 + 2x + 1 \quad + 9 - 1$$
$$4y = (x+1)^2 + 8$$
$$y = \frac{1}{4}(x+1)^2 + 2$$

Use the information provided to write each equation in standard form

17. Circle with center (-10, -3) and radius of 6

$$(x+10)^2 + (y+3)^2 = 36$$

18. Circle with endpoints of diameter (-5, -8) and (-9, 6)

$$\text{center}\left(\frac{-5+-9}{2}, \frac{-8+6}{2}\right)$$
$$(-7, -1)$$

7

2

$$(x+7)^2 + (y+1)^2 = 53$$

$$2^2 + 7^2$$
$$4 + 49$$

$10^2 + 8^2 = r^2$ $\Delta x = 10,\ \Delta y = 8$

19. Circle with center at (-6, 0) and point (-16, 8)

$(x+6)^2 + y^2 = 164$

20. Ellipse with Vertices: $(-2, -10), (-18, -10)$ Co-vertices: $(-10, -3), (-10, -17)$

center $(-10, -10)$ Δx

$a^2 = 64$
$c^2 = 49$

$$\frac{(x+10)^2}{64} + \frac{(y+10)^2}{25} = 1$$

$c^2 = a^2 - b^2$
$49 = 64 - b^2$
$25 = b^2$

Δx

21. Ellipse with Vertices: $(-2, -4), (-12, -4)$ Foci: $(-3, -4), (-11, -4)$

center $(-7, -4)$

$a^2 = 25$
$c^2 = 16$

$$\frac{(x+7)^2}{25} + \frac{(y+4)^2}{9} = 1$$

$c^2 = a^2 - b^2$
$16 = 25 - b^2$
$b^2 = 9$

22. Ellipse with Foci: $(4, 11), (4, 1)$ Co-vertices: $(16, 6), (-8, 6)$

Δy $c^2 = \cancel{100}\ 25$
$b^2 = 144$

center $(4, 6)$

$$\frac{(x-4)^2}{144} + \frac{(y-6)^2}{\cancel{244}\ 169} = 1$$

$c^2 = a^2 - b^2$
$\cancel{100}\ 25 = a^2 - 144$
$\cancel{244}\ 169 = a^2$

23. Hyperbola with Vertices: $(-8, 10), (-8, 2)$ Foci: $(-8, 11), (-8, 1)$

center $(-8, 6)$ Δy

$$\frac{(y-6)^2}{16} - \frac{(x+8)^2}{9} = 1$$

$c^2 = 25$
$a^2 = 16$
$c^2 = a^2 + b^2$
$25 = 16 + b^2$
$b^2 = 9$

24. Hyperbola with Vertices (8,1) (-4,1) Foci: (12,1) (-8,1)

Δx center $(2, 1)$

$a^2 = 36$
$c^2 = 100$

$c^2 = a^2 + b^2$
$100 = 36 + b^2$
$64 = b^2$

$$\frac{(x-2)^2}{36} - \frac{(y-1)^2}{64} = 1$$

25. Parabola with Vertex: (-8,-4) and Focus: (-8,1)

Δy

$$y = \frac{1}{20}(x+8)^2 - 4$$

26. Parabola with Vertex: (10,-5) and Directrix: x = 6

Δx

$$x = \frac{1}{16}(y+5)^2 + 10$$

Targeted Review Problems

1. $\frac{(x+2)^2}{25} + \frac{(y+3)^2}{16} = 1$

$a = 5$
$b = 4$

Horizontal because larger denominator with x^2

ELLIPSE
+ SIGN
different coefficients

Center $(-2, -3)$

add/subtract "a" value

Vertices $(-2 \pm 5, -3)$

Go on horizontal axis

$\boxed{(3,-3) \text{ and } (-7,-3)}$

add/subtract "b" value

Co-vertices $(-2, -3 \pm 4)$

Go on vertical axis

$\boxed{(-2, 1), (-2, -7)}$

Foci : Find c value first!

Go on horizontal axis

$c^2 = a^2 - b^2$

$c^2 = 25 - 16$

$c^2 = 9 \therefore c = 3$

add/subtract "c" value

$(-2 \pm 3, -3)$

$\boxed{(1,-3) \text{ and } (-5,-3)}$

Go in this order to classify:

Is it a parabola?

NO (either x^2 or y^2, not both)

Is it a hyperbola?

No (either x^2 or y^2 is negative when on same side of equation)

Is it an ellipse?

NO ($x^2 + y^2$ have different coefficients)

Is it a circle?

($x^2 + y^2$ have same coefficients).

2. $\dfrac{(x+3)^2}{4} - \dfrac{(y+4)^2}{1} = 1$

- NOT a parabola
- hyperbola because of $-$ sign
- Graph opens horizontally because x^2 is the leading term (positive)

Center $(-3,-4)$

$a=2$
$b=1$

$c^2 = a^2 + b^2$
$c^2 = 4+1$
$c^2 = 5$
$c = \sqrt{5}$

add/subtract "a"

Vertices: $(-3 \pm 2, -4)$

Go on horizontal axis

$(-1,-4)$ and $(-5,-4)$

covertices $(-3, -4 \pm 1)$ add/subtract "b"

Go on vertical axis

$(-3,-3)$ and $(-3,-5)$

foci $(-3 \pm \sqrt{5}, -4)$ add/subtract "c"

Go on horizontal axis

3. $x = \frac{1}{12}(y+2)^2 + 1$

$y=-2$ $\quad$ $x=1$

- parabola since there is no x^2
- Horizontal opening right because $x = +y^2$

vertex $(1,-2)$

focus must be to right of vertex

Let $4p = 12$, then $p = 3$

Add 3 to x-coordinate (of vertex) to get focus $(1+3, -2)$
$(4,-2)$

Subtract 3 from x-coordinate to get equation of directrix $(1, -2)$
-3

$\rightarrow x = -2$

5. $(x-5)^2+(y+7)^2=4$ circle

center $(5,-7)$

radius $= 2$

9. $x^2+y^2-6x-2y+1=0$

circle

x^2+y^2 have same coefficient $(+1)$

$x^2-6x \quad +y^2-2y \quad = -1$

$x^2-6x+9 \quad +y^2-2y+1 \quad = -1$

$\left.\begin{matrix}+9\\+1\end{matrix}\right\}$ balance the equation

$(x-3)^2+(y-1)^2=9$

center $(3,1)$; $r=3$

Ellipse (x^2+y^2 have different coefficients)

11. $9x^2+4y^2+54x-16y+61=0$

Reorder terms

$9x^2+54x \quad +4y^2-16y \quad = -61$

$9(x^2+6x+9)+4(y^2-4y+4)$

$\left.\begin{matrix}+81\\+16\end{matrix}\right\}$ balance equation

factor GCF from x terms

$9(x+3)^2+4(y-2)^2=36$

factor GCF from y terms

$$\frac{(x+3)^2}{4}+\frac{(y-2)^2}{9}=1$$

$c^2=a^2-b^2$
$c^2=9-4$
$c^2=5$
$c=\sqrt{5}$

center $(-3,2)$

Graph is vertical since y^2 has larger denominator

vertices $(-3,2\pm3)$

$(-3,5)$ and $(-3,-1)$

Co-vertices $(-3\pm2,2)$

$(-1,2)$ $(-5,2)$

foci $(-3,2\pm\sqrt{5})$

10. $y^2 - 12x - 72 = 0$

Parabola (no x^2)

$\cancel{y^2 - 12x = 72}$

Isolate the linear term

$\cancel{y^2 = 12x + 72}$

$-\frac{1}{12}\left(-12x = -y^2 + 72\right)$

$x = \frac{1}{12}y^2 - 6$

Vertex: $(-6, 0)$

Same as
$x = \frac{1}{12}(y+0)^2 - 6$

15. $16x^2 - y^2 + 96x + 8y + 112 = 0$

$16x^2 + 96x \quad -y^2 + 8y \quad = -112$

$16(x^2 + 6x + 9) - (y^2 - 8y + 16) = \left.\begin{matrix} -112 \\ +144 \\ -16 \end{matrix}\right\}$ balance the equation

factor out the GCF for $x^2 + y^2$

For hyperbola, at a minimum, factor out the negative sign

$16(x+3)^2 - (y-4)^2 = 16$

$(x+3)^2 - \frac{(y-4)^2}{16} = 1$

Horizontal opening since x^2 is positive (leading term)

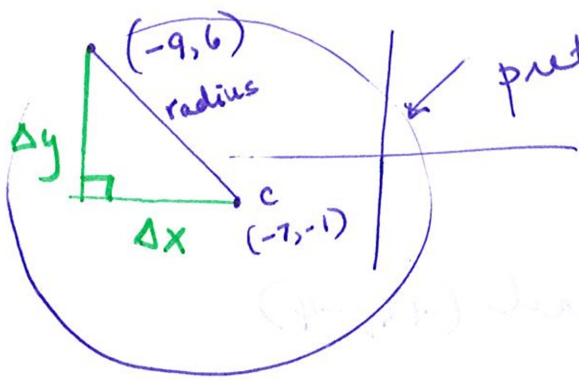
Center $(-3, 4)$

18. $(-5,-8)$ $(-9,6)$ endpoints of diameter

$\left(\frac{-5+-9}{2}, \frac{-8+6}{2}\right)$ find midpoint (center)

$(-7,-1)$ is center

Use the center and either point to find radius



pretend this is the circle with center $(-7,-1)$ and passing through $(-9,6)$

The side lengths of the right triangle are the distances between x values ($\Delta x = 2$) and between y values ($\Delta y = 7$)

Compare

$$\begin{array}{c} (-7,-1) \\ (-9,6) \\ \hline \Delta x = 2,\ \Delta y = 7 \end{array}$$

$a^2 + b^2 = c^2$

is same as

$(\Delta x)^2 + (\Delta y)^2 = r^2$

$2^2 + 7^2 = r^2$

$4 + 49 = r^2$

$53 = r^2$

$(x+7)^2 + (y+1)^2 = 53$

19. circle with center

$(-6,0)$

given a point $(-16,8)$

$\Delta x=10 \quad \Delta y=8$

$10^2+8^2=r^2$

$100+64=r^2$

$164=r^2$

$(x+6)^2+y^2=164$

21. Ellipse given

vertices $(-2,-4)$ and $(-12,-4)$

Foci $(-3,-4)$ and $(-11,-4)$

Center is MIDPOINT OF EITHER pair

$\left(\frac{-2+-12}{2}, \frac{-4+-4}{2}\right)$

$(-7,-4)$

Graph is horizontal because Δx

Distance between vertices is $10(\Delta x)$

$\therefore a=5$ and $a^2=25$

Distance between Foci is $8\ (\Delta x)$

$\therefore c=4$ and $c^2=16$

$c^2=a^2-b^2$

$16=25-b^2$

$-9=-b^2$

$b^2=9$

$\frac{(x+7)^2}{25}+\frac{(y+4)^2}{9}=1$

23. Hyperbola

GRAPH is vertical

vertices $(-8,10)$ $(-8,2)$

Foci $(-8,11)$ $(-8,1)$

$\Delta y = 8$

$\therefore a = 4$

$a^2 = 16$

$\Delta y = 10$

$\therefore c = 5$ and $c^2 = 25$

$c^2 = a^2 + b^2$

$25 = 16 + b^2$

$9 = b^2$

center $\left(\frac{-8+-8}{2}, \frac{10+2}{2}\right)$

$(-8,6)$ y must lead!

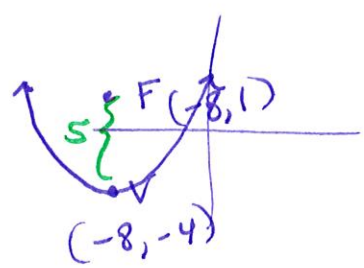
$$\frac{(y-6)^2}{16} - \frac{(x+8)^2}{9} = 1$$

25. parabola

vertex $(-8,-4)$ focus $(-8,1)$

$\Delta y = 5$ this is p!

Graph is vertical; opens upward →



use $y = x^2$

$$y = \frac{1}{4\cdot 5}(x+8)^2 - 4$$

$$y = \frac{1}{20}(x+8)^2 - 4$$