Matrices

- A matrix is a rectangular array of rows and columns.

$$A = \begin{bmatrix} 2 & 6 & -8 \\ 7 & 3 & 4 \end{bmatrix}$$

dimensions are 2×3

rows columns

$$B = \begin{bmatrix} 0 \\ -6 \\ 1 \end{bmatrix}$$

dimensions are 3×1

rows column

$$C = \begin{bmatrix} 8 \end{bmatrix}$$

dimensions are 1×1

Identifying an entry

A_{21} Find the entry in matrix A, row 2, column 1

row 2 column 1

$A_{21} = 7$

Find $B_{31} = 1$

Find $A_{12} = 6$

row 1 column 2

* In order to add/subtract matrices the dimensions must be identical! Simply combine corresponding entries.

Ex $\begin{bmatrix} 2 & 7 \\ -1 & 3 \end{bmatrix} + \begin{bmatrix} 5 & 6 \\ 0 & 4 \end{bmatrix}$

2×2 $\quad$ 2×2

Is the sum defined? yes! the dimensions are the same.

$\begin{bmatrix} 7 & 13 \\ -1 & 7 \end{bmatrix}$ ANSWER

Ex $\begin{bmatrix} 9 \\ 0 \\ -11 \end{bmatrix} - \begin{bmatrix} 5 \\ -4 \\ -2 \end{bmatrix}$

3×1 $\quad$ 3×1

First verify that the dimensions are the same.

$\begin{bmatrix} 9-5 \\ 0--4 \\ -11--2 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \\ -9 \end{bmatrix}$ Answer

Ex $\begin{bmatrix} 4 \\ -7 \end{bmatrix} + \begin{bmatrix} 8 & 1 \end{bmatrix}$ UNDEFINED

The dimensions

Scalar Multiplication

A real number multiplied to a matrix (think distributive property!)

Ex $4\begin{bmatrix} 6 & 7 \\ -2 & -1 \end{bmatrix}$ multiply every entry by 4

scalar

$\begin{bmatrix} 24 & 28 \\ -8 & -4 \end{bmatrix}$ Answer

Ex SIMPLIFY COMPLETELY

$8\begin{bmatrix} 6 & -5 \end{bmatrix} - 4\begin{bmatrix} 3 & 1 \end{bmatrix}$ Follow order of operations

MATRIX ADDITION

do scalar multiplication first

$\begin{bmatrix} 48 & -40 \end{bmatrix} + \begin{bmatrix} -12 & -4 \end{bmatrix}$

$\begin{bmatrix} 36 & -44 \end{bmatrix}$ Answer

$A = \begin{bmatrix} -8 & 6 \\ 2 & -1 \end{bmatrix}$ $B = \begin{bmatrix} -6 & 13 \\ 4 & 7 \end{bmatrix}$ $C = \begin{bmatrix} -1 & 2 \\ 4 & 3 \\ -7 & 6 \end{bmatrix}$ $D = \begin{bmatrix} -5 & -2 & -1 & 0 \\ 3 & -3 & 2 & 4 \end{bmatrix}$

1. What are the dimensions of matrix A? 2x2 matrix B? 2x2

matrix C? 3x2 matrix D? 2x4

2. Identify the following entries.

A_{21} 2 B_{22} 7 C_{12} 2 D_{23} 2

3. Which of the matrices above can be added or subtracted? A and B only

4. Is C - D defined? Explain. no; the dimensions are not the same

Perform the indicated operation.

5. $2\begin{bmatrix} 2 & 8 & -1 \\ 0 & 2 & 0 \end{bmatrix} - \begin{bmatrix} 2 & 6 & 3 \\ -1 & -1 & 3 \end{bmatrix}$

$\begin{bmatrix} 4 & 16 & -2 \\ 0 & 4 & 0 \end{bmatrix} + \begin{bmatrix} -2 & -6 & -3 \\ 1 & 1 & -3 \end{bmatrix}$

$\begin{bmatrix} 2 & 10 & -5 \\ 1 & 5 & -3 \end{bmatrix}$

6. $3\begin{bmatrix} 1 & -4 \\ 3 & 8 \end{bmatrix} - 5\begin{bmatrix} 4 & 7 \\ 0 & -1 \end{bmatrix}$

$\begin{bmatrix} 3 & -12 \\ 9 & 24 \end{bmatrix} + \begin{bmatrix} -20 & -35 \\ 0 & 5 \end{bmatrix}$

$\begin{bmatrix} -17 & -47 \\ 9 & 29 \end{bmatrix}$

7. $3\begin{bmatrix} (x+2) \\ (y-3) \end{bmatrix} + \begin{bmatrix} 5 \\ -4 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 7 \\ 1 \end{bmatrix}$

$3y - 9 - 4 = \frac{1}{2}$

$3y - 13 = \frac{1}{2}$

$3y = \frac{27}{2}$

$y = \frac{27}{6} = \frac{9}{2}$

$\begin{bmatrix} 2 & 1 \end{bmatrix}$ $\begin{bmatrix} 2 & -3 \end{bmatrix}$ $\begin{bmatrix} -1 & 7 \end{bmatrix}$

$X = \begin{bmatrix} 9 & -16 \\ -6 & -7 \end{bmatrix}$

8. $4A + X = B$

$X = B - 4A$

$X = \begin{bmatrix} 5 & -4 \\ 2 & -3 \end{bmatrix} - 4\begin{bmatrix} -1 & 3 \\ 2 & 1 \end{bmatrix}$

$X = \begin{bmatrix} 5 & -4 \\ 2 & -3 \end{bmatrix} + \begin{bmatrix} 4 & -12 \\ -8 & -4 \end{bmatrix}$

$X = \begin{bmatrix} -5 & 6 \\ -7 & 16 \end{bmatrix}$

9. $X + 3B = C$

$X = C - 3B$

$X = \begin{bmatrix} 10 & -6 \\ -1 & 7 \end{bmatrix} - 3\begin{bmatrix} 5 & -4 \\ 2 & -3 \end{bmatrix}$

$\begin{bmatrix} 10 & -6 \\ -1 & 7 \end{bmatrix} + \begin{bmatrix} -15 & 12 \\ -6 & 9 \end{bmatrix}$

$\begin{bmatrix} 12 & -\frac{17}{2} \\ 6 & -7 \end{bmatrix}$

$\frac{1}{2}\begin{bmatrix} 24 & -17 \\ 12 & -14 \end{bmatrix}$

$\frac{1}{2}\left(\begin{bmatrix} 25 & -20 \\ 10 & -15 \end{bmatrix} + \begin{bmatrix} -1 & 3 \\ 2 & 1 \end{bmatrix}\right)$

10. $2X - A = 5B$

$2X = 5B + A$

$X = \dfrac{5B + A}{2}$ OR $X = \frac{1}{2}(5B + A)$

$\frac{1}{2}\left(5\begin{bmatrix} 5 & -4 \\ 2 & -3 \end{bmatrix} + \begin{bmatrix} -1 & 3 \\ 2 & 1 \end{bmatrix}\right)$

Perform the indicated operations. If not possible, give an explanation.

11. $\begin{bmatrix} 2 & -1 & -4 \\ 3 & 8 & 1 \end{bmatrix} - \begin{bmatrix} 6 & 5 & 3 \\ 5 & 5 & -3 \end{bmatrix}$ $\begin{bmatrix} -4 & -6 & -7 \\ -2 & 3 & 4 \end{bmatrix}$	12. $3\begin{bmatrix} -1 & 2 \\ 4 & -2 \end{bmatrix} + 2\begin{bmatrix} 3 & -1 \\ 2 & 3 \end{bmatrix}$ $\begin{bmatrix} -3 & 6 \\ 12 & -6 \end{bmatrix} + \begin{bmatrix} 6 & -2 \\ 4 & 6 \end{bmatrix}$ $\begin{bmatrix} 3 & 4 \\ 16 & 0 \end{bmatrix}$
13. $5\begin{bmatrix} 2 & -1 \\ 4 & 3 \\ -2 & 1 \end{bmatrix} - \begin{bmatrix} -3 & -4 \\ 2 & 1 \\ -1 & 0 \end{bmatrix}$ $\begin{bmatrix} 10 & -5 \\ 20 & 15 \\ -10 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 4 \\ -2 & -1 \\ 1 & 0 \end{bmatrix}$ $\begin{bmatrix} 13 & -1 \end{bmatrix}$	14. $\begin{bmatrix} 6 & -4 \\ 2 & 3 \end{bmatrix} + 4\left(\frac{1}{2}\begin{bmatrix} 10 & -4 \\ -2 & 6 \end{bmatrix} - 3\begin{bmatrix} -1 & 2 \\ -3 & 1 \end{bmatrix}\right)$ start here $4\left(\begin{bmatrix} 5 & -2 \\ -1 & 3 \end{bmatrix} + \begin{bmatrix} 3 & -6 \\ 9 & -3 \end{bmatrix}\right)$ add these $4\left(\begin{bmatrix} 8 & -8 \\ 8 & 0 \end{bmatrix}\right.$

15. $\begin{bmatrix} -1 & 3y \end{bmatrix} - \begin{bmatrix} -1 & 39 \end{bmatrix}$

$2x = -12$

$x = -6$

$4 = 4$ ✓

$-1 = -1$ ✓

$3y = 39$

$y = 13$

16. $4\begin{bmatrix} 1 & y \end{bmatrix} - \begin{bmatrix} 4 & 6y-7 \end{bmatrix}$

Distribute the scalar first

$\begin{bmatrix} 4x-4 & 8 \\ 4 & 4y \end{bmatrix} = \begin{bmatrix} 20 & 8 \\ 4 & 6y-7 \end{bmatrix}$

$4x - 4 = 20$

$4x = 24$

$x = 6$

$8 = 8$ ✓

$4 = 4$ ✓

$4y = 6y - 7$

$-2y = -7$

$y = 3.5$

17. $\begin{bmatrix} 6+4z & 2x+7 \\ 2y+8 & 4w-1 \end{bmatrix} = \begin{bmatrix} 8 & 11 \\ y-3 & 2+\frac{w}{3} \end{bmatrix}$

$6 + 4z = 8$

$4z = 2$

$z = \frac{1}{2}$

$2x + 7 = 11$

$2x = 4$

$x = 2$

$2y + 8 = y - 3$

$y + 8 = -3$

$y = -11$

$4w - 1 = 2 + \frac{w}{3}$

$12w - 3 = 6 + w$

$11w - 3 = 6$

$11w = 9$

$w = \frac{9}{11}$

18. $\begin{bmatrix} 5x-7 & 11 \\ 5 & 23 \end{bmatrix} = \begin{bmatrix} 8 & 21-m \\ r^3-3 & 4y+x \end{bmatrix}$

$5x - 7 = 8$

$5x = 15$

$x = 3$

$11 = 21 - m$

$-10 = -m$

$m = 10$

$5 = r^3 - 3$

$8 = r^3$

$r = 2$

substitute

$23 = 4y + x$

$23 = 4y + 3$

$4y = 20$

$y = 5$

Essential Questions

19. How do we determine if matrices can be added or subtracted?

If their dimensions are the same

20. How do we determine if two matrices are equal?

If their dimensions are the same and corresponding entries are equal.

21. How do we determine the dimensions of a matrix?