

# Applications of Determinants

Ex 1 Finding the area of a triangle

use the formula

$$A = \pm \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

simply choose the sign that gives you a positive value  
divide by 2

augment with a column of 1s

place the x,y pairs into the rows (order top to bottom does not matter)

Ex Find the area of the triangle defined by

(8, 1), (5, -3) and (-7, 2)

$$A = \pm \frac{1}{2} \begin{vmatrix} 8 & 1 & 1 \\ 5 & -3 & 1 \\ -7 & 2 & 1 \end{vmatrix}$$

$$A = \pm \frac{1}{2} [(-24 + 7 + 10) - (21 + 16 + 5)]$$

$$A = \pm \frac{1}{2} [-21 - 42]$$

$$A = \pm \frac{1}{2} (-63)$$

$$A = -\frac{1}{2} (-63) = 31.5 \text{ units}^2$$

choose  $-\frac{1}{2}$  to ensure positive outcome

\* If the determinant is

equal to zero, then no triangle exists.

This means the points are collinear.

Ex 2 Writing the equation of a line in standard form ( $Ax + By = C$ ).

Write the equation of the line

passing through (4, -3) and (8, 1).

$$\left| \begin{array}{ccc|cc} 4 & -3 & 1 & 4 & -3 \\ 8 & 1 & 1 & 8 & 1 \\ x & y & 1 & x & y \end{array} \right| = 0$$

Add a column of 1s and a 3rd coordinate pair (x, y)

$$[(4 - 3x + 8y) - (x + 4y - 24)] = 0$$

$$4 - 3x + 8y - x - 4y + 24 = 0$$

$$-4x + 4y + 28 = 0$$

$$\text{all can divide by } 4 \quad 4x - 4y = 28 \quad \boxed{x - y = 7}$$

#1-9. Evaluate the determinant of the following matrices.

1. 
$$\begin{bmatrix} -4 & 2 \\ 8 & 0 \end{bmatrix}$$

$$(-4)(0) - (8)(2)$$
$$\boxed{\frac{0-16}{-16}}$$

2. 
$$\begin{bmatrix} 1 & 4 \\ 5 & 1 \end{bmatrix}$$

$$(1)(1) - (5)(4)$$
$$\boxed{\frac{1-20}{-19}}$$

3. 
$$\begin{bmatrix} -6 & 5 \\ 8 & 10 \end{bmatrix}$$

$$(-6)(10) - (8)(5)$$
$$\frac{-60-40}{-100}$$

4. 
$$\begin{bmatrix} 5 & 9 \\ 8 & 1 \end{bmatrix}$$

$$(5)(1) - (8)(9)$$
$$\boxed{\frac{5-72}{-67}}$$

5. 
$$\begin{bmatrix} 7 & -7 \\ 11 & 4 \end{bmatrix}$$

$$(7)(4) - (11)(-7)$$
$$\boxed{\frac{28+77}{105}}$$

6. 
$$\begin{bmatrix} 1 & 3 \\ -2 & -6 \end{bmatrix}$$

$$(1)(-6) - (-2)(3)$$
$$\boxed{0}$$

7. 
$$\begin{bmatrix} 3 & 2 & -5 \\ 6 & 0 & -1 \\ 0 & -1 & 3 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 6 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\left[ (3 \cdot 0 \cdot 3) + (2 \cdot -1 \cdot 0) + (-5 \cdot 6 \cdot -1) \right] -$$
$$\left[ (0 \cdot 0 \cdot 5) + (-1 \cdot -1 \cdot 3) + (3 \cdot 6 \cdot 2) \right]$$
$$\left[ 0 + 0 + 30 \right] - \left[ 0 + 3 + 36 \right]$$
$$\boxed{\frac{30-39}{-9}}$$

8. 
$$\begin{bmatrix} -1 & 2 & 7 \\ 2 & -1 & -1 \\ 3 & 5 & 2 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ 2 & -1 \\ 3 & 5 \end{bmatrix}$$

9. 
$$\begin{bmatrix} 1 & 2 & 1 \\ 6 & 5 & 0 \\ 1 & 4 & -2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 6 & 5 \\ 1 & 4 \end{bmatrix}$$

$$\left[ (-1 \cdot -1 \cdot 2) + (2 \cdot -1 \cdot 3) + (7 \cdot 2 \cdot 5) \right] -$$

$$\left[ -10 + 0 + 24 \right] - [5 + 0 - 24]$$

$$\left[ (3 \cdot -1 \cdot 7) + (5 \cdot -1 \cdot -1) + (2 \cdot 2 \cdot 2) \right]$$

$$\boxed{\frac{14-19}{-5}}$$

$$\left[ 2 + -6 + 70 \right] - \left[ -21 + 5 + 8 \right] \rightarrow \boxed{74}$$

#10-12. Find the value of the variable in each of the following equations.

10. 
$$\begin{vmatrix} 2 & 6 \\ 1 & x \end{vmatrix} = 2$$

$$(2)(x) - (1)(6) = 2$$

$$2x - 6 = 2$$

$$2x = 8$$

$$x = 4$$

11. 
$$\begin{vmatrix} x & 3 \\ -4 & x \end{vmatrix} = 7x$$

$$(x)(x) - (-4)(3) = 7x$$

$$x^2 + 12 = 7x$$

$$\cancel{x^2 - 7x + 12 = 0}$$

$$(x-4)(x-3) = 0$$

$$x = 4, x = 3$$

12. 
$$\begin{vmatrix} x & 3 & -1 \\ 2 & 1 & -2 \\ 4 & 1 & x \end{vmatrix} = 10$$

$$\begin{array}{ccc|cc} x & 3 & -1 & x & 3 \\ 2 & 1 & -2 & 2 & 1 \\ 4 & 1 & x & 4 & 1 \end{array} = 10$$

$$\left[ x^2 - 24 - 2 \right] - \left[ -4 - 2x + 6x \right] = 10$$
$$\left[ x^2 - 26 \right] - \left[ -4 + 4x \right] = 10$$

$$\rightarrow x^2 - 26 + 4 - 4x = 10$$

$$x^2 - 22 - 4x = 10$$

$$x^2 - 4x - 32 = 0$$

$$(x-8)(x+4) = 0$$

$$\boxed{\begin{array}{l} x=8 \\ x=-4 \end{array}}$$

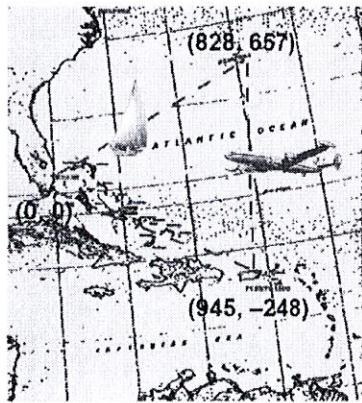
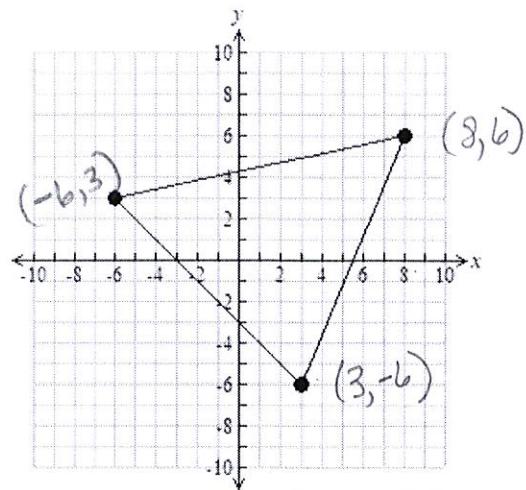
Show all work in the space provided. Answers alone are unacceptable.

1. Find the area of the triangle shown at right.

$$A = \pm \frac{1}{2} \begin{vmatrix} -6 & 3 & 1 \\ 8 & 6 & 1 \\ 3 & -6 & 1 \end{vmatrix} = \begin{vmatrix} -6 & 3 \\ 8 & 6 \\ 3 & -6 \end{vmatrix}$$

$$A = \pm \frac{1}{2} [(-36 + 9 - 48) - (18 + 36 + 24)]$$

$$A = \pm \frac{1}{2} [(-75) - (78)] = 76.5 \text{ units}^2$$



2. Throughout history, ships and planes have been disappearing without a trace in an area known as the Bermuda Triangle. The boundary lines of the Bermuda Triangle connect Miami, Florida, San Juan, Puerto Rico, and the island of Bermuda.

Let Miami, Florida, be represented on the map as the origin (0, 0). If Bermuda is located 828 miles east and 657 miles north and San Juan is located 945 miles east and 248 miles south, estimate, to the nearest mile, the area of the Bermuda Triangle.

$$(x, y) \leftarrow \begin{array}{l} +y = \text{NORTH} \\ -y = \text{SOUTH} \end{array}$$

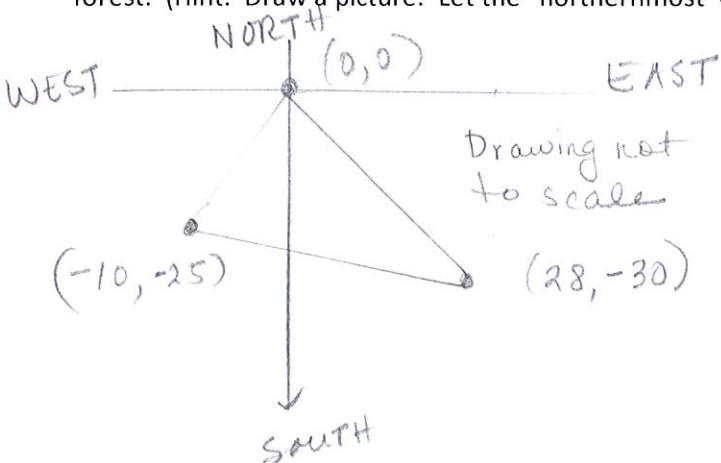
$$\begin{array}{l} +x = \text{EAST} \\ -x = \text{WEST} \end{array}$$

$$A = \pm \frac{1}{2} \begin{vmatrix} 0 & 0 & 1 \\ 828 & 657 & 1 \\ 945 & -248 & 1 \end{vmatrix} = \begin{vmatrix} 0 & 0 \\ 828 & 657 \\ 945 & -248 \end{vmatrix} - \begin{vmatrix} 945 & 657 & 1 \\ -248 & 1 & 0 \\ 1 & 828 & 0 \end{vmatrix}$$

$$A = \pm \frac{1}{2} [(-205344) - (620865)]$$

$$\boxed{A \approx 413,105 \text{ miles}^2} \quad A = \pm \frac{1}{2} (-826209)$$

3. A large region of forest has been infested with gypsy moths. The region is roughly triangular. From the northernmost vertex of the region, the distance to the other vertices is 28 miles east and 30 miles south (for the easternmost vertex) and 10 miles west and 25 miles south (for the westernmost vertex). Approximate the number of square miles of infested forest. (Hint: Draw a picture. Let the "northernmost vertex" be located at (0, 0).



$$A = \pm \frac{1}{2} \begin{vmatrix} 0 & 0 & 1 \\ -10 & -25 & 1 \\ 28 & -30 & 1 \end{vmatrix} = \begin{vmatrix} 0 & 0 \\ -10 & -25 \\ 28 & -30 \end{vmatrix} - \begin{vmatrix} 28 & -30 & 1 \\ -10 & -25 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$A = \pm \frac{1}{2} [(0 + 0 + 300) - (-700 + 0 + 0)]$$

$$A = \pm \frac{1}{2} [300 - (-700)]$$

$$A = \pm \frac{1}{2} (1000) \quad A \approx 500 \text{ miles}^2$$

If the determinant is equal to zero, then the points are collinear!

Are the following sets of points for *collinear*? Prove using a matrix determinant.

4.  $(-8, 1), (9, 5)$ , and  $(-25, -3)$

$$\begin{vmatrix} -8 & 1 & 1 \\ 9 & 5 & 1 \\ -25 & -3 & 1 \end{vmatrix} \begin{matrix} -8 & 1 \\ 9 & 5 \\ -25 & -3 \end{matrix}$$

$$[(-40 + -25 + -27) - (-125 + 24 + 9)]$$

$$(-92) - (-92)$$

$$-92 + 92 = 0 \therefore \text{these points are collinear}$$

5.  $(6, 4), (-2, 5)$  and  $(3, 8)$

$$\begin{vmatrix} 6 & 4 & 1 \\ -2 & 5 & 1 \\ 3 & 8 & 1 \end{vmatrix} \begin{matrix} 6 & 4 \\ -2 & 5 \\ 3 & 8 \end{matrix}$$

$$[(30 + 12 - 16) - (15 + 48 - 8)]$$

$$26 - 55$$

-29 ∵ These points are not collinear i.e. they form a triangle

Use a matrix determinant to write the equation of the line passing through the points given.

6.  $(1, 4)$  and  $(7, -3)$

$$\begin{vmatrix} 1 & 4 & 1 \\ 7 & -3 & 1 \\ x & y & 1 \end{vmatrix} \begin{matrix} 1 & 4 \\ 7 & -3 \\ x & y \end{matrix} = 0$$

7.  $(0, 2)$  and  $(5, -1)$

$$\begin{vmatrix} 0 & 2 & 1 \\ 5 & -1 & 1 \\ x & y & 1 \end{vmatrix} \begin{matrix} 0 & 2 \\ 5 & -1 \\ x & y \end{matrix} = 0$$

$$(-3 + 4x + 7y) - (-3x + y + 28) = 0$$

$$-3 + 4x + 7y + 3x - y - 28 = 0$$

$$7x + 6y - 31 = 0$$

$$7x + 6y = 31$$

$$(0 + 2x + 5y) - (-x + 0 + 10) = 0$$

$$2x + 5y + x - 10 = 0$$

$$3x + 5y = 10$$