Precalculus
Matrices Review

Name_____

Determine the dimensions of the matrices:

1. $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \\ 0 & 1 & 2 \\ 3 & 5 & 4 \end{bmatrix}$ 5 × 3

2. $\begin{bmatrix} 5 & 3 & 0 \end{bmatrix}$ 1 × 3

3. Find x & y $\begin{bmatrix} 4 & 9 & 4 \\ 13 & 15 & 6 \\ 2 & 7 & 0 \end{bmatrix} = \begin{bmatrix} 4 & 2x+1 & 4 \\ 13 & 15 & 6 \\ 2 & 3y-5 & 0 \end{bmatrix}$

Set corresponding entries equal to each other

$9 = 2x+1$ $\quad 3y-5=7$

$8 = 2x$ $\quad 3y = 12$

$4 = x$ $\quad y = 4$

Perform the indicated operation if possible:

4. $6\begin{bmatrix} 4 & 6 \\ 2 & 0 \end{bmatrix}$ $\begin{bmatrix} 24 & 36 \\ 12 & 0 \end{bmatrix}$

5. $\begin{bmatrix} 3 & 4 & -1 \\ 0 & 5 & 2 \end{bmatrix} - \frac{3}{2}\begin{bmatrix} 1 & 10 & 5 \\ -1 & 2 & 7 \end{bmatrix}$

Distribute -1.5 to all entries; replace - sign with +

$\begin{bmatrix} 3 & 4 & -1 \\ 0 & 5 & 2 \end{bmatrix} + \begin{bmatrix} -1.5 & -15 & -7.5 \\ 0 & -3 & -10.5 \end{bmatrix}$

$\begin{bmatrix} 1.5 & -11 & -8.5 \\ 0 & 2 & -8.5 \end{bmatrix}$

6. $\begin{bmatrix} -2 & 0 & 2 \\ -4 & 0 & 4 \end{bmatrix} * \begin{bmatrix} 1 & 7 \\ 4 & 5 \\ 10 & -1 \end{bmatrix}$

2 × 3 and 3 × 2

C1 C2

$= \begin{bmatrix} -2+0+20 & -14+0+-2 \\ -4+0+40 & -28+0+-4 \end{bmatrix}$ R1 R2

$\rightarrow \begin{bmatrix} 18 & -16 \\ 36 & -32 \end{bmatrix}$

7. Find A^{-1} if $A = \begin{bmatrix} -5 & 1 \\ -2 & -3 \end{bmatrix}$

Step 1: Find $|A|$: $(-5)(-3)-(-2)(1)$

$15+2$

17

Step 2: $\frac{1}{17}\begin{bmatrix} -3 & -1 \\ 2 & -5 \end{bmatrix}$

Step 3: $\begin{bmatrix} -\frac{3}{17} & -\frac{1}{17} \\ \frac{2}{17} & -\frac{5}{17} \end{bmatrix}$

8. Find A^{-1}, $A = \begin{bmatrix} 2 & 4 \\ 0 & -7 \end{bmatrix}$

Step 1

$|A| = (2)(-7)-(0)(4)$

$-14-0$

-14

$\rightarrow \frac{1}{-14}\begin{bmatrix} -7 & -4 \\ 0 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} \frac{7}{14} & \frac{4}{14} \\ 0 & -\frac{2}{14} \end{bmatrix} \rightarrow \begin{bmatrix} \frac{1}{2} & \frac{2}{7} \\ 0 & -\frac{1}{7} \end{bmatrix}$

9. Show that $A = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$ are Inverses

Show that $AB = I_{2\times 2}$

$\begin{bmatrix} 2-1 & 1-1 \\ -2+2 & -1+2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ✓

10. When would a matrix not have an inverse?

If it is not square; if the determinant = 0

11. Set up a Matrix equation for the system of equations and solve:

$7x - 2y = 4$

$2x - 3y = 12$

$\underset{A}{\begin{bmatrix} 7 & -2 \\ 2 & -3 \end{bmatrix}} \underset{X}{\begin{bmatrix} x \\ y \end{bmatrix}} = \underset{B}{\begin{bmatrix} 4 \\ 12 \end{bmatrix}}$ ← matrix of constants

matrix of coefficients; matrix of variables

find $A^{-1}B$

1. $|A| = (7)(-3) - (2)(-2)$
$-21 + 4$
-17

$X = \frac{1}{-17}\begin{bmatrix} -3 & 2 \\ -2 & 7 \end{bmatrix}\begin{bmatrix} 4 \\ 12 \end{bmatrix}$

$X = \frac{1}{-17}\begin{bmatrix} -12 + 24 \\ -8 + 84 \end{bmatrix}$

$X = \frac{-1}{17}\begin{bmatrix} 12 \\ 76 \end{bmatrix}$

$X = \begin{bmatrix} -\frac{12}{17} \\ \frac{76}{17} \end{bmatrix}$

$\therefore x = \frac{-12}{17}$

$y = \frac{76}{17}$

12. Evaluate $\begin{vmatrix} 4 & 7 \\ 2 & 0 \end{vmatrix}$ $(4)(0) - (2)(7)$

$0 - 14$

-14

13. Evaluate det $\begin{bmatrix} 2 & -3 & 1 \\ 0 & 6 & 0 \\ -1 & 1 & 2 \end{bmatrix}$ $\begin{matrix} 2 & -3 \\ 0 & 6 \\ -1 & 1 \end{matrix}$

$[(2\cdot 6\cdot 2) + (-3\cdot 0\cdot -1) + (1\cdot 0\cdot 1)] - [(-1\cdot 6\cdot 1) + (1\cdot 0\cdot 2) + (2\cdot 0\cdot -3)]$

$[24 + 0 + 0] - [-6 + 0 + 0]$

$24 + 6$

30

14. Find the Area of the region bounded by the given coordinates:

(-13, 3) (5, 20) (19, -4)

$A = \pm\frac{1}{2}\begin{bmatrix} -13 & 3 & 1 \\ 5 & 20 & 1 \\ 19 & -4 & 1 \end{bmatrix}\begin{matrix} -13 & 3 \\ 5 & 20 \\ 19 & -4 \end{matrix}$

$[(-13\cdot 20\cdot 1) + (3\cdot 1\cdot 19) + (1\cdot 5\cdot -4)] - [(19\cdot 20\cdot 1) + (-4\cdot 1\cdot -13) + (1\cdot 5\cdot 3)]$

$A = \pm\frac{1}{2}[(-260 + 57 + -20) - (380 + 52 + 15)]$

$A = \pm\frac{1}{2}[(-223) - (447)]$

$A = \pm\frac{1}{2}(-670)$ → $A = 335$ units^2

15. Determine if the points are collinear (-3,-5) (6, 1) (10, 2)

They are collinear if the determinant is equal to 0.

$\begin{bmatrix} -3 & -5 & 1 \\ 6 & 1 & 1 \\ 10 & 2 & 1 \end{bmatrix}\begin{matrix} -3 & -5 \\ 6 & 1 \\ 10 & 2 \end{matrix}$

$[(-3\cdot 1\cdot 1) + (-5\cdot 1\cdot 10) + (1\cdot 6\cdot 2)] - [(10\cdot 1\cdot 1) + (2\cdot 1\cdot -3) + (1\cdot 6\cdot -5)]$

$[-3 + -50 + 12] - [10 + -6 + -30]$

$-41 - (-26)$

-15 ∴ not collinear

16. The cities of San Francisco (-30, 32), Oakland (-16, 40), and San Jose (0, 0) form a triangular shaped area. Use a determinant to estimate the area of the region formed by the three cities.

$$A = \pm\frac{1}{2}\begin{vmatrix} -30 & 32 & 1 \\ -16 & 40 & 1 \\ 0 & 0 & 1 \end{vmatrix} \quad \begin{matrix} -30 & 32 \\ -16 & 40 \\ 0 & 0 \end{matrix}$$

$$A = \pm\frac{1}{2}\left[(-1200) - (-512)\right]$$

$$A = \pm\frac{1}{2}[-688]$$

$A = 344$ units^2

$$[(-30 \cdot 40 \cdot 1) + (32 \cdot 1 \cdot 0) + (1 \cdot -16 \cdot 0)] - [(0 \cdot 40 \cdot 1) + (0 \cdot 1 \cdot -30) + (1 \cdot -16 \cdot 32)]$$

17. A fruit grower raises two crops, which are shipped to three outlets. This is represented by the matrix A $\begin{bmatrix} 125 & 100 & 75 \\ 100 & 175 & 125 \end{bmatrix}$. The profit per unit is represented by the matrix B $\begin{bmatrix} 3.50 & 6.50 \end{bmatrix}$. Find BA and state what the answer matrix represents.

$$\begin{bmatrix} 3.50 & 6.50 \end{bmatrix}\begin{bmatrix} 125 & 100 & 75 \\ 100 & 175 & 125 \end{bmatrix} =$$

1×2 $\quad$ 2×3

$$\begin{bmatrix} \$1087.50 & \$1487.50 & \$1075 \end{bmatrix}$$

The answer matrix represents the dollar amount of both products stocked at 3 stores

18. The state fair is a popular field trip destination. This year the senior class at High School A and the senior class at High School B both planned trips there. The senior class at High School A rented and filled 8 vans and 8 buses with 240 students. High School B rented and filled 4 vans and 1 bus with 54 students. Every van had the same number of students in it as did the buses. Find the number of students in each van and in each bus by using matrices to solve a system of equations.

$$8V + 8B = 240$$
$$4V + 1B = 54$$

There were 8 students per van; 22 students per bus

$$\begin{bmatrix} 8 & 8 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} V \\ B \end{bmatrix} = \begin{bmatrix} 240 \\ 54 \end{bmatrix}$$

$A \quad X = B$ $\quad$ find $A^{-1}B$

By hand

1. find $|A| = (8 \cdot 1) - (4 \cdot 8)$
$8 - 32$
-24

2. $X = \frac{1}{-24}\begin{bmatrix} 1 & -8 \\ -4 & 8 \end{bmatrix}\begin{bmatrix} 240 \\ 54 \end{bmatrix}$

$$X = -\frac{1}{24}\begin{bmatrix} 240 - 432 \\ -960 + 432 \end{bmatrix}$$

$$X = -\frac{1}{24}\begin{bmatrix} -192 \\ -528 \end{bmatrix}$$

$$X = \begin{bmatrix} 8 \\ 22 \end{bmatrix}$$

Using the given matrices, evaluate the expression.

$$A = \begin{bmatrix} -1 & 3 \\ -4 & 5 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix} \quad C = \begin{bmatrix} -7 & -2 \\ 0 & -3 \end{bmatrix}$$

$$D = \begin{bmatrix} 3 & -1 & 2 \\ 4 & -2 & 0 \\ 0 & -3 & -4 \end{bmatrix} \quad E = \begin{bmatrix} 2 & 5 & -3 \\ 7 & 2 & -6 \\ 1 & -5 & -1 \end{bmatrix}$$

19. A + 2B

$$\begin{bmatrix} -1 & 3 \\ -4 & 5 \end{bmatrix} + \begin{bmatrix} 0 & 4 \\ 6 & -8 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 7 \\ 2 & -3 \end{bmatrix}$$

20. 4D – E

$$\begin{bmatrix} 12 & -4 & 8 \\ 16 & -8 & 0 \\ 0 & -12 & -16 \end{bmatrix} - \begin{bmatrix} 2 & 5 & -3 \\ 7 & 2 & -6 \\ 1 & -5 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 10 & -9 & 11 \\ 9 & -10 & 6 \\ -1 & -7 & -15 \end{bmatrix}$$

21. A(B + C)

22. 2AC + 3AB

Please use your calculator for all these :)

23. Use a matrix determinant to write the standard form equation of the line passing through the points (12, -7) and (5, 3).

$$\begin{vmatrix} 12 & -7 & 1 \\ 5 & 3 & 1 \\ x & y & 1 \end{vmatrix} \quad \begin{matrix} 12 & -7 \\ 5 & 3 \\ x & y \end{matrix} = 0$$

$$[36 - 7x + 5y] - [3x + 12y - 35] = 0$$

$$36 - 7x + 5y - 3x - 12y + 35 = 0$$

$$-10x - 7y + 71 = 0$$

OR $\quad 10x + 7y - 71 = 0$

$10x + 7y = 71$