How Do I Graph an Ellipse?

Horizontal Major Axis	Vertical Major Axis
$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$	$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$

(Note: $a > b > 0$) To find the foci, use the equation $\boxed{c^2 = a^2 - b^2}$

1. $\frac{x^2}{9}+\frac{y^2}{4}=1$ larger denominator associated to x^2

H or V? Horizontal Center $(0,0)$

a = $\sqrt{9}=3$ b = $\sqrt{4}=2$ c = $c^2=a^2-b^2$, $c^2=9-4$, $c^2=5$, $c=\sqrt{5}$

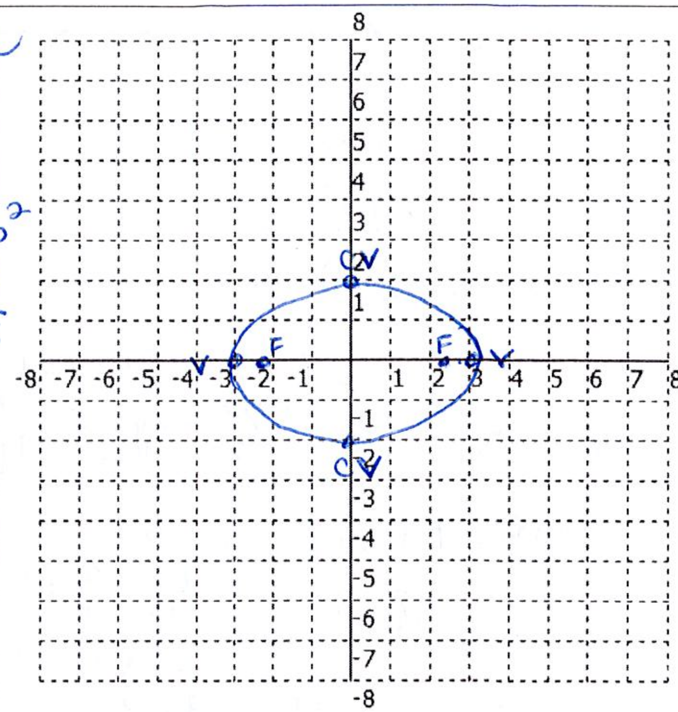
Endpoints of major axis $(0\pm3, 0)$

Endpoints of minor axis $(0, 0\pm2)$

Coordinates of foci $(0\pm\sqrt{5}, 0)$

Length of major axis $2(3)=6$

Length of minor axis $2(2)=4$



2. $x^2+\frac{y^2}{25}=1$ larger denominator associated to y^2

H or V? Vertical Center $(0,0)$

a = $\sqrt{25}=5$ b = $\sqrt{1}=1$ c = $c^2=25-1$, $c^2=24$, $c=2\sqrt{6}$ $c\approx4.9$

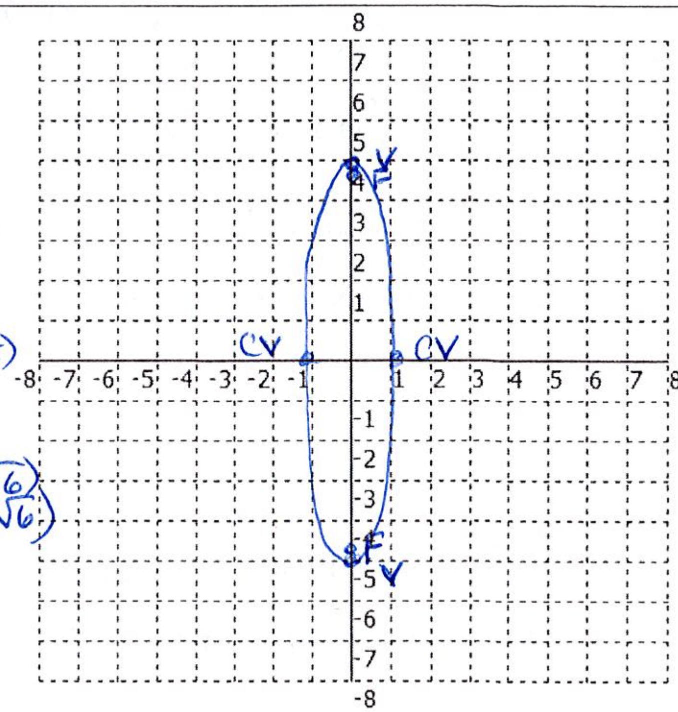
Endpoints of major axis $(0, 0\pm5)\rightarrow(0,5), (0,-5)$

Endpoints of minor axis $(0\pm1, 0)\rightarrow(1,0), (-1,0)$

Coordinates of foci $(0, 0\pm2\sqrt{6})\rightarrow (0, 2\sqrt{6}), (0, -2\sqrt{6})$

Length of major axis $2(5)=10$

Length of minor axis $2(1)=2$



Steps for graphing:

1. Determine whether horizontal or vertical
2. Plot center
3. Plot vertices, co-vertices & foci
4. Sketch

3. $\frac{(x+3)^2}{4}+\frac{(y-1)^2}{16}=1$ larger denominator associated to y^2

H or V? Vertical Center $(-3,1)$

a = 4 b = 2 c = $c^2=16-4$, $c^2=12$, $c=2\sqrt{3}$ or ≈ 3.5

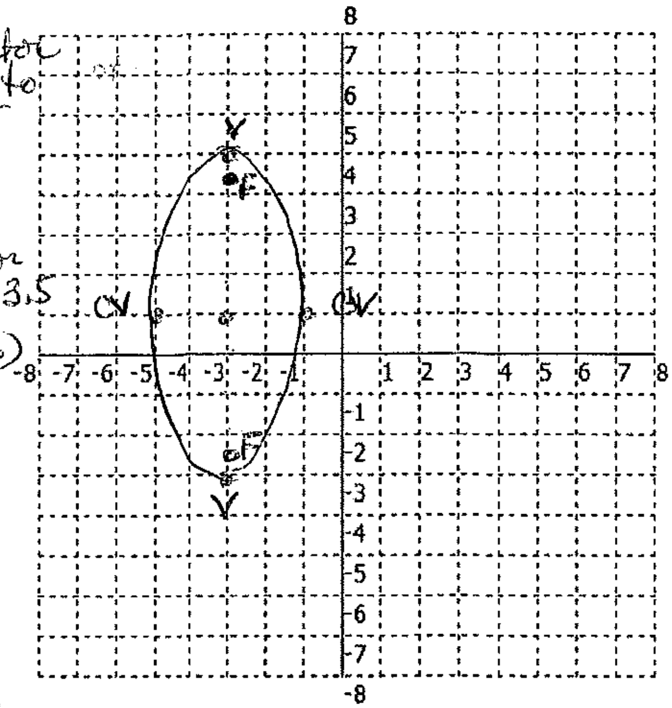
Endpoints of major axis $(-3, 1\pm 4) \to (-3,5)$, $(-3,-3)$

Endpoints of minor axis $(-3\pm 2, 1) \to (-1,1)$, $(-5,1)$

Coordinates of foci $(-3, 1\pm 2\sqrt{3})$

Length of major axis $2(4)=8$

Length of minor axis $2(2)=4$



4. $\frac{2x^2}{18}+\frac{3(y+2)^2}{18}=\frac{18}{18}$ set equation = 1 $\boxed{\frac{x^2}{9}+\frac{(y+2)^2}{6}=1}$

H or V? Horizontal Center $(0,-2)$

a = 3 b = $\sqrt{6}\approx 2.4$ c = $c^2=9-6$, $c^2=3$, $c=\sqrt{3}$ or ≈ 1.7

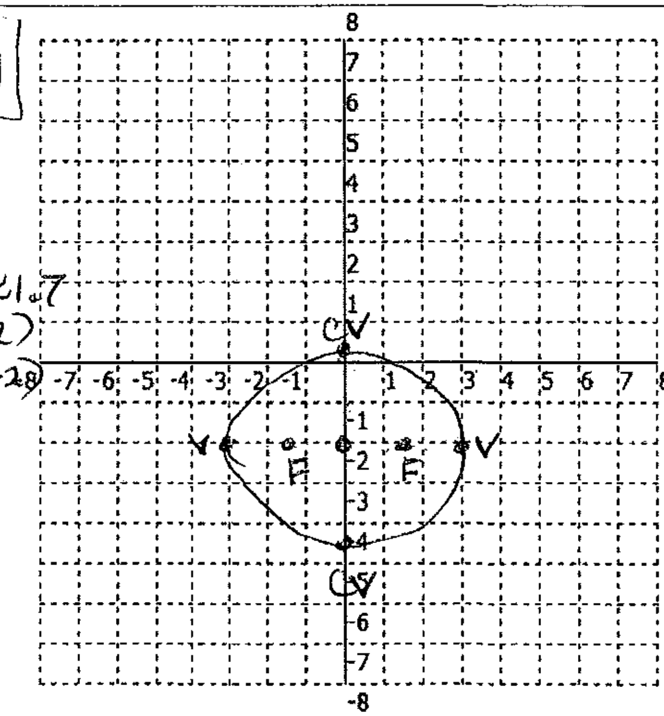
Endpoints of major axis $(0\pm 3,-2) \to (3,-2)$, $(-3,-2)$

Endpoints of minor axis $(0,-2\pm\sqrt{6})$

Coordinates of foci $(0\pm\sqrt{3},-2)$

Length of major axis $2(3)=6$

Length of minor axis $2(\sqrt{6})=2\sqrt{6}$



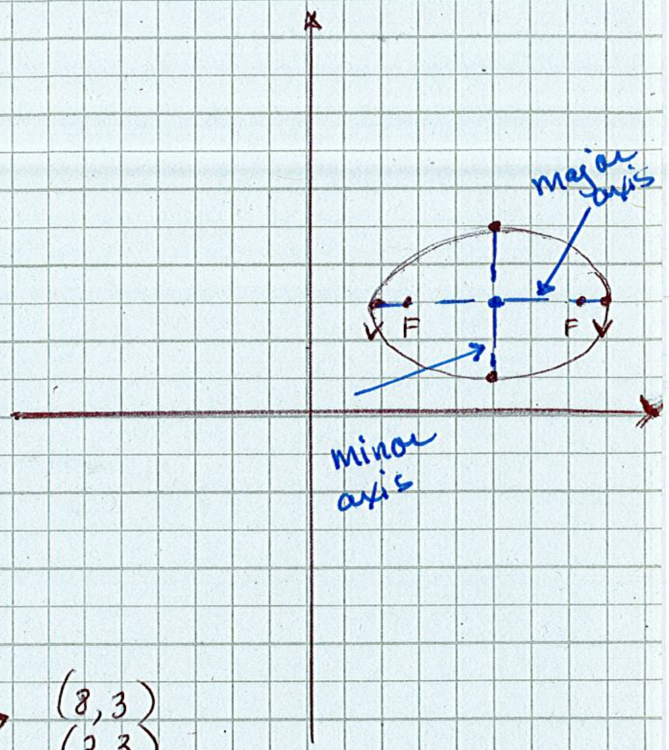
How can you tell by looking at the equation of an ellipse whether the graph will be horizontal or vertical?

If the larger denominator is with x^2, it's horizontal; If it's with y^2, it's vertical

How to graph/analyze an ellipse

EX
$$\frac{(x-5)^2}{9} + \frac{(y-3)^2}{4} = 1$$
$a=3$ $\quad$ $b=2$

1. Determine whether horizontal or vertical (bigger denominator belongs to which variable?)
HORIZONTAL



2. Identify center (h,k) and plot $(5,3)$

3. Because major axis is horizontal, go "a" spaces left & right. Plot these "vertices" (endpoints of major axis). To find their coordinates, add/subtract a value from x coordinate of center:

center $(5 \overset{+3}{\underset{-3}{,}} 3) \to (8,3)$, $(2,3)$

4. Add/subtract "b" value from y-coordinate of center. This gives you the covertices (endpoints of minor axis).

Center $(5,3)\overset{+2}{\underset{-2}{}} \to (5,5)$, $(5,1)$

5. Add/subtract "c" value from x-coordinate of center. This gives you the coordinates of the foci. Approximate placement of points using $\sqrt{5} \approx 2.2$

center $(5,3)\overset{+\sqrt{5}}{\underset{-\sqrt{5}}{}} \to (5+\sqrt{5},3)$, $(5-\sqrt{5},3)$

Remember, foci must go on <u>major</u> axis. They always lie inside the ellipse.

6. Verify the lengths of your axes. There should be $2 \times 3 = 6$ spaces on your major axis and $2 \times 2 = 4$ spaces on your minor axis.

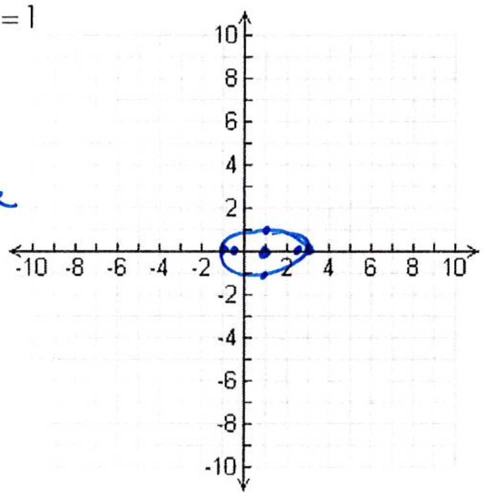
Precalculus Ellipse Practice Name: ____________________

For problems 1-6, complete the chart with the appropriate information.

	Equation	Center	Length of Major Axis	Length of Minor Axis
1.	$\frac{(x+3)^2}{9}+\frac{(y+1)^2}{4}=1$ $a^2=9$ $\therefore a=3$ $b^2=4$ $\therefore b=2$	$(-3,-1)$	$2(3)=6$	$2(2)=4$
2.	$\frac{(x+3)^2}{9}+y^2=1$ $a=3, b=1$	$(-3,0)$	$2(3)=6$	$2(1)=2$
3.	$\frac{x^2}{16}+\frac{(y+2)^2}{25}=1$ $b=4, a=5$	$(0,-2)$	$2(5)=10$	$2(4)=8$
4.	$\frac{(x-3)^2}{4}+\frac{(y-1)^2}{81}=1$ $b=2, a=9$	$(3,1)$	$2(9)=18$	$2(2)=4$
5.	$(x+1)^2+\frac{(y+4)^2}{144}=1$ $b=1, a=12$	$(-1,-4)$	$2(12)=24$	$2(1)=2$
6.	$\frac{(y+6)^2}{49}+\frac{(x-7)^2}{25}=1$ $a=7, b=5$	$(7,-6)$	$2(7)=14$	$2(5)=10$

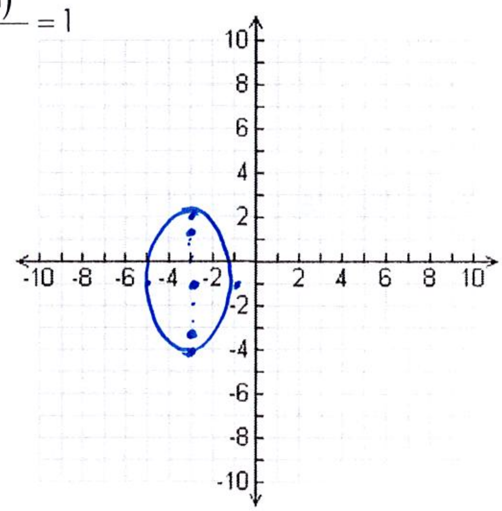
7. $\frac{(x-1)^2}{4}+y^2=1$

$c^2=4-1$
$c^2=3$
$c=\sqrt{3}$ or
≈ 1.7



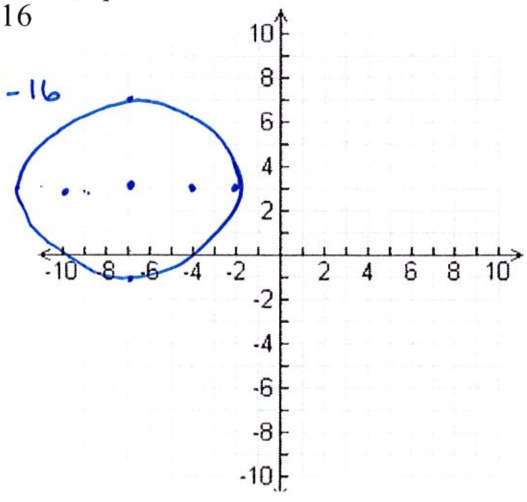
8. $\frac{(x+3)^2}{4}+\frac{(y+1)^2}{9}=1$

$c^2=9-4$
$c^2=5$
$c=\sqrt{5}$
or ≈ 2.2



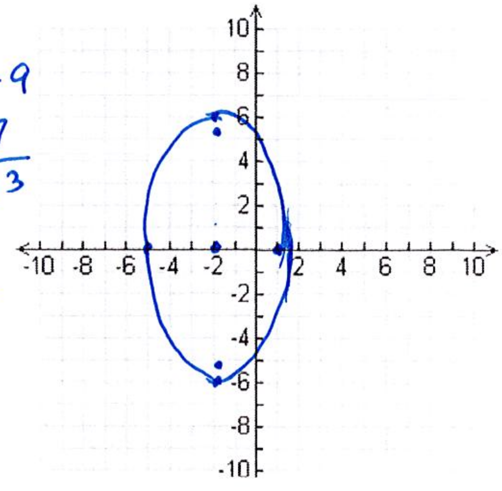
9. $\frac{(x+7)^2}{25}+\frac{(y-3)^2}{16}=1$

$c^2=25-16$
$c^2=9$
$c=3$



10. $\frac{(x+2)^2}{9}+\frac{y^2}{36}=1$

$c^2=36-9$
$c^2=27$
$c=3\sqrt{3}$
or
≈ 5.2



Math 3

Worksheet: Equations and Graphs of Ellipses

Rewrite these general form equations in standard form. Identify the center and the values of a, b and c.

1. $4x^2 + y^2 - 32x + 4y + 64 = 0$ center $(4,-2)$ a 2 b 1 c $\sqrt{3}$

$4x^2 - 32x \quad + y^2 + 4y = -64$

$4(x^2 - 8x + 16) + y^2 + 4y + 4 = -64 + 64 + 4$

$4(x-4)^2 + (y+2)^2 = 4$

$(x-4)^2 + \frac{(y+2)^2}{4} = 1$

$c^2 = 4 - 1$

$c^2 = 3$

$c = \sqrt{3}$

2. $4x^2 + 9y^2 - 8x - 36y + 4 = 0$ center $(1,2)$ a 3 b 2 c $\sqrt{5}$

$4x^2 - 8x \quad + 9y^2 - 36y = -4$

$4(x^2 - 2x + 1) + 9(y^2 - 4y + 4) = -4 + 4 + 36$

$4(x-1)^2 + 9(y-2)^2 = 36$

$\frac{(x-1)^2}{9} + \frac{(y-2)^2}{4} = 1$

$c^2 = 9 - 4$

$c^2 = 5$

$c = \sqrt{5}$

3. $4x^2 + y^2 + 24x - 10y + 45 = 0$ center $(-3,5)$ a 4 b 2 c $2\sqrt{3}$

$4x^2 + 24x \quad + y^2 - 10y = -45$

$4(x^2 + 6x + 9) + y^2 - 10y + 25 = -45 + 36 + 25$

$4(x+3)^2 + (y-5)^2 = 16$

$\frac{4(x+3)^2}{16} + \frac{(y-5)^2}{16} = 1$

$\frac{(x+3)^2}{4} + \frac{(y-5)^2}{16} = 1$

$b^2 \quad a^2$

4. $9x^2 + 4y^2 - 18x + 16y = 11$ center $(1,-2)$ a 3 b 2 c $\sqrt{5}$

$9x^2 - 18x \quad + 4y^2 + 16y = 11$

$9(x^2 - 2x + 1) + 4(y^2 + 4y + 4) = 11 + 9 + 16$

$9(x-1)^2 + 4(y+2)^2 = 36$

$\frac{(x-1)^2}{4} + \frac{(y+2)^2}{9} = 1$

5. $3x^2 + 7y^2 - 12x - 28y = -19$ center $(2,2)$ a $\sqrt{7}$ b $\sqrt{3}$ c 2

$3x^2 - 12x + 7y^2 - 28y = -19$

$3(x^2 - 4x + 4) + 7(y^2 - 4y + 4) = -19 + 12 + 28$

$3(x-2)^2 + 7(y-2)^2 = 21$

$\frac{(x-2)^2}{7} + \frac{(y-2)^2}{3} = 1$

$a^2 = 7$

$b^2 = 3$

$c^2 = a^2 - b^2$

$c^2 = 7 - 3$

$c^2 = 4$

$c = 2$

6. $4x^2 + 9y^2 - 48x + 72y + 144 = 0$ center $(6,-4)$ a 6 b 4 c $2\sqrt{5}$

$4x^2 - 48x + 9y^2 + 72y = -144$

$4(x^2 - 12x + 36) + 9(y^2 + 8y + 16) = -144 + 144 + 144$

$4(x-6)^2 + 9(y+4)^2 = 144$

$\frac{4(x-6)^2}{144} + \frac{9(y+4)^2}{144} = \frac{144}{144}$

$a^2 = 36 \quad b^2 = 16$

$c^2 = 36 - 16 = 20$

$c = 2\sqrt{5}$

$\frac{(x-6)^2}{36} + \frac{(y+4)^2}{16} = 1$