Notes: Right Triangle Trig

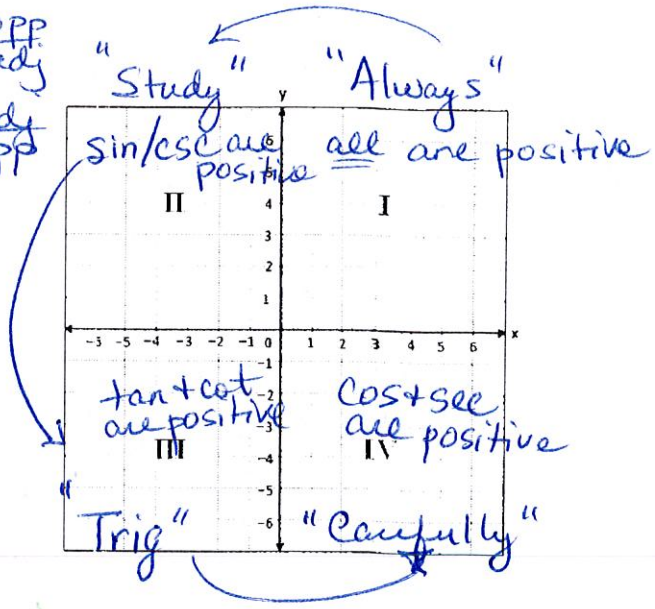
Important things to remember:

- The Pythagorean Theorem is $a^2+b^2=c^2$.
- The six trig ratios are:

$\sin\theta = \frac{opp}{hyp}$ $\quad \cos\theta = \frac{adj}{hyp}$ $\quad \tan\theta = \frac{opp}{adj}$

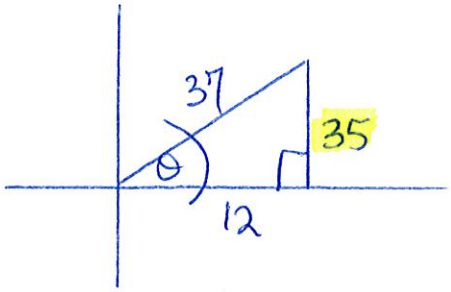
$\csc\theta = \frac{hyp}{opp}$ $\quad \sec\theta = \frac{hyp}{adj}$ $\quad \cot\theta = \frac{adj}{opp}$

- Use the acronym "Always Study Trig Carefully" to remember which trig functions are positive in the quadrants:



Problem type 1: Finding a trig value given a trig value (no constraints)

Given the $\cos\theta = \frac{12}{37}$, find $\csc\theta$



$12^2+b^2=37^2$

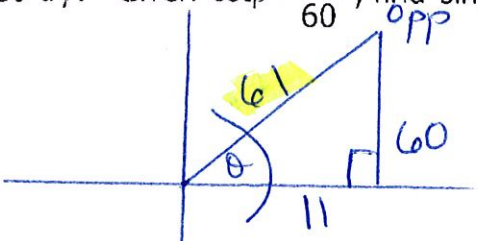
$144+b^2=1369$

$b^2=1225$

$b=35$

$\csc\theta = \frac{hyp}{opp} = \frac{37}{35}$

You try: Given $\cot\beta = \frac{11}{60}$ (adj/opp), find $\sin\beta$



$11^2+60^2=c^2$

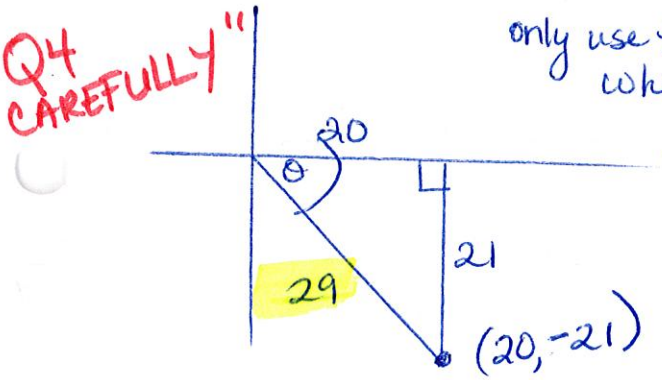
$121+3600=c^2$

$c=61$

$\sin B = \frac{60}{61}$

Problem type 2: Finding trig values given a coordinate point (the constraint is the quadrant in which the point is located!)

EX 1: The point (20, -21) is on the terminal side of an angle in standard position. Determine the exact values of the six trigonometric functions of the angle.



only use the absolute values of the (x, y) coordinates when substituting into Pythagorean Thm.

$20^2+21^2=c^2$

$400+441=c^2$

$841=c^2$

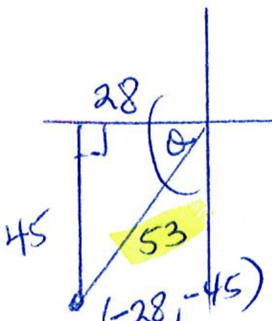
$c=29$

only cosine & secant are positive in Q4

$\sin\theta = \frac{-21}{29}$ $\quad \csc\theta = \frac{-29}{21}$ $\quad \tan\theta = \frac{-21}{20}$

$\cos\theta = \frac{20}{29}$ $\quad \sec\theta = \frac{29}{20}$ $\quad \cot\theta = \frac{-20}{21}$

You try: The point (-28, -45) is on the terminal side of an angle in standard position. Determine the exact values of the six trigonometric functions of the angle.



$28^2 + 45^2 = c^2$

$c = 53$

$\sin\theta = \frac{-45}{53}$ $\csc\theta = \frac{-53}{45}$

$\cos\theta = \frac{-28}{53}$ $\sec\theta = \frac{-53}{28}$

$\tan\theta = \frac{45}{28}$ $\cot\theta = \frac{28}{45}$

Q4 = "TRIG"

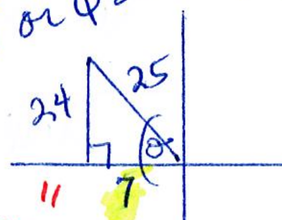
only tan + cot are positive

Problem type 3: Given a function value and an additional constraint

EX 1: Given $\sin\theta = \frac{24}{25}$; constraint: θ lies in Quadrant 2. Find the other five trig values.

If sin is positive, the angle lies in either Q1 or Q2

DONE. The angle lies in Q2



$25^2 = 24^2 + b^2$

$625 = 576 + b^2$

$49 = b^2$

$b = 7$

$\sin\theta = \frac{24}{25}$ $\csc\theta = \frac{25}{24}$

$\cos\theta = \frac{-7}{25}$ $\sec\theta = \frac{-25}{7}$

$\tan\theta = \frac{-24}{7}$ $\cot\theta = \frac{-7}{24}$

Q2 = "Study"

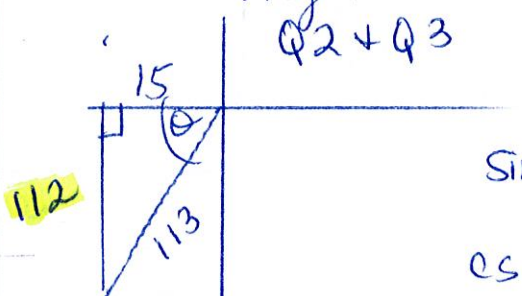
only sine + cosecant are positive

EX 2: Given $\sec\theta = -\frac{113}{15}$ and $\tan\theta > 0$, find the other five trig values.

(cos θ) are negative in Q2 + Q3

Of these 2 quadrants, tan is positive in Q3 ∴ we must graph in Q3

Q3 = "Trig" tan + cot are positive



$\sin\theta = \frac{-112}{113}$ $\cos\theta = \frac{-15}{113}$ $\tan\theta = \frac{112}{15}$

$\csc\theta = \frac{-113}{112}$ $\sec\theta = \frac{-113}{15}$ $\cot\theta = \frac{15}{112}$

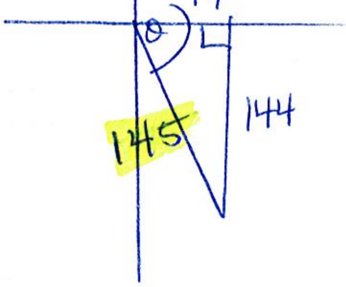
You try: Given $\cot\theta = \frac{-17}{144}$ and $\cos\theta < 0$, find the other five trig values.

Q2 or Q4

~~Q1 or Q3~~

not Q3 ∴ not Q2 ∴ Q4

Q4 "Carefully" ∴ cos + sec are positive

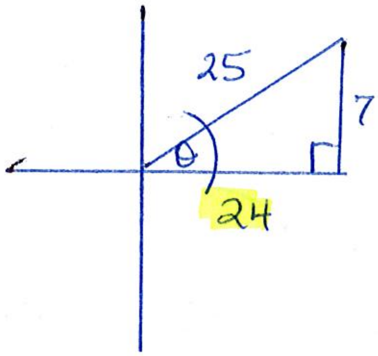


$\sin\theta = \frac{-144}{145}$ $\cos\theta = \frac{17}{145}$ $\tan\theta = \frac{-144}{17}$

$\csc\theta = \frac{-145}{144}$ $\sec\theta = \frac{145}{17}$ $\cot\theta = \frac{-17}{144}$

Find all 6 trig ratios

1. $\sin\theta = \frac{7}{25}$

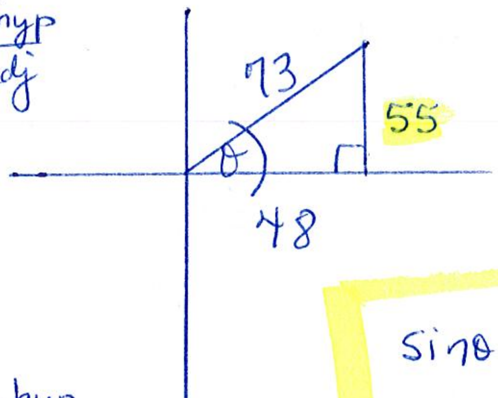


$25^2 = 7^2 + b^2$
$625 = 49 + b^2$
$576 = b^2$
$b = 24$

$\sin\theta = \frac{7}{25}$ $\csc\theta = \frac{25}{7}$
$\cos\theta = \frac{24}{25}$ $\sec\theta = \frac{25}{24}$
$\tan\theta = \frac{7}{24}$ $\cot\theta = \frac{24}{7}$

2. $\sec\theta = \frac{73}{48} = \frac{hyp}{adj}$

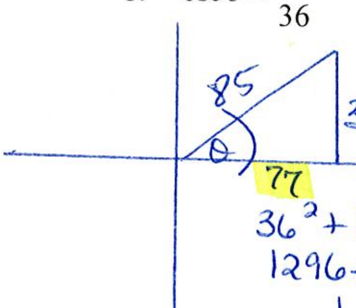
$\cos = \frac{1}{\sec}$



$48^2 + b^2 = 73^2$
$2304 + b^2 = 5329$
$3025 = b^2$
$b = 55$

$\sin\theta = \frac{55}{73}$ $\csc\theta = \frac{73}{55}$
$\cos\theta = \frac{48}{73}$ $\sec\theta = \frac{73}{48}$
$\tan\theta = \frac{55}{48}$ $\cot\theta = \frac{48}{55}$

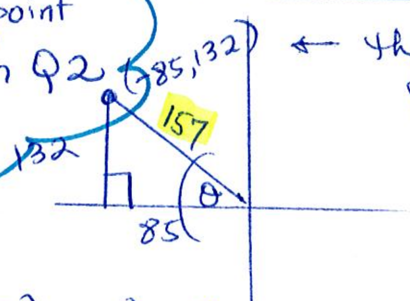
3. $\csc\theta = \frac{85}{36} = \frac{hyp}{opp}$



$36^2 + b^2 = 85^2$
$1296 + b^2 = 7225$
$b^2 = 5929$
$b = 77$

4. (-85, 132)

"STUDY": this point lies in Q2; sin + csc are positive

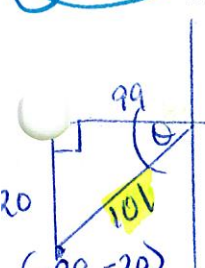


← the leg measures of the right Δ are the absolute values of the coordinates.

$85^2 + 132^2 = c^2$
$7225 + 17{,}424 = c^2$
$24{,}649 = c^2$
$c = 157$

$\sin\theta = \frac{132}{157}$
$\cos\theta = \frac{-85}{157}$
$\tan\theta = \frac{-132}{85}$
$\csc\theta = \frac{157}{132}$
$\sec\theta = \frac{-157}{85}$
$\cot\theta = \frac{-85}{132}$

5. (-99, -20)



$99^2 + 20^2 = c^2$
$10{,}201 = c^2$
$c = 101$

Q3 "Trig": tan + cot are positive

$\sin\theta = \frac{-20}{101}$ $\cos\theta = \frac{-99}{101}$ $\tan\theta = \frac{20}{99}$
$\csc\theta = \frac{101}{-20} = \frac{-101}{20}$ $\sec\theta = \frac{-101}{99}$ $\cot\theta = \frac{99}{20}$

Q4

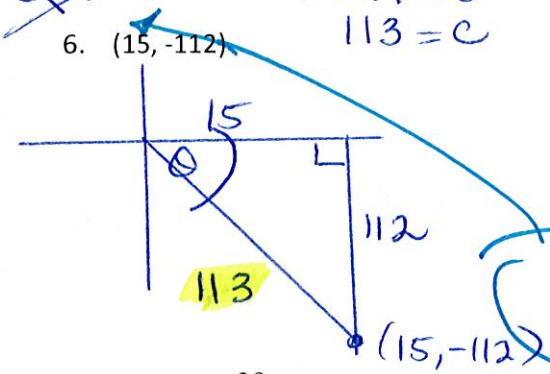
$15^2 + 112^2 = c^2$

$225 + 12{,}544 = c^2$

$12769 = c^2$

$113 = c$

6. (15, -112)



$\sin\theta = \frac{-112}{113}$ $\quad \csc\theta = \frac{-113}{112}$

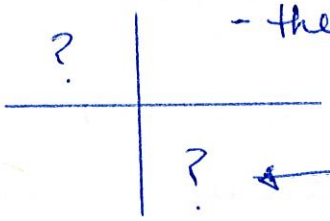
$\cos\theta = \frac{15}{113}$ $\quad \sec\theta = \frac{113}{15}$

$\tan\theta = \frac{-112}{15}$ $\quad \cot\theta = \frac{-15}{112}$

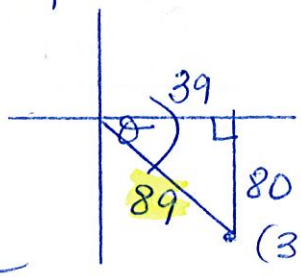
"Carefully": cos+sec are positive

7. $\cot\theta = \frac{-39}{80}$, $\cos\theta > 0$

this confines us to Q2 + Q4 where cot is negative - the 2nd constraint requires us to graph where cos is positive. This eliminates Q2.



θ must lie in Q4!



$\sin\theta = \frac{-80}{89}$ $\quad \csc\theta = \frac{-89}{80}$

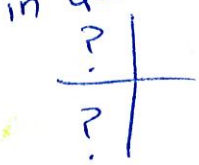
$\cos\theta = \frac{39}{89}$ $\quad \sec\theta = \frac{89}{39}$

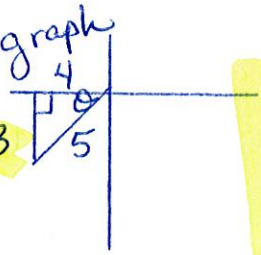
$\tan\theta = \frac{-80}{39}$ $\quad \cot\theta = \frac{-39}{80}$

8. $\cos\theta = -\frac{4}{5}$, $\tan\theta > 0$

cos is neg. in Q2+Q3

tan is positive in Q3 ∴ we must graph in Q3





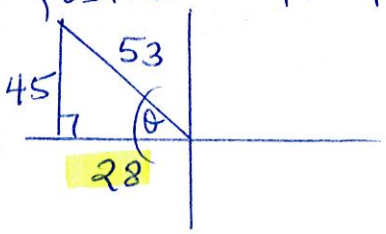
$\sin\theta = \frac{3}{5}$ $\quad \csc\theta = \frac{-5}{3}$

$\cos\theta = \frac{-4}{5}$ $\quad \sec\theta = \frac{-5}{4}$

$\tan\theta = \frac{3}{4}$ $\quad \cot\theta = \frac{4}{3}$

9. $\csc\theta = \frac{53}{45}$, $\tan\theta < 0$ ← we must graph in Q2!

sin is positive in Q1 + Q2



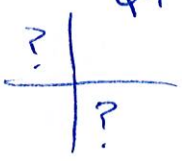
$\sin\theta = \frac{45}{53}$ $\quad \csc\theta = \frac{53}{45}$

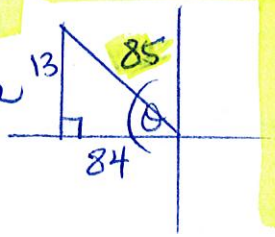
$\cos\theta = \frac{-28}{53}$ $\quad \sec\theta = \frac{-53}{28}$

$\tan\theta = \frac{-45}{28}$ $\quad \cot\theta = \frac{-28}{45}$

10. $\tan\theta = -\frac{13}{84}$, $\sin\theta > 0$

Q2 or Q4? We must use Q2





$\sin\theta = \frac{13}{85}$ $\quad \cos\theta = \frac{-84}{85}$ $\quad \tan\theta = \frac{-13}{84}$

$\csc\theta = \frac{85}{13}$ $\quad \sec\theta = \frac{-85}{84}$ $\quad \cot\theta = \frac{-84}{13}$