

## Sec and Csc Graphing

Sketch the graph of the function.

1.  $y = -\frac{1}{2}\sec\theta$

2.  $y = 4\csc 4\theta$

3.  $y = -2 + 4\sec(4\theta - \pi)$

4.  $y = 5\csc\left(\frac{1}{2}\theta - 2\pi\right) - 3$

5.  $y = -2 \csc\left(5\theta - \frac{3\pi}{2}\right) + 3$

6.  $y = 6 + 3 \sec\left(\frac{2}{3}\theta - 4\pi\right)$

1.  $y = -\frac{1}{2} \sec \theta \rightarrow$  use  $-\cos \theta$  to help  
construct graph

period:  $2\pi$

intervals:  $\frac{2\pi}{4} = \frac{\pi}{2}$

phase shift: none

midline:  $y = 0$

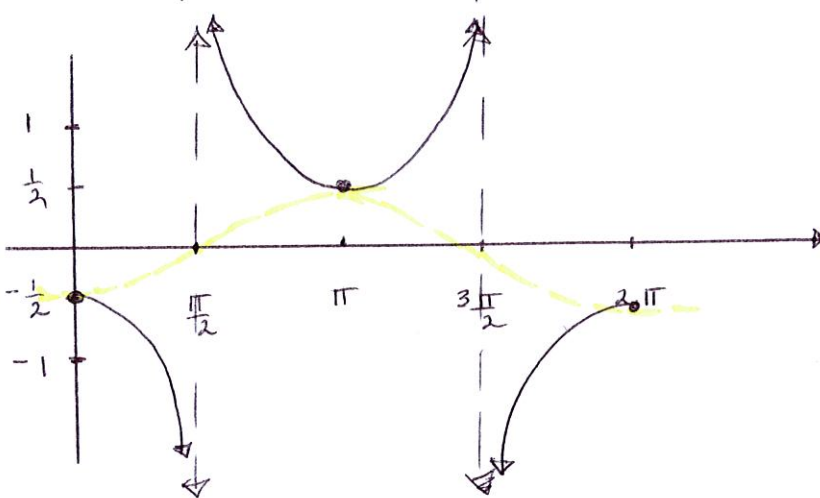
MAX:  $0 + \frac{1}{2} = \frac{1}{2}$

MIN:  $0 - \frac{1}{2} = -\frac{1}{2}$

← Add and subtract amplitude  
from midline value

	$\theta$	$y$ (critical values for $y = -\cos \theta$ )
no phase shift $\rightarrow$	0	MIN $-\frac{1}{2}$
shift $+\frac{\pi}{2}$	$\frac{\pi}{2}$	MID 0
$+\frac{\pi}{2}$	$\pi$	MAX $\frac{1}{2}$
$+\frac{\pi}{2}$	$\frac{3\pi}{2}$	MID 0
$+\frac{\pi}{2}$	$2\pi$	MIN $-\frac{1}{2}$

For reciprocal functions, midline values become asymptotes. Minimums become maximums and maximums become minimums (relative max/min).



If you need to,  
LIGHTLY sketch  
the reciprocal  
function

2.  $y = 4 \csc 4\theta \rightarrow$  use  $y = +\sin\theta$  to help construct graph

period :  $\frac{2\pi}{4} = \frac{\pi}{2}$

intervals :  $\frac{\pi}{2} \div 4 \quad \text{OR} \quad \frac{\pi}{2 \cdot 4} = \frac{\pi}{8}$

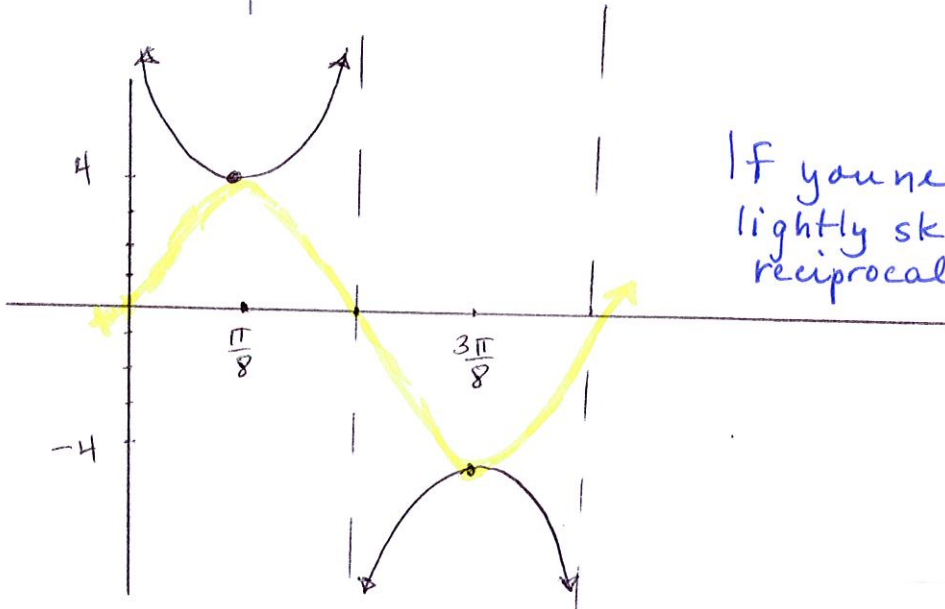
phase shift : none

midline :  $y = 0$

MAX :  $0 + 4 = 4$

MIN :  $0 - 4 = -4$

$\theta$	$y = \sin\theta$		
0	MID	0	(asymptote)
$+\frac{\pi}{8}$	$\frac{\pi}{8}$	MAX	4
$+\frac{\pi}{8}$	$\frac{\pi}{4}$	MID	0 (asymptote)
$+\frac{\pi}{8}$	$\frac{3\pi}{8}$	MIN	-4
$+\frac{\pi}{8}$	$\frac{\pi}{2}$	MID	0 (asymptote)



If you need to, lightly sketch the reciprocal function

3.  $y = -2 + 4 \sec(4\theta - \pi)$

→ use  $y = \cos \theta$

Same as  $y = 4 \sec(4\theta - \pi) - 2$

↙ the constant is the midline

period:  $\frac{2\pi}{4} = \frac{\pi}{2}$

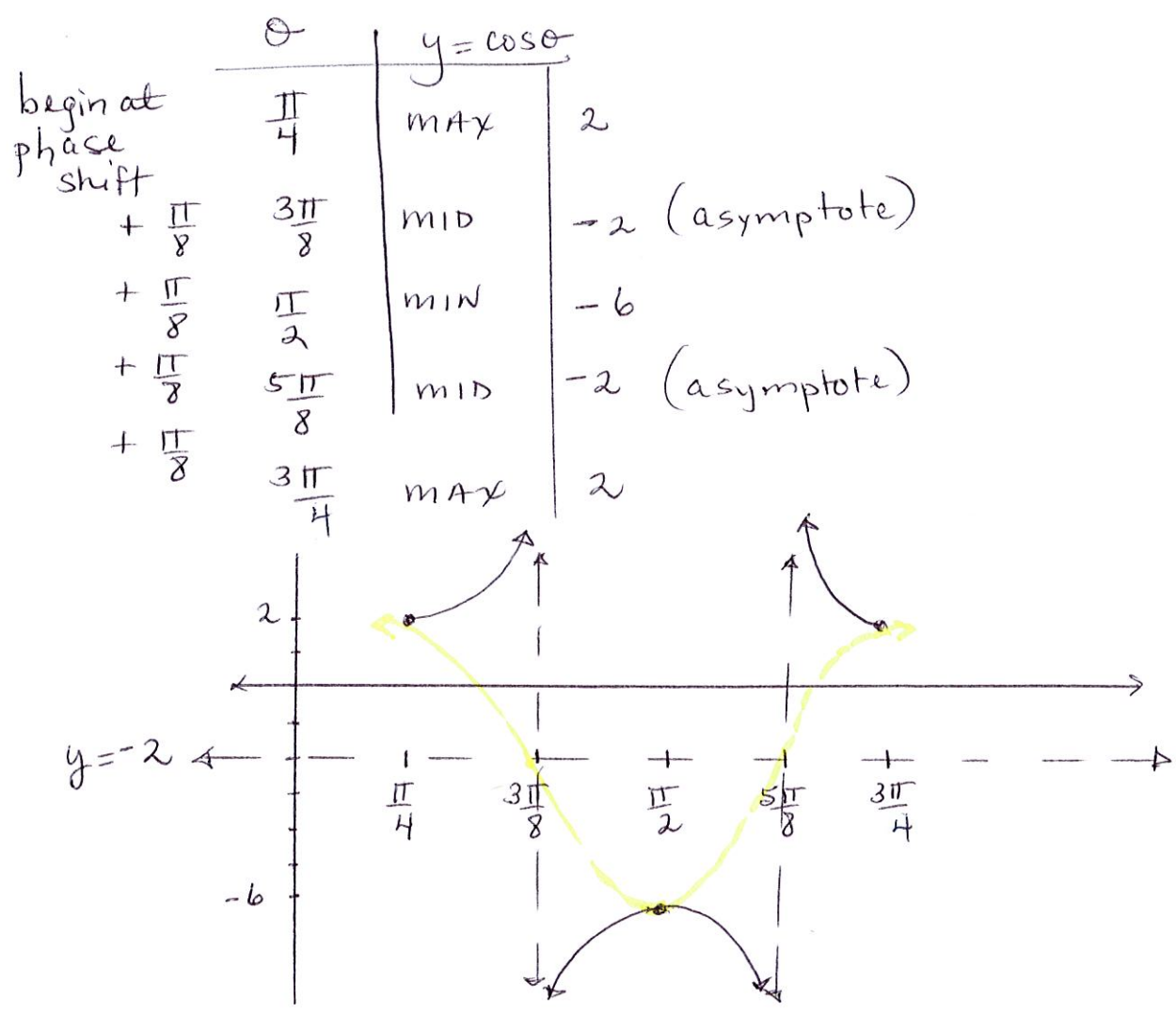
intervals:  $\frac{\pi}{2 \cdot 4} = \frac{\pi}{8}$

phase shift:  $4\theta - \pi = 0$   
 $4\theta = \pi$   
 $\theta = \frac{\pi}{4}$

MIDLINE:  $y = -2$

MAX:  $-2 + 4 = 2$

MIN:  $-2 - 4 = -6$



4.  $y = 5 \csc\left(\frac{1}{2}\theta - 2\pi\right) - 3$

use  $y = \sin\theta$

period:  $\frac{2\pi}{\frac{1}{2}} = 4\pi$

intervals:  $\frac{4\pi}{4} = \pi$

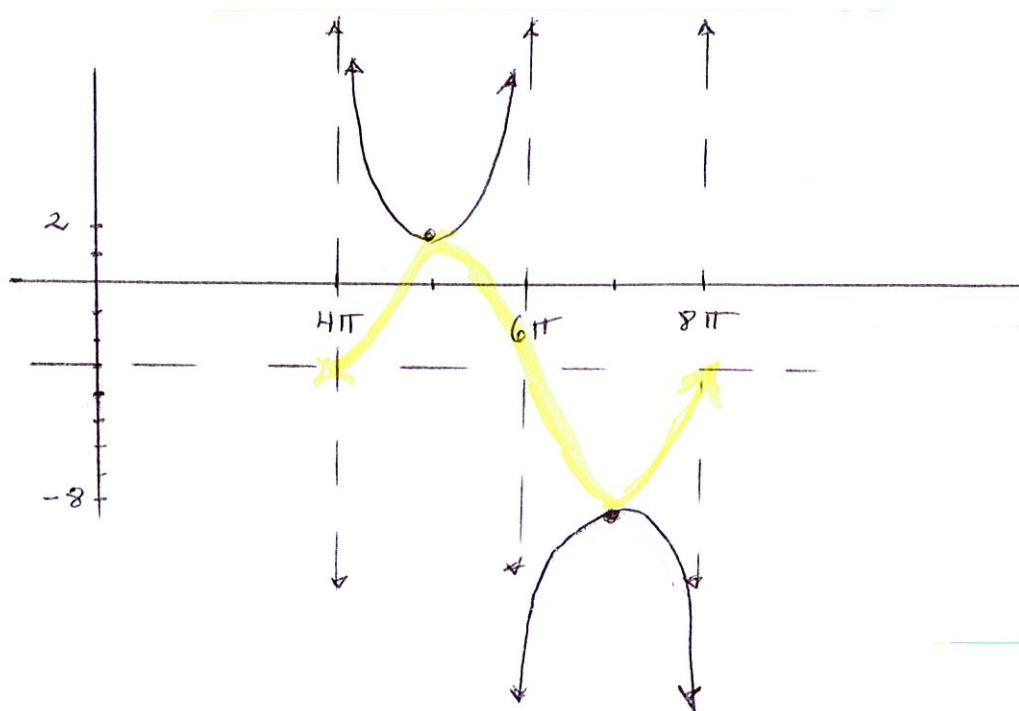
phase shift:  $\frac{1}{2}\theta - 2\pi = 0$   
 $\frac{1}{2}\theta = 2\pi$   
 $\theta = 4\pi$

MIDLINE:  $y = -3$

MAX:  $-3 + 5 = 2$


MIN:  $-3 - 5 = -8$

$\theta$		$y = \sin\theta$
$4\pi$	MID	$-3$ (asymptote)
$5\pi$	MAX	$2$
$6\pi$	MID	$-3$ (asymptote)
$7\pi$	MIN	$-8$
$8\pi$	MID	$-3$ (asymptote)



$$5. \quad y = -2 \csc\left(5\theta - \frac{3\pi}{2}\right) + 3$$

Graph  $y = -\sin\theta$



period :  $\frac{2\pi}{5}$

intervals :  $\frac{2\pi}{5 \cdot 4} = \frac{2\pi}{20} = \frac{\pi}{10}$

phase shift :  $5\theta - \frac{3\pi}{2} = 0$

$$5\theta = \frac{3\pi}{2}$$

$$\theta = \frac{3\pi}{10}$$

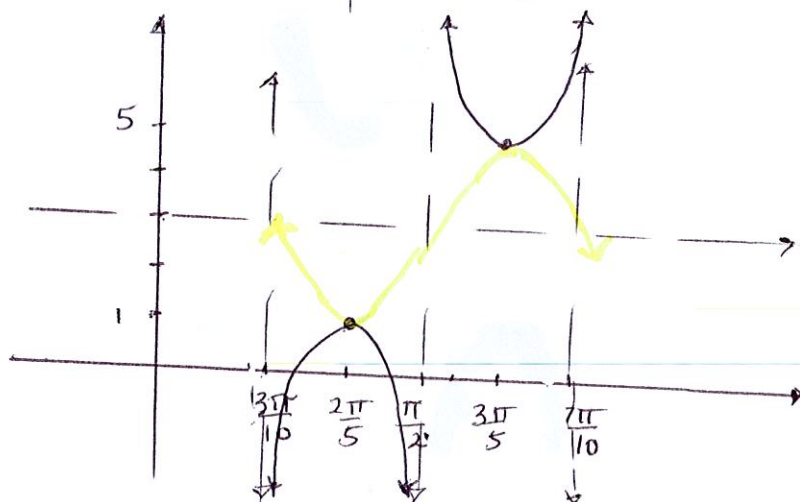
MIDLINE :  $y = 3$

MAX :  $3 + 2 = 5$

MIN :  $3 - 2 = 1$

$\theta$	$y = -\sin\theta$	
$\frac{3\pi}{10}$	MID	3 (asymptote)
$\frac{2\pi}{5}$	MIN	1
$\frac{\pi}{2}$	MID	3 (asymptote)
$\frac{3\pi}{5}$	MAX	5
$\frac{7\pi}{10}$	MID	3 (asymptote)

$+\frac{\pi}{10}$   
 $+\frac{\pi}{10}$   
 $+\frac{\pi}{10}$   
 $+\frac{\pi}{10}$



$$6. \quad y = 6 + 3 \sec\left(\frac{2}{3}\theta - 4\pi\right)$$

use  $y = \cos\theta$   
to graph

$$\text{period: } \frac{2\pi}{\frac{2}{3}} \rightarrow 2\pi \times \frac{3}{2} = \frac{6\pi}{2} = 3\pi$$

$$\text{intervals: } \frac{3\pi}{4}$$

$$\text{phase shift: } \frac{2}{3}\theta - 4\pi = 0$$

$$\frac{2}{3}\theta = 4\pi$$

$$\theta = 4\pi \times \frac{3}{2}$$

$$\theta = \frac{12\pi}{2} = 6\pi$$

$$\text{MIDLINE: } y = 6$$

$$\text{MAX: } 6 + 3 = 9$$

$$\text{MIN: } 6 - 3 = 3$$

$\theta$	$y = \cos\theta$
$6\pi$	MAX 9
$+ \frac{3\pi}{4}$ $\frac{27\pi}{4}$	MID 6
$+ \frac{3\pi}{4}$ $\frac{15\pi}{2}$	MIN 3
$+ \frac{3\pi}{4}$ $\frac{33\pi}{4}$	MID 6
$+ \frac{3\pi}{4}$ $9\pi$	MAX 9

