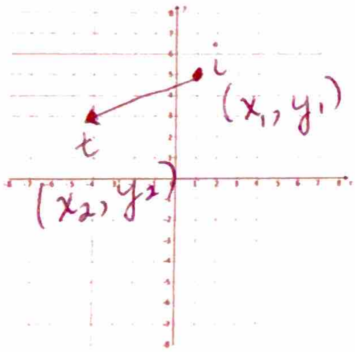
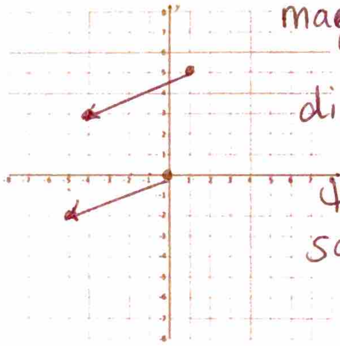
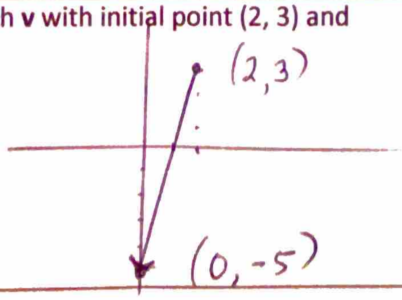
Precalculus: Vectors Notes

What is a *scalar*?	Examples of scalars:
A quantity that has magnitude (size) only	Length, volume, area, height
What is a *vector*?	**Examples of vectors:**
A quantity that has magnitude + direction	displacement, acceleration, velocity, weight ↓

There are different ways to represent a vector.

<table>
<tr>
<td>Sketch the vector v with initial point (1, 5) and terminal point (-4, 3)

</td>
<td>Find the component form of the vector v by subtracting the initial point from the terminal point.
$\langle x_2 - x_1, y_2 - y_1 \rangle$
$\langle -4-1, 3-5 \rangle$
$\langle -5, -2 \rangle$ component
begin at (0,0)</td>
<td>Sketch the component vector of v. What is the relationship between the component vector and the initial vector?

the magnitude + direction are the same</td>
</tr>
<tr>
<td colspan="3">To change to linear combinations form, use $(x_2 - x_1)i + (y_2 - y_1)j$
$-5i - 2j$ linear combinations</td>
</tr>
</table>

Try these:

<table>
<tr>
<td>EX 1: Draw a vector with v with initial point (2, 3) and terminal point (0, -5)

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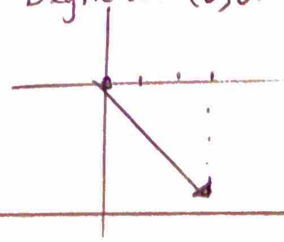
<td>EX 2: Draw the vector given its component form
w = $\langle 3, -4 \rangle$ begin at (0,0)
</td>
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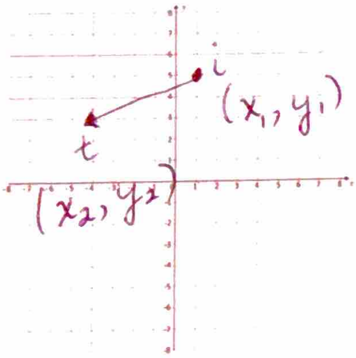
Precalculus: Vectors Notes

What is a *scalar*? A quantity that has magnitude (size) only	Examples of scalars: Length, volume, area, height
What is a *vector*? A quantity that has magnitude + direction	Examples of vectors: displacement, acceleration, velocity, weight ↓

There are different ways to represent a vector.

Sketch the vector **v** with initial point (1, 5) and terminal point (-4, 3)



Find the *component form* of the vector **v** by subtracting the initial point from the terminal point.

$\langle x_2 - x_1, y_2 - y_1 \rangle$

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$\langle -5, -2 \rangle$ component

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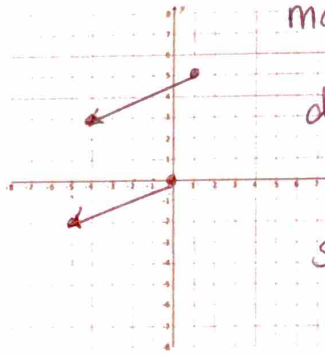

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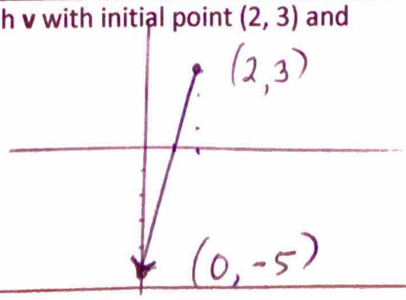
To change to *linear combinations* form, use $(x_2 - x_1)i + (y_2 - y_1)j$

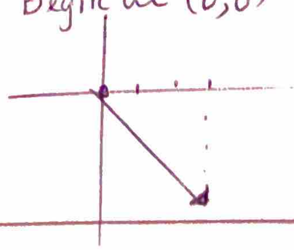
$-5i - 2j$ linear combinations

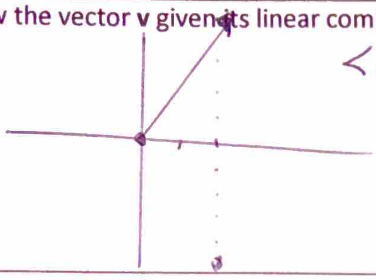
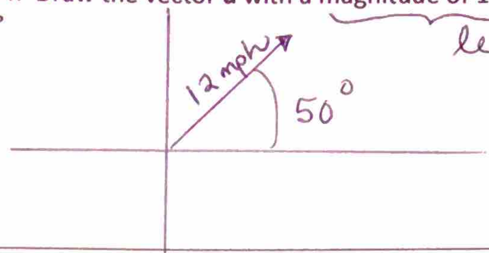
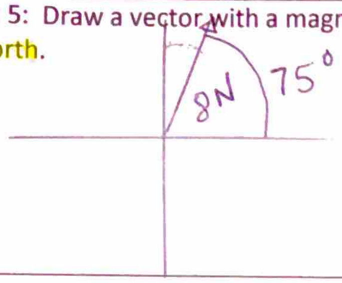
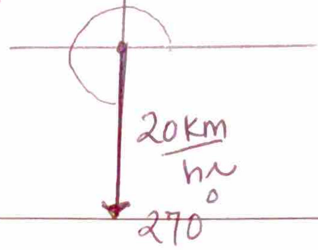
Try these:

EX 1: Draw a vector with **v** with initial point (2, 3) and terminal point (0, -5)



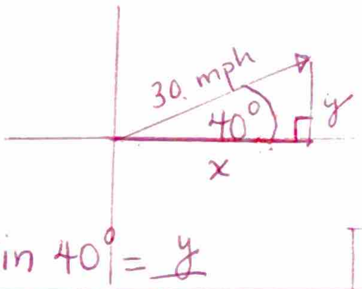
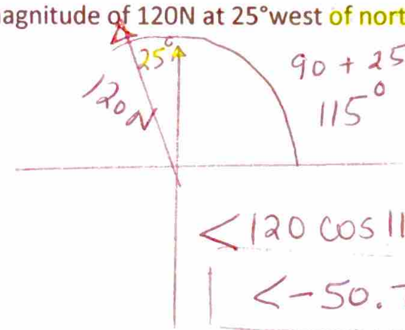
EX 2: Draw the vector given its component form **w** = $\langle 3, -4 \rangle$ begin at (0,0)



EX 3: Draw the vector **v** given its linear combination form $v = 2i + 5j$	EX 4: Draw the vector **u** with a magnitude of 12 mph at 50°
 $\langle 2, 5 \rangle$	 $\underbrace{\text{magnitude of 12 mph}}_{\text{length}}$
Ex 5: Draw a vector with a magnitude of 8N at 15° east of north.	EX 6: Draw a vector with a magnitude of 20 km/h due south.
 ↑ newton	

Find the component form of a vector given its magnitude and direction: use $\langle A\cos\theta, A\sin\theta \rangle$

< magnitude · cos, mag. · sin >

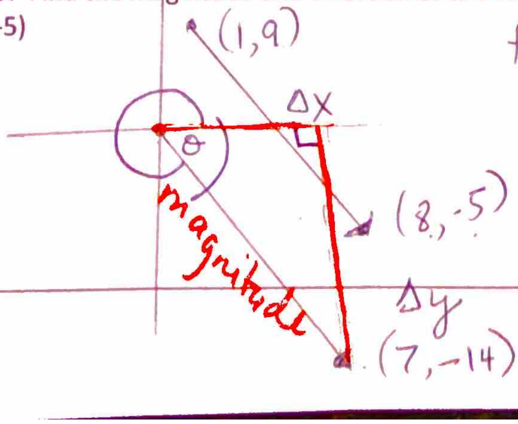
Ex 7: Find the component form of a vector with a magnitude of 30 mph at 40°	Ex 8: Find the component form of a vector with magnitude of 120N at 25°west of north.
 $\langle x, y \rangle$ $\cos 40° = \frac{x}{30}$ $30\cos 40° = x$ $x \approx 23.0$ $\sin 40° = \frac{y}{30}$ $\boxed{\langle 23.0, 19.3 \rangle}$	 90 + 25 115° $\langle 120\cos 115°, 120\sin 115° \rangle$ $\boxed{\langle -50.7, 108.8 \rangle}$

$30\sin 40° = y$ $y \approx 19.3$

Find the magnitude and direction of a vector given its component form: use the formulas

To find magnitude, use the Pythagorean Theorem!	To find direction use right triangle trig!
Magnitude: $\lVert v \rVert = \sqrt{x^2 + y^2}$ ↑ shortcut formula	Direction: $\theta = \tan^{-1}\left(\frac{y}{x}\right)$ If your angle is in Q1, use the value in your calculator. If your angle is in Q2 or Q3, add 180° If your angle is in Q4, add 360°

EX 9: Find the magnitude and direction of the vectors ~~shown~~ whose initial point is (1, 9) and whose terminal points is (8, -5)



find component form!

$\langle 8-1, -5-9 \rangle$

$\langle 7, -14 \rangle$

$\sqrt{7^2 + 14^2} = \lVert v \rVert$

$\sqrt{49 + 196} \approx \boxed{15.7}$

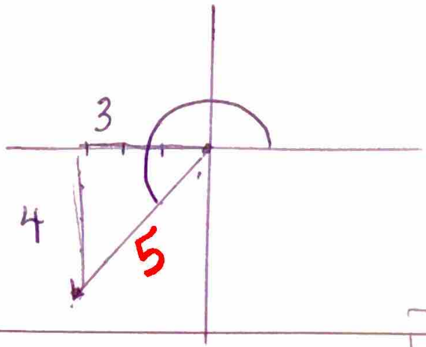
Q4

angle + 360°

$\tan^{-1}\left(\frac{-14}{7}\right)$

$\boxed{\theta \approx 296.6°}$

Ex 10: Find the magnitude and direction of the vector $w = \langle -3, -4 \rangle$



magnitude =

$\sqrt{3^2 + 4^2}$

$\sqrt{9+16}$

$\sqrt{25}$

$\boxed{5}$

Q3 → $+180°$

$\tan^{-1}\left(\frac{-4}{-3}\right) + 180°$

$\boxed{233.1°}$

Operations with vectors:

Vectors can be multiplied by a scalar, added and subtracted. These operations can change the magnitude and direction of the vector. The answer is called the *resultant*. Please keep consistent form.

linear combination

Given the linear combinations form of the vector $v = -2i + 5j$ and $w = i - 3j$, find all the following:

$-4v$

Distribute

$-4(-2i + 5j)$

$\boxed{8i - 20j}$

$3v - 2w$

$3(-2i + 5j) - 2(i - 3j)$

$-6i + 15j - 2i + 6j$

$\boxed{-8i + 21j}$

$v + 8w$

$(-2i + 5j) + 8(i - 3j)$

$-2i + 5j + 8i - 24j$

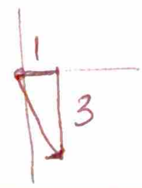
$\boxed{6i - 19j}$

$\|w\|$

magnitude

$\sqrt{1^2 + 3^2}$

$\sqrt{10} \approx 3.2$



component form

Given the component form of the vectors $u = \langle 6, 8 \rangle$ and $v = \langle -1, 0 \rangle$, find all the following:

$3v - u$

$3\langle -1, 0 \rangle - \langle 6, 8 \rangle$

$\langle -3, 0 \rangle + \langle -6, -8 \rangle$

$\boxed{\langle -9, -8 \rangle}$

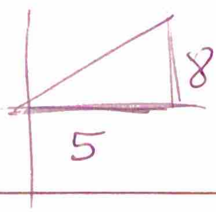
$5v$

$5\langle -1, 0 \rangle$

$\boxed{\langle -5, 0 \rangle}$

$\|u + v\|$

$\| 5, 8 \|$



$\sqrt{5^2 + 8^2}$

$\sqrt{25 + 64} = \sqrt{89} \approx 9.4$

$-\frac{1}{2}u$

$-\frac{1}{2}\langle 6, 8 \rangle$

$\boxed{\langle -3, -4 \rangle}$