

Worksheet A

1. $\sin^2 \theta - 1 = -2 \sin \theta + 2 \sin^2 \theta$

$0 = \sin^2 \theta - 2 \sin \theta + 1$

$(\sin \theta - 1)(\sin \theta - 1) = 0$ ~~$\begin{matrix} 1 \\ -2 \\ -1 \end{matrix}$~~

$\sin \theta = 1$
 $\theta = \frac{\pi}{2}$

2. $\sqrt{3} \sin \theta - \sin^2 \theta = \sin^2 \theta$

$\sqrt{3} \sin \theta - 2 \sin^2 \theta = 0$

$\sin \theta (\sqrt{3} - 2 \sin \theta) = 0$

$\sin \theta = 0$ $\sqrt{3} - 2 \sin \theta = 0$

$\theta = 0$

$\sqrt{3} = 2 \sin \theta$

$\frac{\sqrt{3}}{2} = \sin \theta$

$\left\{ 0, \frac{\pi}{3} \right\}$

$\theta = \frac{\pi}{3}$

3. $1 = 3 \cos \theta - 2 \cos^2 \theta$

$2 \cos^2 \theta - 3 \cos \theta + 1 = 0$

$(\cos \theta - 2) (\cos \theta - 1) = 0$ ~~$\begin{matrix} 2 \\ -2 \\ -3 \end{matrix}$~~

$(\cos \theta - 1) (2 \cos \theta - 1) = 0$

$\cos \theta - 1 = 0$
 $\cos \theta = 1$ $\theta = 0$

$2 \cos \theta - 1 = 0$

$2 \cos \theta = 1$

$\cos \theta = \frac{1}{2}$ $\theta = \frac{\pi}{2}$

$\left\{ 0, \frac{\pi}{3} \right\}$

$$4. \quad 5 \cos^2 \theta = \sqrt{2} \cos \theta + 3 \cos^2 \theta$$

$$2 \cos^2 \theta - \sqrt{2} \cos \theta = 0$$

$$\cos \theta (2 \cos \theta - \sqrt{2}) = 0$$

$$\cos \theta = 0 \quad 2 \cos \theta - \sqrt{2} = 0$$

$$\theta = \frac{\pi}{2}$$

$$2 \cos \theta = \sqrt{2}$$

$$\cos \theta = \frac{\sqrt{2}}{2}$$

$$\left\{ \frac{\pi}{2}, \frac{\pi}{4} \right\}$$

$$\theta = \frac{\pi}{4}$$

$$5. \quad -\cos^2 \theta = \cos \theta + \cos^2 \theta$$

$$0 = 2 \cos^2 \theta + \cos \theta$$

$$0 = \cos \theta (2 \cos \theta + 1)$$

$$\cos \theta = 0 \quad 2 \cos \theta + 1 = 0$$

$$\theta = \frac{\pi}{2}$$

$$2 \cos \theta = -1$$

$$\cos \theta = -\frac{1}{2}$$

$$\theta = \frac{2\pi}{3}$$

$$\left\{ \frac{\pi}{2}, \frac{2\pi}{3} \right\}$$

$$6. \quad 2 \tan \theta = 1 + \tan^2 \theta$$

$$0 = \tan^2 \theta - 2 \tan \theta + 1$$

$$0 = (\tan \theta - 1)(\tan \theta - 1)$$

$$\tan \theta - 1 = 0$$

$$\tan \theta = 1$$

$$\boxed{\theta = \frac{\pi}{4}}$$

$$7. \quad -3 = -2 \cos^2 \theta - 2 + \cos \theta$$

$$2 \cos^2 \theta - \cos \theta - 1 = 0$$

$$(\cos \theta - 2)(\cos \theta + 1) = 0$$

$$\begin{array}{c} \swarrow \\ \frac{2}{2} \\ \cos \theta - 1 = 0 \end{array}$$

$$\cos \theta = 1$$

$$\theta = 0$$

$$\begin{array}{c} \searrow \\ \frac{2}{2} \\ \cos \theta + \frac{1}{2} = 0 \end{array}$$

$$\cos \theta = -\frac{1}{2}$$

$$\theta = \frac{2\pi}{3}$$

$$\boxed{\left\{ 0, \frac{2\pi}{3} \right\}}$$

$$8. \quad -2 \sin^2 \theta = -\sin \theta - 4 \sin^2 \theta$$

$$2 \sin^2 \theta + \sin \theta = 0$$

$$\sin \theta (2 \sin \theta + 1) = 0$$

$$\sin \theta = 0$$

$$\theta = 0$$

$$2 \sin \theta + 1 = 0$$

$$2 \sin \theta = -1$$

$$\sin \theta = -\frac{1}{2}$$

$$\theta = -\frac{\pi}{6}$$

$$\boxed{\left\{ 0, -\frac{\pi}{6} \right\}}$$

$$9. \cos^2 \theta + 2 = 2 \cos \theta + 1$$

$$\cos^2 \theta - 2 \cos \theta + 1 = 0$$

$$(\cos \theta - 1)(\cos \theta - 1) = 0$$

$$\cos \theta - 1 = 0$$

$$\cos \theta = 1$$

$$\boxed{\theta = 0}$$

$$10. \quad 2 = -4 \sin^2 \theta + 5$$

$$-3 = -4 \sin^2 \theta$$

$$\frac{3}{4} = \sin^2 \theta$$

$$\sin \theta = \pm \sqrt{\frac{3}{4}} \text{ or } \pm \frac{\sqrt{3}}{2}$$

$$\boxed{\theta = \frac{\pi}{3}, \theta = -\frac{\pi}{3}}$$

$$11. \quad 3 \csc \theta = 0$$

$$\csc \theta = \frac{0}{1}$$

$$\sin \theta = \frac{1}{0}$$

undefined

$$12. \quad -6 \csc \theta = -4\sqrt{3}$$

$$\csc \theta = \frac{-4\sqrt{3}}{-6}$$

$$\csc \theta = \frac{2\sqrt{3}}{3}$$

$$\sin \theta = \frac{3}{2\sqrt{3}} \left(\frac{\sqrt{3}}{\sqrt{3}} \right)$$

$$= \frac{3\sqrt{3}}{6}$$

$$\sin \theta = \frac{\sqrt{3}}{2} \quad \boxed{\theta = \frac{\pi}{3} \quad | \quad 60^\circ}$$

$$13. \quad -\frac{1}{4} \csc \theta = \frac{1}{2}$$

$$\csc \theta = \frac{1}{2} \cdot -\frac{4}{1}$$

$$\csc \theta = -\frac{4}{2}$$

$$\csc \theta = -2$$

$$\sin \theta = -\frac{1}{2}$$

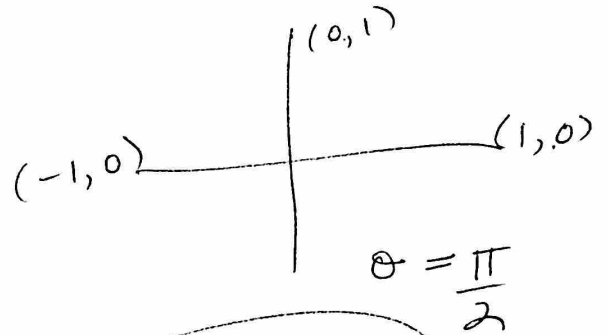
$$\theta = -\frac{\pi}{6}$$

$$\theta = -30^\circ$$

$$14. \quad -2 \cot \theta = 0$$

$$\cot \theta = 0$$

best to use
unit circle



$$\theta = 90^\circ$$

$$15. \quad -4\sqrt{3} = -4 \cot \theta$$

$$\sqrt{3} = \cot \theta$$

$$\text{or } \tan \theta = \frac{\sqrt{3}}{3}$$

$$\theta = \frac{\pi}{6} \text{ or } 30^\circ$$

$$16. \quad \frac{1}{4} \cot \theta = \frac{1}{4}$$

$$\cot \theta = \frac{1}{4} \cdot \frac{4}{1}$$

$$\cot \theta = 1$$

$$\theta = \frac{\pi}{4} \text{ or } 45^\circ$$

$$17. \quad -2\sqrt{3} = 3 \csc \theta$$

$$\frac{-2\sqrt{3}}{3} = \csc \theta$$

$$\frac{-3 \cdot \sqrt{3}}{2\sqrt{3}\sqrt{3}} = \sin \theta$$

$$-\frac{\sqrt{3}}{2} = \sin \theta$$

$$\theta = -\frac{\pi}{3} \text{ or } -60^\circ$$

$$18. \quad 5 + \sec \theta = \frac{15 + 2\sqrt{3}}{3}$$

$$\sec \theta = \frac{15}{3} + \frac{2\sqrt{3}}{3} - 5 \cdot \frac{3}{3}$$

$$\sec \theta = \frac{2\sqrt{3}}{3}$$

$$\cos \theta = \frac{3}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} \rightarrow \frac{3\sqrt{3}}{2 \cdot 3}$$

$$\cos \theta = \frac{\sqrt{3}}{2}$$

$$\theta = \frac{\pi}{6} \text{ or } 30^\circ$$

$$20. \quad -4\sqrt{2} = 4 \sec \theta$$

$$-\sqrt{2} = \sec \theta$$

$$\cos \theta = -\frac{\sqrt{2}}{2}$$

$$\theta = \frac{3\pi}{4} \text{ or } 135^\circ$$

$$19. \quad \frac{\sqrt{3}}{5} = \frac{3}{5} \cot \theta$$

$$\frac{5}{3} \cdot \frac{\sqrt{3}}{5} = \cot \theta$$

$$\frac{\sqrt{3}}{3} = \cot \theta$$

$$\text{or } \tan \theta = \sqrt{3}$$

$$\theta = \frac{\pi}{3} \text{ or } 60^\circ$$