

$$1. \quad 5 + \csc(\theta + 270^\circ) = \frac{15 + 2\sqrt{3}}{3}$$

$$\csc(\theta + 270^\circ) = \frac{15 + 2\sqrt{3}}{3} - \frac{15}{3}$$

$$\csc(\theta + 270^\circ) = \frac{2\sqrt{3}}{3}$$

$$\sin \theta = \frac{\sqrt{3}}{2}$$

$$\theta = 60^\circ$$

$$\theta + 270^\circ = 60^\circ$$

$$\boxed{\theta = -210^\circ}$$

$$2. \quad \frac{-3 + 2\sqrt{3}}{3} = -1 + \csc(\theta + 30^\circ)$$

$$\frac{-3 + 2\sqrt{3}}{3} + \frac{3}{3} = \csc(\theta + 30^\circ)$$

$$\frac{2\sqrt{3}}{3} = \csc(\theta + 30^\circ)$$

$$\frac{\sqrt{3}}{2} = \sin \theta$$

$$\theta = 60^\circ$$

$$\theta + 30^\circ = 60^\circ$$

$$\boxed{\theta = 30^\circ}$$

$$3. \quad 4 \sin(\theta + 60^\circ) = -2\sqrt{3}$$

$$\sin(\theta + 60^\circ) = \frac{-2\sqrt{3}}{4}$$

$$\sin\theta = -\frac{\sqrt{3}}{2}$$

$$\theta = -60^\circ$$

$$\theta + 60^\circ = -60^\circ$$

$$\boxed{\theta = -120^\circ}$$

$$4. \quad -\frac{1}{2} \cdot \sin(-3\theta) = -\frac{1}{4}$$

$$\sin(-3\theta) = \frac{-1}{4} \cdot \frac{-2}{1}$$

$$\sin(-3\theta) = \frac{1}{2}$$

$$\theta = \frac{30^\circ}{-3}$$

$$B = -3$$

$$\theta = -10^\circ$$

$$5. \quad 4 + \sin 3\theta = \frac{8 - \sqrt{2}}{2}$$

$$\sin 3\theta = \frac{8}{2} - \frac{\sqrt{2}}{2} - \frac{8}{2}$$

$$\sin 3\theta = -\frac{\sqrt{2}}{2}$$

$$\theta = \frac{-45^\circ}{3} \rightarrow \boxed{\theta = -15^\circ}$$

$$6. \quad 2 \csc(-3\theta + \pi) = 2$$

$$\csc(-3\theta + \pi) = 1$$

$$\sin\theta = 1$$

$$\theta = 90^\circ$$

$$-3\theta + \pi = \cancel{90^\circ} \frac{\pi}{2}$$

$$-3\theta = -\frac{\pi}{2}$$

$$\boxed{\theta = \frac{\pi}{6}}$$

$$7. \quad \sqrt{3} = 3 \tan\left(\frac{\theta}{3} + \frac{7\pi}{4}\right)$$

$$\frac{\sqrt{3}}{3} = \tan\left(\frac{\theta}{3} + \frac{7\pi}{4}\right)$$

$$\tan\theta = \frac{\sqrt{3}}{3}$$

$$\theta = \frac{\pi}{6}$$

$$\frac{\theta}{3} + \frac{7\pi}{4} = \frac{\pi}{6}$$

$$\frac{\theta}{3} = \frac{2\pi}{12} - \frac{21\pi}{12}$$

$$\frac{\theta}{3} = -\frac{19\pi}{12}$$

$$\theta = -\frac{57\pi}{12}$$

$$\boxed{\theta = -\frac{19\pi}{4}}$$

$$8. \quad 4 \cos(-3\theta + 4\frac{\pi}{3}) = 2$$

$$\cos(-3\theta + 4\frac{\pi}{3}) = \frac{1}{2}$$

$$\theta = \frac{\pi}{3}$$

$$-3\theta + 4\frac{\pi}{3} = \frac{\pi}{3}$$

$$-3\theta = -\frac{3\pi}{3}$$

$$-3\theta = -\pi$$

$$\theta = \frac{\pi}{3}$$

$$9. \quad 4 \cos(2\theta + 3\frac{\pi}{4}) = 4$$

$$\cos(2\theta + 3\frac{\pi}{4}) = 1$$

$$\cos \theta = 1$$

$$\theta = 0$$

$$2\theta + 3\frac{\pi}{4} = 0$$

$$2\theta = -3\frac{\pi}{4}$$

$$\theta = -\frac{3\pi}{8}$$

$$10. \quad 5 + \sin(-2\theta + \frac{\pi}{3}) = \frac{11}{2}$$

$$\sin(-2\theta + \frac{\pi}{3}) = \frac{1}{2}$$

$$\sin \theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{6}$$

$$-2\theta + \frac{\pi}{3} = \frac{\pi}{6}$$

$$-2\theta = -\frac{\pi}{6}$$

$$\theta = \frac{\pi}{12}$$