

1. $2 \cos^2 \frac{\theta}{3} - 1 = 0$ *square root prop.*

$$2 \cos^2 \frac{\theta}{3} = 1$$

$$\cos^2 \frac{\theta}{3} = \frac{1}{2} \quad B = \frac{1}{3}$$

$$\cos \theta = \pm \sqrt{\frac{1}{2}}$$

$$\text{or } \cos \theta = \pm \frac{\sqrt{2}}{2}$$

$$\theta = \frac{\pi}{4}, \quad \theta = \frac{3\pi}{4}$$

$\frac{1}{3}$ $\frac{1}{3}$

$$\theta = \frac{3\pi}{4}, \quad \theta = \frac{9\pi}{4}$$

same as

$$\theta = 135^\circ, \quad \theta = 405^\circ$$

3. $2 \cos^2 \theta = \cos \theta + 1$ *BIGX*

$$2 \cos^2 \theta - \cos \theta - 1 = 0$$

$$-2 \begin{array}{c} -2 \\ \times \\ -1 \end{array} + 1$$

$$\frac{(\cos \theta - 2)}{2} \frac{(\cos \theta + 1)}{2} = 0$$

$$\cos \theta - 1 = 0 \quad \cos \theta + \frac{1}{2} = 0$$

$$\cos \theta = 1 \quad \cos \theta = -\frac{1}{2}$$

$$\theta = 0 \text{ and } \theta = 120^\circ$$

2. $2 \cos^2 \left(\frac{2\theta}{3}\right) + \cos \left(\frac{2\theta}{3}\right) = 0$ *factor BIGX*

GCF $\cos \theta (2 \cos \theta + 1) = 0$

$$\cos \theta = 0 \quad 2 \cos \theta + 1 = 0$$

$$\theta = 90^\circ$$

$\frac{2}{3}$

$$2 \cos \theta = -1$$

$$\cos \theta = -\frac{1}{2}$$

$$\theta = 120^\circ$$

$$\theta = 135^\circ, \quad \theta = 180^\circ$$

4. $3 \sec^2 5\theta = 2 \sec^2 5\theta + 2$ *B = 5*

$$\sec^2 5\theta = 2$$

$$\cos^2 \theta = \frac{1}{2}$$

$$\cos \theta = \pm \sqrt{\frac{1}{2}} = \pm \frac{\sqrt{2}}{2}$$

$$\theta = 45^\circ$$

$\frac{5}{5}$

$$\theta = 135^\circ$$

$\frac{5}{5}$

$$\theta = 9^\circ, \quad \theta = 27^\circ$$

$$5. 3 \tan^2 \theta (\theta + 10^\circ) - 1 = 0$$

$$3 \tan^2 (\theta + 10^\circ) = 1$$

Square root

$$\tan^2 (\theta + 10^\circ) = \frac{1}{3}$$

$$\tan \theta = \pm \sqrt{\frac{1}{3}} = \pm \frac{\sqrt{3}}{3}$$

$$\theta = 30^\circ, \theta = -30^\circ$$

$$\theta + 10^\circ = 30^\circ \quad \theta + 10^\circ = -30^\circ$$

$$\boxed{\theta = 20^\circ \quad \theta = -40^\circ}$$

$$6. \csc^2 \theta + 4 \csc \theta + 4 = 0$$

Binomial

$$2 \begin{array}{c} 4 \\ \times \\ 2 \\ 4 \end{array}$$

$$(\csc \theta + 2)(\csc \theta + 2) = 0$$

$$\csc \theta + 2 = 0$$

$$\csc \theta = -2$$

$$\sin \theta = -\frac{1}{2}$$

$$\boxed{\theta = -30^\circ}$$

$$7. \cos \theta \tan^2 \theta = 3 \cos \theta$$

ACF/

$$\cos \theta \tan^2 \theta - 3 \cos \theta = 0$$

Square root

$$\cos \theta (\tan^2 \theta - 3) = 0$$

$$\cos \theta = 0$$

$$\tan^2 \theta - 3 = 0$$

$$\tan^2 \theta = 3$$

$$\tan \theta = \pm \sqrt{3}$$

$$\boxed{\theta = 90^\circ \quad \theta = 60^\circ, \theta = -60^\circ}$$

$$8. \sin 2\theta = \sqrt{2} - \sin 2\theta$$

$$2 \sin 2\theta = \sqrt{2} \quad \text{linear}$$

$$\sin 2\theta = \frac{\sqrt{2}}{2}$$

$$\theta = \frac{45^\circ}{2}$$

$$\boxed{\theta = 22.5^\circ}$$

$$9. 8 \sec(\theta - 30^\circ) = 8\sqrt{2}$$

$$\sec(\theta - 30^\circ) = \sqrt{2} \quad \text{linear}$$

$$\cos \theta = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\theta = 45^\circ$$

$$\theta - 30^\circ = 45^\circ$$

$$\boxed{\theta = 75^\circ}$$

$$10. 4 \sin^2 \theta (2\theta + 15^\circ) = 1$$

Square root

$$\sin^2 \theta (2\theta + 15^\circ) = \frac{1}{4}$$

$$\sin \theta = \pm \sqrt{\frac{1}{4}} = \pm \frac{1}{2}$$

$$\theta = 30^\circ, \theta = -30^\circ$$

$$2\theta + 15^\circ = 30^\circ$$

$$2\theta = 15^\circ$$

$$\theta = 7.5^\circ$$

$$2\theta + 15^\circ = -30^\circ$$

$$2\theta = -45^\circ$$

$$\theta = -22.5^\circ$$