

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$\cot^2 \theta + 1 = \csc^2 \theta$$

### Trig Identities

#### Simplify each expression

$$1. \sin^2 x + \sin^2 x \cot^2 x$$

$$\sin^2 x (1 + \cot^2 x)$$

$$\sin^2 x (\csc^2 x)$$

$$\frac{\sin^2 x}{\cancel{\sin^2 x}} \left( \frac{1}{\cancel{\sin^2 x}} \right)$$

$$\boxed{1}$$

$$2. \frac{\sin^2 \theta + \cos^2 \theta}{\sin^2 \theta}$$

$$\frac{1}{\sin^2 \theta}$$

$$\boxed{\csc^2 \theta}$$

$$3. \frac{\csc^2 x}{1 + \tan^2 x}$$

$$\frac{\csc^2 x}{\sec^2 x}$$

$$\csc^2 x \cdot \frac{1}{\sec^2 x}$$

$$\frac{1}{\sin^2 x} \cdot \cos^2 x$$

$$5. \frac{\sin^2 x - \cos^2 x \sin^2 x}{\sin^2 x}$$

$$\frac{\cos^2 x}{\sin^2 x}$$

$$\boxed{\cot^2 x}$$

$$4. \sin \theta (1 + \csc \theta)$$

$$\sin \theta + \sin \theta \csc \theta$$

$$\sin \theta + \sin \theta \cdot \frac{1}{\sin \theta}$$

$$\boxed{\sin \theta + 1}$$

$$6. 1 + \frac{\tan^2 x}{1 + \sec x}$$

$$\frac{1 + \sec x}{1 + \sec x} + \frac{\tan^2 x}{1 + \sec x}$$

$$\frac{1 + \sec x + \tan^2 x}{1 + \sec x} \rightarrow \frac{\sec^2 x + \sec x}{1 + \sec x}$$

$$\frac{\sin^2 x (1 - \cos^2 x)}{\sin^2 x}$$

$$\boxed{\sin^2 x}$$

$$8. \cos^2 \theta (1 + \tan^2 \theta)$$

$$\cos^2 \theta \cdot \sec^2 \theta$$

$$\cos^2 \theta \cdot \frac{1}{\cos^2 \theta}$$

$$\boxed{1}$$

$$\frac{\sec x (\sec x + 1)}{1 + \sec x}$$

$$\boxed{\sec x}$$

$$7. (1 - \cos \theta)(1 + \cos \theta)$$

$$1 - \cos^2 \theta$$

$$\boxed{\sin^2 \theta}$$

#### Verify each identity

$$9. \frac{\sec^2 \theta - 1}{\sec^2 \theta} = \sin^2 \theta$$

$$\frac{\tan^2 \theta}{\sec^2 \theta}$$

$$\tan^2 \theta \cdot \frac{1}{\sec^2 \theta}$$

$$\frac{\sin^2 \theta \cdot \cos^2 \theta}{\cos^2 \theta}$$

$$\sin^2 \theta //$$

$$10. \cos^2 \theta + \tan^2 \theta \cos^2 \theta = 1$$

$$\cos^2 \theta (1 + \tan^2 \theta)$$

$$\cos^2 \theta \cdot \sec^2 \theta$$

$$\cos^2 \theta \cdot \frac{1}{\cos^2 \theta}$$

$$1 //$$

11.  $\sec x - \cos x = \sin x \tan x$

$$\frac{1}{\cos x} - \cos x$$

$$\frac{1 - \cos^2 x}{\cos x}$$

$$\frac{\sin^2 x}{\cos x}$$

$$\frac{\sin x \cdot \sin x}{\cos x}$$

$$\sin x \tan x //$$

13.  $\frac{\csc^2 \theta - \cot^2 \theta}{1 - \sin^2 \theta} = \sec^2 \theta$

$$\frac{1}{\cos^2 \theta}$$

$$\sec^2 \theta //$$

12.  $\frac{\sec \theta}{\sin \theta} - \frac{\sin \theta}{\cos \theta} = \cot \theta$

$$\sec \theta \cdot \frac{1}{\sin \theta} - \frac{\sin \theta}{\cos \theta}$$

$$\frac{1}{\cos \theta} \cdot \frac{1}{\sin \theta} - \frac{\sin \theta}{\cos \theta}$$

$$\frac{1}{\cos \theta \sin \theta} - \frac{\sin^2 \theta}{\cos \theta \sin \theta}$$

$$\frac{1 - \sin^2 \theta}{\cos \theta \sin \theta}$$

$$\frac{\cos^2 \theta}{\cos \theta \sin \theta}$$

$$\frac{\cos \theta}{\sin \theta}$$

$$\cot \theta //$$

14.  $\frac{1}{\sec^2 \theta} + \frac{1}{\csc^2 \theta} = 1$

$$\cos^2 \theta + \sin^2 \theta$$

$$1 //$$

15.  $\sec \theta = \sin \theta (\tan \theta + \cot \theta)$

$$\sin \theta \left( \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} \right)$$

$$\frac{\sin^2 \theta}{\cos \theta} + \frac{\sin \theta \cos \theta}{\sin \theta}$$

$$\frac{\sin^2 \theta}{\cos \theta} + \frac{\cos \theta \cdot \cos \theta}{\cos \theta}$$

16.  $\frac{1 + \tan \theta}{\tan \theta} = 1 + \cot \theta$

$$\frac{1}{\tan \theta} + \frac{\tan \theta}{\tan \theta}$$

$$\cot \theta + 1 //$$

17.  $\sin \theta + \cot \theta \cos \theta = \csc \theta$

$$\sin \theta + \frac{\cos \theta \cdot \cos \theta}{\sin \theta}$$

$$\sin \theta + \frac{\cos^2 \theta}{\sin \theta}$$

$$\frac{\sin^2 \theta}{\sin \theta} + \frac{\cos^2 \theta}{\sin \theta}$$

$$\frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta}$$

$$\frac{1}{\sin \theta}$$

$$\csc \theta //$$

18.  $\csc \theta - \cot \theta = \frac{\sin \theta}{1 + \cos \theta} \cdot \frac{(1 - \cos \theta)}{(1 - \cos \theta)}$

$$\frac{\sin \theta (1 - \cos \theta)}{1 - \cos^2 \theta}$$

$$\frac{\sin \theta (1 - \cos \theta)}{\sin^2 \theta}$$

$$\frac{\sin \theta (1 - \cos \theta)}{1 - \cos \theta}$$

$$\frac{1}{\sin \theta}$$

19.  $\frac{\tan^2 \theta}{\tan^2 \theta + 1} = \sin^2 \theta$

$$\frac{\tan^2 \theta}{\sec^2 \theta}$$

$$\tan^2 \theta \cdot \frac{1}{\sec^2 \theta}$$

$$\frac{\tan^2 \theta \cdot \cos^2 \theta}{\cos^2 \theta}$$

$$\sin^2 \theta //$$

20.  $\csc x - \sin x = \cot x \cos x$

$$\frac{1}{\sin x} - \sin x$$

$$\frac{1}{\sin x} - \frac{\sin^2 x}{\sin x}$$

$$\frac{1 - \sin^2 x}{\sin x}$$

$$\frac{\cos^2 x}{\sin x}$$

$$\frac{\cos x \cdot \cos x}{\sin x}$$

$$\cot x \cos x //$$