

1. Compute $u \cdot v$

a. $u = \langle 2, 3 \rangle$ and $v = \langle 0, 5 \rangle$

$$\begin{aligned} & (2 \cdot 0) + (3 \cdot 5) \\ & 0 + 15 \\ & 15 \end{aligned}$$

b. $u = \langle 3, 0 \rangle$ and $v = \langle 0, -2 \rangle$

$$\begin{aligned} & (3 \cdot 0) + (0 \cdot -2) \\ & 0 + 0 \\ & 0 \end{aligned}$$

c. $u = \langle -2, 1 \rangle$ and $v = \langle 2, 4 \rangle$

$$\begin{aligned} & (-2 \cdot 2) + (1 \cdot 4) \\ & -4 + 4 \\ & 0 \end{aligned}$$

d. $u = 2i - 5j$ and $v = -4i + 10j$

$$\begin{aligned} & (2 \cdot -4) + (-5 \cdot 10) \\ & -8 + -50 \\ & -58 \end{aligned}$$

~~X~~ Which of the vectors in #1 are orthogonal? Which of them are parallel?

Use the vectors $u = \langle 1, -2 \rangle$, $v = \langle -9, -1 \rangle$ and $w = \langle 3, 4 \rangle$ to find the indicated quantity. Show all work.

3. $(u \cdot v)v$ $\langle 63, 7 \rangle$

$$\begin{aligned} & (1 \cdot -9) + (-2 \cdot -1) \quad \langle -9, -1 \rangle \\ & (-9 + 2) \quad \langle -9, -1 \rangle \\ & -7 \quad \langle -9, -1 \rangle \end{aligned}$$

4. $4u \cdot v$ -28

$$\begin{aligned} & \overset{4u}{\langle 4, -8 \rangle} \cdot \langle -9, -1 \rangle \\ & (4 \cdot -9) + (-8 \cdot -1) \\ & -36 + 8 \end{aligned}$$

5. $-3w \cdot v$ 93

$$\begin{aligned} & \langle -9, -12 \rangle \cdot \langle -9, -1 \rangle \\ & (-9 \cdot -9) + (-12 \cdot -1) \\ & 81 + 12 \end{aligned}$$

6. $(v \cdot u) - (w \cdot v)$ 24

$$\begin{aligned} & (1 \cdot -9) + (-2 \cdot -1) \quad (3 \cdot -9) + (4 \cdot -1) \\ & -9 + 2 \quad -27 + -4 \\ & -7 \quad -31 \\ & 24 \end{aligned}$$

7. $-3(w \cdot v)$ 93

$$\begin{aligned} & -3[(-9 \cdot 3) + (-1 \cdot 4)] \\ & -3(-27 + -4) \end{aligned}$$

8. $u(v \cdot v)$ $\langle 82, -164 \rangle$

$$\begin{aligned} & \langle 1, -2 \rangle [(-9 \cdot -9) + (-1 \cdot -1)] \\ & \langle 1, -2 \rangle [81 + 1] \\ & \langle 1, -2 \rangle (82) \end{aligned}$$

Find the angle between the vectors u and v

9. $u = \langle 1, 4 \rangle$ and $v = \langle 5, -3 \rangle$

$$\begin{aligned} & \cos^{-1} \left(\frac{(1 \cdot 5) + (4 \cdot -3)}{\sqrt{1^2 + 4^2} \sqrt{5^2 + 3^2}} \right) \\ & \cos^{-1} \left(\frac{-7}{\sqrt{17} \sqrt{34}} \right) \approx 106.9^\circ \end{aligned}$$

10. $u = -8i + j$ and $v = 2i + 6j$

$$\begin{aligned} & \cos^{-1} \left(\frac{(-8 \cdot 2) + (1 \cdot 6)}{\sqrt{8^2 + 1^2} \sqrt{2^2 + 6^2}} \right) \\ & \cos^{-1} \left(\frac{-10}{\sqrt{65} \sqrt{40}} \right) \approx 101.3^\circ \end{aligned}$$

Determine whether the following vectors are orthogonal, parallel, or neither.

1. $u = \langle -12, 30 \rangle$ and $v = \langle 8, -20 \rangle$ **NOT ORTHOGONAL**
 $u \cdot v = (-12 \cdot 8) + (30 \cdot -20) = -96 + -600 = -696$
 $\sqrt{12^2 + 30^2} \cdot \sqrt{8^2 + 20^2} = 696$
PARALLEL
2. $u = \langle 2, -3 \rangle$ and $v = \langle 6, 4 \rangle$
 $u \cdot v = (2 \cdot 6) + (-3 \cdot 4) = 12 + -12 = 0$
ORTHOGONAL
3. $u = \langle -2, 1 \rangle$ and $v = \langle 2, 4 \rangle$
 $u \cdot v = (-2 \cdot 2) + (1 \cdot 4) = -4 + 4 = 0$
ORTHOGONAL
4. $u = 2i - 5j$ and $v = 4i + 7j$
 $u \cdot v = (2 \cdot 4) + (-5 \cdot 7) = 8 + -35 = -27$
NEITHER
 $\sqrt{2^2 + 5^2} \sqrt{4^2 + 7^2} = \sqrt{29} \sqrt{65} \neq 27$
NOT PARALLEL

Find the work done using the formula $work = force \cdot displacement$

5. A constant force $F = \langle 2, 5 \rangle$ moves an object along the vector $\langle -1, 5 \rangle$. Find the work done during this process.

$$\begin{aligned} & \langle 2, -1 \rangle + \langle 5, 5 \rangle \\ & -2 + 25 \\ & \boxed{23 \text{ joules}} \end{aligned}$$

6. Find the work done by the force $F = -2i + 3j$ in moving an object from the point $(1, 4)$ to the point $(3, -5)$

$$\begin{aligned} & \langle -2, 3 \rangle \cdot \langle 2, -9 \rangle \\ & (-2 \cdot 2) + (3 \cdot -9) \\ & -4 + -27 \\ & \boxed{-31 \text{ joules}} \end{aligned}$$

$$\begin{aligned} & i + \\ & \langle 3 - 1, -5 - 4 \rangle \\ & \langle 2, -9 \rangle \end{aligned}$$

7. A 36 N force acting at 36° moves a box 20 m horizontally. Find the work done.

$$\begin{aligned} & \langle 36 \cos 36^\circ, 36 \sin 36^\circ \rangle \\ & \langle 29.1, 21.2 \rangle \cdot \langle 20, 0 \rangle \\ & (29.1 \cdot 20) + (21.2 \cdot 0) \\ & \boxed{582 \text{ joules}} \end{aligned}$$

8. Bob is carrying a pack of books from the table to the shelf with a force of 35 pounds at a 28° angle. If the distance from the table to the shelf is 20 feet horizontally. How much work has Bob done when he carries these books?

$$\begin{aligned} & \langle 35 \cos 28^\circ, 35 \sin 28^\circ \rangle \\ & \langle 30.9, 16.4 \rangle \cdot \langle 20, 0 \rangle \\ & (30.9 \cdot 20) + (16.4 \cdot 0) \\ & \boxed{618 \text{ joules}} \end{aligned}$$

9. How much work is done by a force $F = \langle 9, 7 \rangle$ in moving an object from $(-1, 6)$ to $(2, 9)$?

$$\begin{aligned} & \langle 9, 7 \rangle \cdot \langle 3, 3 \rangle \\ & (9 \cdot 3) + (7 \cdot 3) \\ & 27 + 21 \\ & \boxed{48 \text{ joules}} \end{aligned}$$

$$\begin{aligned} & \langle 2 - (-1), 9 - 6 \rangle \\ & \langle 3, 3 \rangle \end{aligned}$$

10. A 50 N force acting at 260° moves an object 14 m at 215° . Find the work done.

$$\begin{aligned} & \langle 50 \cos 260^\circ, 50 \sin 260^\circ \rangle \quad \langle 14 \cos 215^\circ, 14 \sin 215^\circ \rangle \\ & \langle -8.7, -49.2 \rangle \cdot \langle -11.5, -8.0 \rangle \\ & (-8.7 \cdot -11.5) + (-49.2 \cdot -8.0) \\ & 100.05 + 393.6 \\ & \boxed{493.65 \text{ joules}} \end{aligned}$$