

# How Do I Graph an Ellipse?

$a^2 = \text{larger denominator}$

Horizontal Major Axis

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

Vertical Major Axis

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

**Focus**

(Note:  $a > b > 0$ )

To find the foci, use the equation  $c^2 = a^2 - b^2$

$$1. \frac{x^2}{9} + \frac{y^2}{4} = 1$$

Hor V? **H**

$a = 3$     $b = 2$     $c = \sqrt{c^2 = a^2 - b^2} = \sqrt{9-4} = \sqrt{5}$

Vertices: Endpoints of major axis  $(-3, 0), (3, 0)$

CO-Vertices: Endpoints of minor axis  $(0, 2), (0, -2)$

Coordinates of foci: Place on major axis!  $(-\sqrt{5}, 0), (\sqrt{5}, 0)$

Length of major axis: 6 (double "a" value)

Length of minor axis: 4 (double the "b" value)

**HORIZONTAL  
MAJOR**

Center  $(0, 0)$

$$c^2 = a^2 - b^2$$

$$c^2 = 9 - 4$$

$$c^2 = 5$$

$$c = \sqrt{5}$$

$$r \approx 2.2$$

$$2. \frac{x^2}{25} + \frac{y^2}{1} = 1$$

Hor V? **V** Center  $(0, 0)$

$a = 5$     $b = 1$     $c = \sqrt{24}$

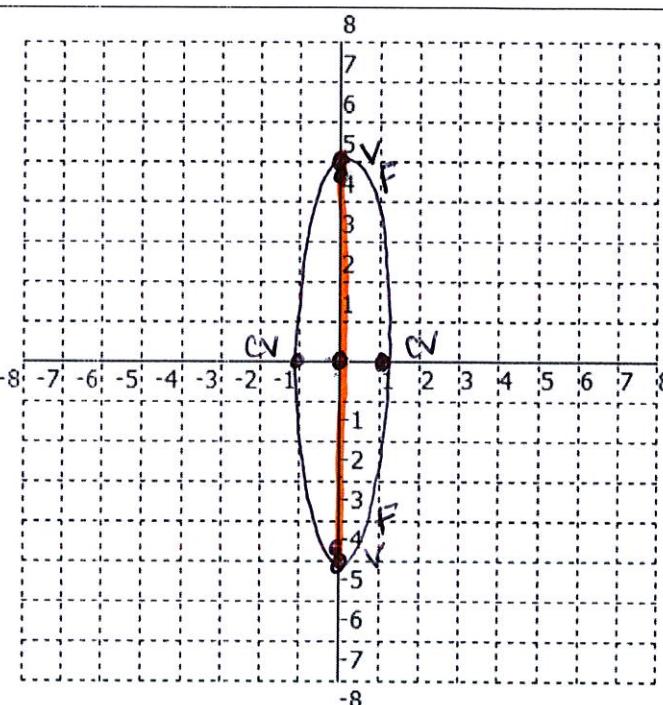
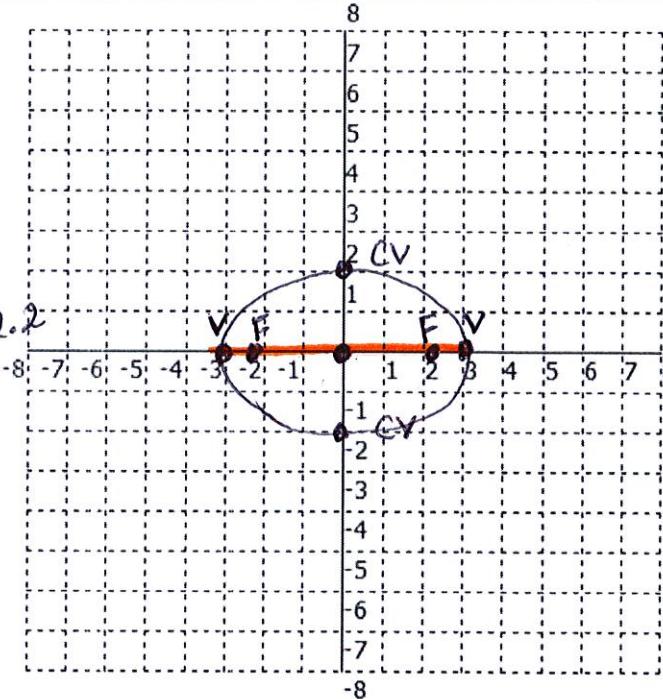
Endpoints of major axis:  $(0, 5), (0, -5)$

Endpoints of minor axis:  $(-1, 0), (1, 0)$

Coordinates of foci:  $(0, \sqrt{24}), (0, -\sqrt{24})$

Length of major axis:  $2(5) = 10$

Length of minor axis:  $2(1) = 2$



$$C^2 = 16 - 4$$

$$3. \frac{(x+3)^2}{4} + \frac{(y-1)^2}{16} = 1 \quad C^2 = 12 \quad C = \sqrt{12} \approx 3.$$

H or V? V Center (-3, 1)

$a = 4$   $b = 2$   $c = \sqrt{12}$  or  $2\sqrt{3}$

Endpoints of major axis  $(-3, 1) \rightarrow (-3, 5)$   
 $\begin{array}{r} +4 \\ \hline -3 \end{array}$   $\begin{array}{r} 2\sqrt{3} \text{ OR} \\ -4 \end{array}$

Endpoints of minor axis  $(-3, 1) \rightarrow (-1, 1), (-5, 1)$

Coordinates of foci  $(-3, 1 \pm 2\sqrt{3})$

Length of major axis 8

Length of minor axis 4

$$C^2 = 9 - 6$$

$$C^2 = 3$$

$$C = \sqrt{3}$$

$$4. \frac{2x^2}{18} + \frac{3(y+2)^2}{18} = \frac{18}{18} \quad \left| \frac{x^2}{9} + \frac{(y+2)^2}{6} = 1 \right. \quad \boxed{9} \quad \boxed{6}$$

H or V? H Center (0, -2)

$a = \sqrt{9} = 3$   $b = \sqrt{6} \approx 2.4$   $c = \sqrt{3} \approx 1.7$

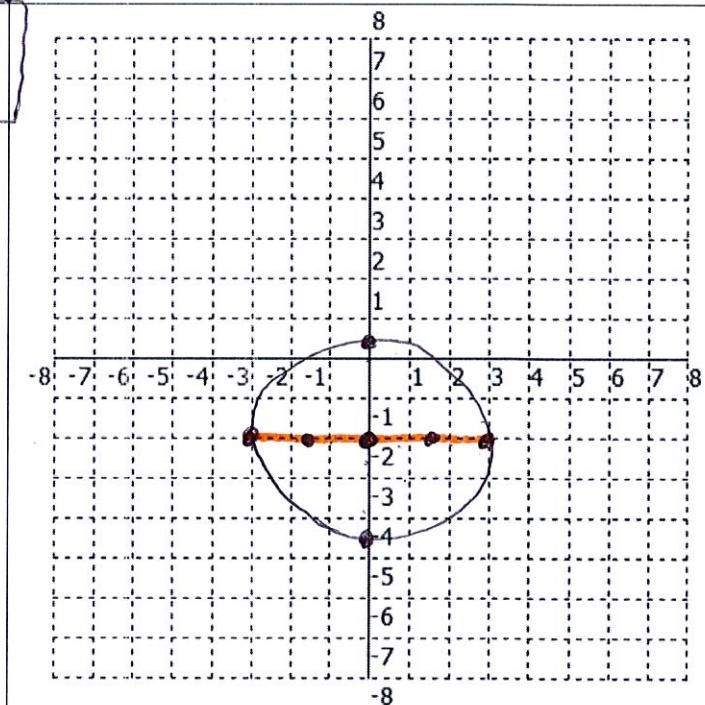
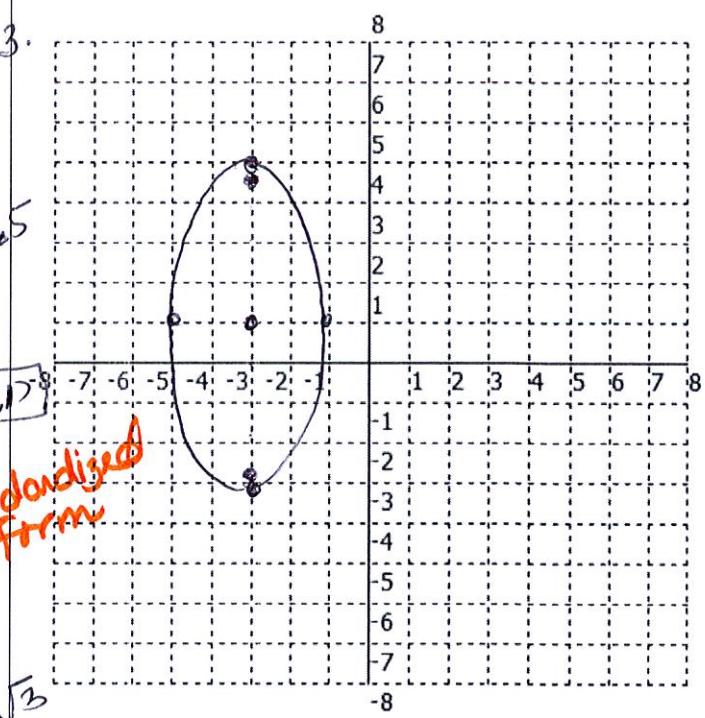
Endpoints of major axis  $(3, -2), (-3, -2)$

Endpoints of minor axis  $(0, -2 \pm \sqrt{6})$

Coordinates of foci  $(\pm\sqrt{3}, 0)$

Length of major axis 6

Length of minor axis  $2\sqrt{6}$



How can you tell by looking at the equation of an ellipse whether the graph will be horizontal or vertical?

If the larger denominator belongs to  $x^2$ , it's horizontal;  
 if it belongs to  $y^2$ , it's vertical

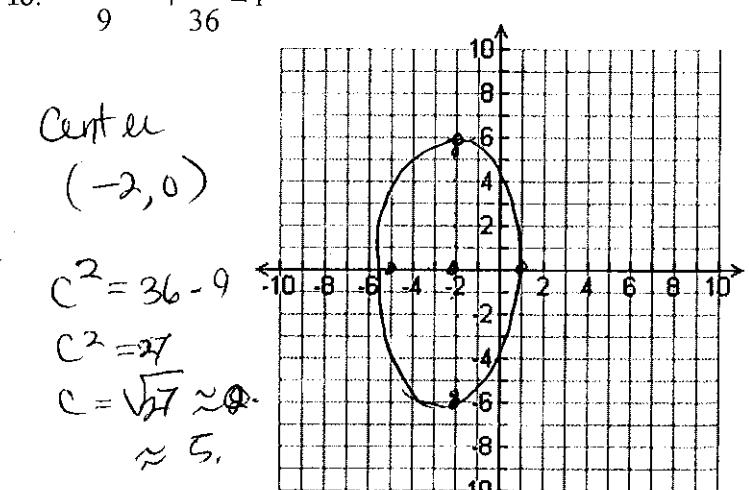
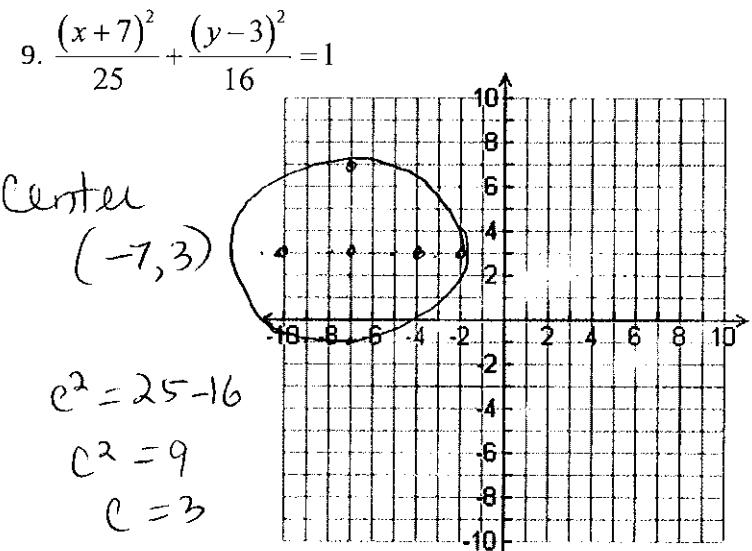
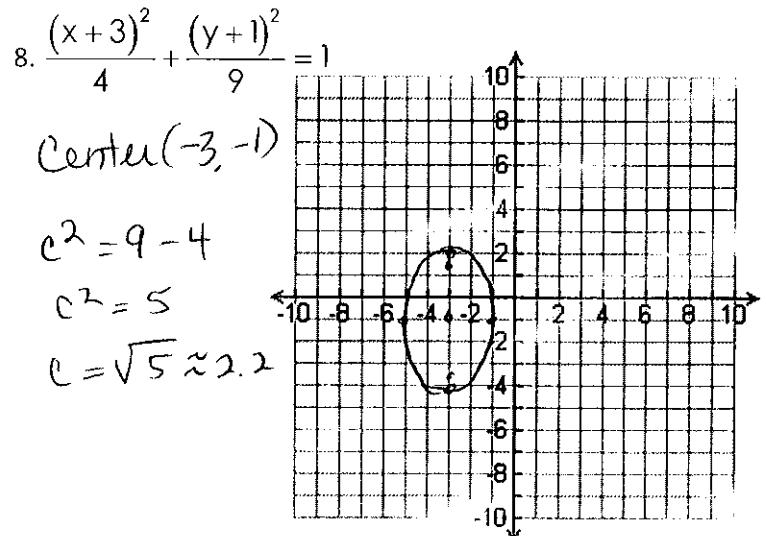
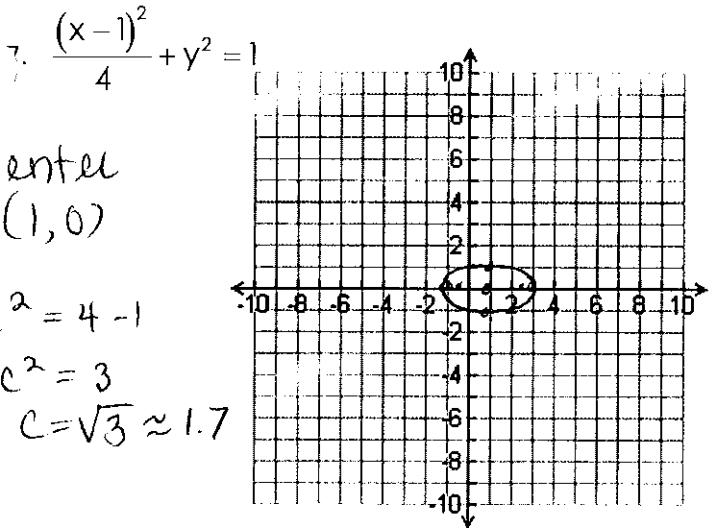
$\checkmark$  Center  $(0, -2)$   $\rightarrow$   $(3, -2), (-3, -2)$

$\times \vee$  Center  $(0, -2)$   $\rightarrow$   $(0, -2 \pm \sqrt{6})$

$F$   $(0, -2)$   $\rightarrow$   $(\pm\sqrt{3}, -2)$

For problems 1-6, complete the chart with the appropriate information.

Equation	Center	Length of Major Axis	Length of Minor Axis
1. $\frac{(x+3)^2}{9} + \frac{(y+1)^2}{4} = 1$	(-3, -1)	6	4
2. $\frac{(x+3)^2}{9} + y^2 = 1$	(-3, 0)	6	2
3. $\frac{x^2}{16} + \frac{(y+2)^2}{25} = 1$	(0, -2)	10	8
4. $\frac{(x-3)^2}{4} + \frac{(y-1)^2}{81} = 1$	(3, 1)	18	4
5. $(x+1)^2 + \frac{(y+4)^2}{144} = 1$	(-1, -4)	24	2
6. $\frac{(y+6)^2}{49} + \frac{(x-7)^2}{25} = 1$	(7, -6)	14	10



Rewriting in standard form

$$18. \quad 9x^2 + 25y^2 - 36x + 50y - 164 = 0$$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

Rearrange terms

$$\underbrace{9x^2 - 36x}_{\text{factor out the GCF for each variable; complete the square}}$$

$$\underbrace{+ 25y^2 + 50y}_{\text{factor out the GCF for each variable; complete the square}}$$

$$\begin{cases} = 164 \\ + 36 \\ + 25 \end{cases}$$

$$9 \left( x^2 - 4x + \frac{4}{9} \right) + 25 \left( y^2 + 2y + \frac{1}{25} \right)$$

$$\frac{9(x-2)^2}{225} + \frac{25(y+1)^2}{225} = \frac{225}{225}$$

Rewrite in binomial form

$$\frac{(x-2)^2}{25} + \frac{(y+1)^2}{9} = 1$$

$$19. \quad x^2 + 4y^2 + 10x - 8y + 13 = 0$$

$$x^2 + 10x + \frac{25}{16} + 4(y^2 - 2y + \frac{1}{4}) = \frac{-13}{16} + \frac{25}{16}$$

$$\frac{(x+5)^2}{16} + \frac{4(y-1)^2}{16} = \frac{16}{16}$$

$$\boxed{\frac{(x+5)^2}{16} + \frac{(y-1)^2}{4} = 1}$$

$$20. \quad 4x^2 + y^2 + 16x - 6y - 39 = 0$$

$$4x^2 + 16x + y^2 - 6y = 39$$

$$4(x^2 + 4x + 4) + y^2 - 6y + 9 = 39$$

$$\frac{4(x+2)^2}{64} + \frac{(y-3)^2}{64} = \frac{64}{64} + 9$$

$$\boxed{\frac{(x+2)^2}{16} + \frac{(y-3)^2}{64} = 1}$$

$$21. \quad 4x^2 + 25y^2 - 24x + 100y + 36 = 0$$

$$4x^2 - 24x + 25y^2 + 100y = -36$$

$$4(x^2 - 6x + 9) + 25(y^2 + 4y + 4) = -36$$

$$\frac{4(x-3)^2}{100} + \frac{25(y+2)^2}{100} = \frac{100}{100} + 36$$

$$\boxed{\frac{(x-3)^2}{25} + \frac{(y+2)^2}{4} = 1}$$

