

$$1. \frac{\cot\theta}{\tan\theta}$$

$$\cot\theta = \frac{1}{\tan\theta}$$

$$\cot\theta \cdot \cot\theta$$

$$\boxed{\cot^2\theta}$$

$$2. \frac{\sin^2\theta}{\cos^2\theta} + \sec\theta \csc\theta$$

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{1}{\csc\theta} \cdot \csc\theta$$

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\csc\theta}{\csc\theta} \quad (\csc\theta) \quad (\csc\theta)$$

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta}$$

$$3. \sec(\csc\theta + \sin\theta)$$

$$\frac{1}{\csc\theta} (\csc\theta + \sin\theta)$$

$$\frac{\csc\theta}{\csc\theta} + \frac{\sin\theta}{\csc\theta}$$

$$\boxed{1 + \tan\theta}$$

$$\frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta}$$

$$\frac{1}{\cos^2\theta}$$

$$\frac{1}{\csc^2\theta}$$

$$\boxed{\sec^2\theta}$$

$$4. (\cos\theta + \sin\theta)^2$$

$$(\cos\theta + \sin\theta)(\cos\theta + \sin\theta)$$

$$\cos^2\theta + \cos\theta \sin\theta + \sin\theta \cos\theta + \sin^2\theta$$

$$\cos^2\theta + 2\cos\theta \sin\theta + \sin^2\theta$$

$$\boxed{1 + 2\cos\theta \sin\theta}$$

Pythagorean Identity

$$5. \sec \theta + \tan \theta$$

$$\csc \theta$$

$$\sec \theta + \tan \theta \cdot \frac{1}{\csc \theta}$$

$$\frac{1}{\cos \theta} \cdot \frac{\sin \theta}{\cos \theta} \cdot \frac{\sin \theta}{\cos \theta}$$

$$\frac{\sin^2 \theta}{\cos^2 \theta} = \boxed{\tan^2 \theta}$$

$$8. \sin \theta = \frac{1}{\csc \theta}$$

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\cos \theta = \frac{1}{\sec \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\tan \theta = \frac{1}{\cot \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$y = x^2$
vertical opening
 $p = 4$

$$6. \sin^2 \theta + \cot^2 \theta + \cos^2 \theta$$

Pythagorean Identity

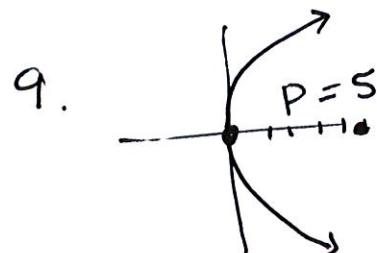
$$\frac{1 + \cot^2 \theta}{\csc^2 \theta}$$

$$7. \boxed{\cos^2 \theta + \sin^2 \theta = 1}$$

$$\frac{\cos^2 \theta}{\sin^2 \theta} \quad \frac{\cos^2 \theta}{\sin^2 \theta} \quad \frac{\cos^2 \theta}{\sin^2 \theta}$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$\boxed{\cot^2 \theta + 1 = \csc^2 \theta}$$



Horizontal opening right

$$x = y^2$$

$$x = \frac{1}{20} y^2$$

$$10. (x+3)^2 = 16(y+4)$$

vertex $(-3, -4)$
focus $(-3, 0)$

$$\frac{1}{16}(x+3)^2 = y+4$$

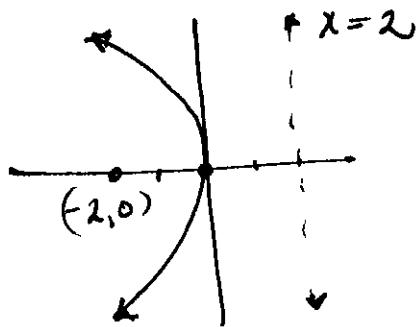
$$\frac{1}{16}(x+3)^2 - 4 = y$$

directrix: $y = -8$

11. Graph $x = -\frac{1}{8}y^2$

Horizontal,
opens to
the left
vertex $(0, 0)$

$$P = 2$$

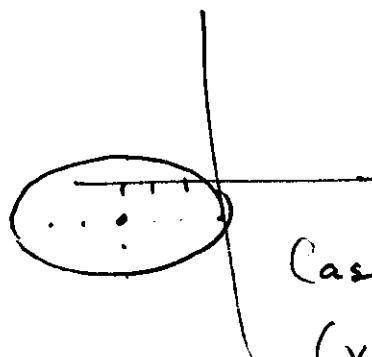


12. ellipse

center $(-3, -1)$

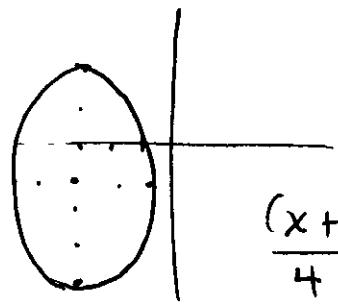
$$a = 3, b = 2$$

ORIENTATION
not
specified



Case I:

$$\frac{(x+3)^2}{9} + \frac{(y+1)^2}{4} = 1$$



Case II

$$\frac{(x+3)^2}{4} + \frac{(y+1)^2}{9} = 1$$

13. circle

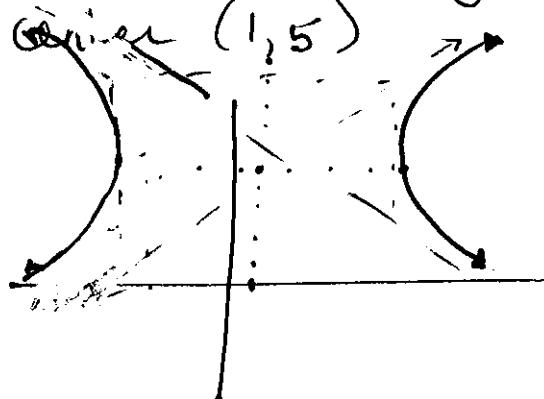
center $(12, -9)$; $r=4$

$$(x-12)^2 + (y+9)^2 = 16$$

14. $\frac{(x-1)^2}{16} - \frac{(y-5)^2}{25} = 1$

Horizontal opening

center $(1, 5)$



$$15. \quad 9x^2 + 25y^2 - 36x + 50y - 164 = 0$$

$$9x^2 - 36x + 25y^2 + 50y = 164$$

$$9(x^2 - 4x + 4) + 25(y^2 + 2y + 1) = 164$$

+ 36

+ 25

$$9(x-2)^2 + 25(y+1)^2 = 225$$

$$\frac{(x-2)^2}{25} + \frac{(y+1)^2}{9} = 1$$

Horizontal ellipse
center (2, -1)

$$16. \quad y^2 + 12x - 2y - 35 = 0$$

$$12x = -y^2 + 2y + 35$$

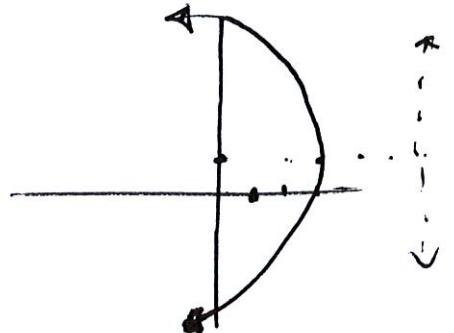
Graph: parabola
vertex (3, 1)

opens left

$$12x = -(y^2 - 2y + 1) + 35$$

$$12x = -(y-1)^2 + 36$$

$$x = -\frac{1}{12}(y-1)^2 + 3$$



$$17. \quad 16x^2 - y^2 + 32x + 6y + 39 = 0$$

$$16x^2 + 32x - y^2 + 6y = -39$$

$$16(x^2 + 2x + 1) - (y^2 - 6y + 9) = -39$$

$$16(x+1)^2 - (y-3)^2 = -32$$

$$-\frac{(x+1)^2}{2} + \frac{(y-3)^2}{32} = 1$$

Vertical
hyperbola
center
(-1, 3)

$$\frac{(y-3)^2}{32} - \frac{(x+1)^2}{2} = 1$$

18. $4x^2 = 4y^2 + 16y - 15$

$$-4y^2 - 4y^2$$

hyperbola Rearrange to get x^2 & y^2 on same side

$$4x^2 - 4y^2 - 16y = -15$$

19. $(x-8)^2 + y^2 = 625$

center $(8, 0)$; $r = 25$

20. $\frac{(x-7)^2}{81} + \frac{y^2}{25} = 1$ center $(7, 0)$

$a=9$: major axis
is 18 units long

$b=5$; minor axis
is 10 units long

21. $u = \langle -2, 5 \rangle, v = \langle 5, 13 \rangle$

$3u - 4v$

$3\langle -2, 5 \rangle - 4\langle 5, 13 \rangle$

$\langle -6, 15 \rangle + \langle -20, -52 \rangle$

$\langle -26, 37 \rangle$

Change problem
to dot product

23. Find the
dot product

22. $\langle 15, -12 \rangle$

$\langle 8, -3 \rangle, \langle -2, 4 \rangle$

$\sqrt{15^2 + 12^2}$

$\sqrt{225 + 144}$

$\sqrt{369}$ or ≈ 19.2

$-16 + -12$

-28

$$24. \quad u = \langle -7, 6 \rangle$$

$$\begin{aligned} & \sqrt{7^2 + 6^2} \\ & \sqrt{49 + 36} \\ & \sqrt{85} \end{aligned}$$

unit vector:

$$\left\langle \frac{-7}{\sqrt{85}}, \frac{6}{\sqrt{85}} \right\rangle$$

linear combination
form:

$$\frac{-7}{\sqrt{85}} i + \frac{6}{\sqrt{85}} j$$

$$\text{OR } \frac{-7\sqrt{85}}{85} i + \frac{6\sqrt{85}}{85} j$$

not rationalized

$$25. \quad \text{Balloon: } \langle 100.4 \cos 150^\circ, 100.4 \sin 150^\circ \rangle \quad \langle -86.9, 50.2 \rangle$$

$$\text{wind } \langle 40.9 \cos 45^\circ, 40.9 \sin 45^\circ \rangle \quad \begin{matrix} + \\ \hline \end{matrix} \quad \langle 28.9, 28.9 \rangle$$

$$\tan^{-1}\left(\frac{79.1}{-58}\right) + 180^\circ = 126.3^\circ \quad \begin{matrix} \text{Resultant} \\ \text{lies in Q2} \\ \text{so add } 180^\circ \\ \text{to angle} \end{matrix} \quad \langle -58, 79.1 \rangle$$

$$\text{magnitude } \sqrt{79.1^2 + 58^2} \approx 98.1 \text{ mi/hr} \quad \begin{matrix} \text{(speed)} \\ \text{in a cloud} \end{matrix}$$

$$26. \quad \text{force } \langle 120 \cos 70^\circ, 120 \sin 70^\circ \rangle$$

$$\text{displacement } \langle 95, 0 \rangle$$

$$\begin{matrix} \cancel{\langle 41.0, 0 \rangle} \\ \cancel{\langle 41.0, 112.8 \rangle} \\ \underline{\langle 95, 0 \rangle} \end{matrix}$$

~~Resultant~~

WORK = force \cdot displacement

$$\langle 41.0, 112.8 \rangle \cdot \langle 95, 0 \rangle$$

$$(41.0 \times 95) + (112.8 \times 0)$$

$$3895 \text{ joules}$$

$$27. \quad \langle 1350 \cos 50^\circ, 1350 \sin 50^\circ \rangle$$

Component vector $\langle 867.8, 1034.2 \rangle$

$$28. \text{ initial point } (-4, 3)$$

$$\text{terminal point } (-7, 11)$$

position vector : terminal - initial

$$(-7, 11) - (-4, 3)$$

$$\langle -7 - -4, 11 - 3 \rangle$$

$\langle -3, 8 \rangle$