

GRAPHING TRANSFORMATIONS

$$y = A \sin(Bx + C) + D$$

A = amplitude (always positive)

B = affects the length of the graph
period: $\frac{2\pi}{B}$

D = midline. This is a vertical shift.

$$\text{MAX} = D + A$$

$$\text{MIN} = D - A$$

EX $y = 2 \sin 4\theta + 3$

amplitude $A = 2$

midline $D = 3$ int

maximum: $D + A$
 $3 + 2$

5 MAX

minimum: $D - A$

$3 - 2$

1 MIN

period: $\frac{2\pi}{4} = \frac{\pi}{2}$

intervals: $\frac{\pi}{4}$ or $\frac{\pi}{2} \times \frac{1}{4} = \frac{\pi}{8}$

no parentheses = no phase shift begin at 0!

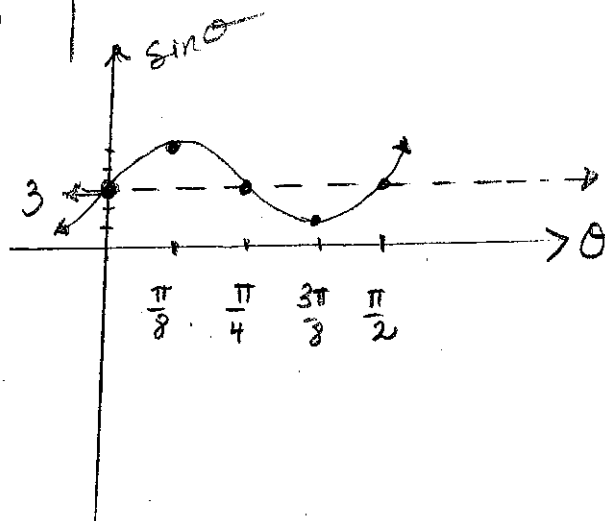
$$y = A \cos(Bx + C) + D$$

- if there are parentheses, there is a horizontal translation (phase shift)

- Set $(Bx + C) = 0$

Solve for x; this is where you begin the x/y table.

θ	$\sin \theta$
0	INT 3
$\frac{\pi}{8}$	MAX 5
$\frac{2\pi}{8}$	INT 3
$\frac{3\pi}{8}$	MIN 1
$\frac{4\pi}{8}$	INT 3



these are equal!

Ex $y = 2 \sin \left(3\theta - \frac{\pi}{2} \right) + 0$ ← no vertical shift

Amplitude = 2

MAX = 2

MIN = -2

period : $\frac{2\pi}{3}$

intervals : $\frac{\frac{2\pi}{3}}{4} = \frac{\pi}{6}$

Horizontal shift

Where do we start?

$$3\theta - \frac{\pi}{2} = 0$$

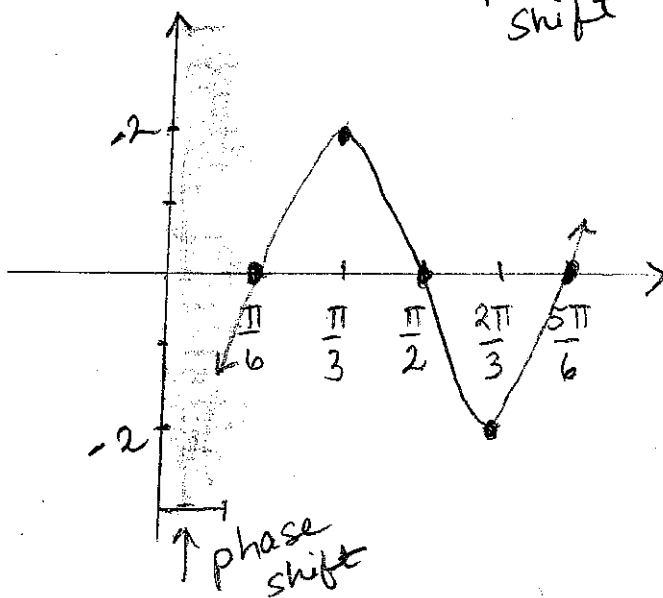
$$3\theta = \frac{\pi}{2}$$

$$\theta = \frac{\frac{\pi}{2}}{3} = \frac{\pi}{6}$$

Do not start at 0!
Start at $\frac{\pi}{6}$

Phase shift

θ	$\sin \theta$
$\frac{\pi}{6}$	INT 0
$+\frac{\pi}{6}$ $\frac{2\pi}{6}$	MAX 2
$+\frac{\pi}{6}$ $\frac{3\pi}{6}$	INT 0
$+\frac{\pi}{6}$ $\frac{4\pi}{6}$	MIN -2
$+\frac{\pi}{6}$ $\frac{5\pi}{6}$	INT 0



Ex $y = -1 + 3 \cos(2\theta - \pi)$

↙ midline (D-value)

↖ phase shift

amplitude = 3

midline = -1 INT

MAX = $-1 + 3 = 2$ MAX

MIN = $-1 - 3 = -4$ MIN

period : $\frac{2\pi}{2} = \pi$

intervals : $\frac{\pi}{4}$

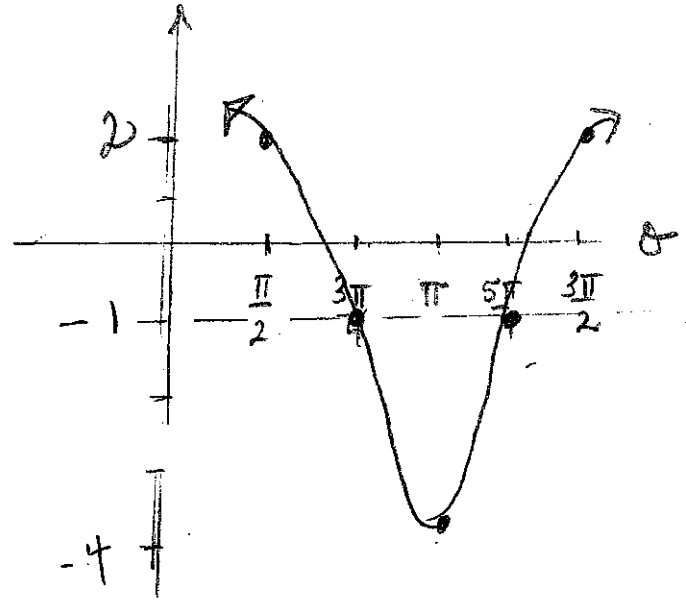
$2\theta - \pi = 0$

$2\theta = \pi$

$\theta = \frac{\pi}{2}$

start your x/y table here

θ	$\cos \theta$
$\frac{\pi}{2}$	0
$\frac{3\pi}{4}$	$-\frac{1}{\sqrt{2}}$
π	-1
$\frac{5\pi}{4}$	$-\frac{1}{\sqrt{2}}$
$\frac{3\pi}{2}$	0
$\frac{7\pi}{4}$	$\frac{1}{\sqrt{2}}$
2π	1



Ex $y = -3 \sin\left(\frac{3}{2}\theta + \pi\right) - 2$

amplitude = 3

midline = -2 INT

MAX : $-2 + 3 = 1$ MAX

MIN : $-2 - 3 = -5$ MIN

period: $\frac{2\pi}{\frac{3}{2}} = \frac{4\pi}{3}$

intervals: $\frac{4\pi}{3} \cdot \frac{1}{4} = \frac{\pi}{3}$

phase shift: $\frac{3}{2}\theta + \pi = 0$

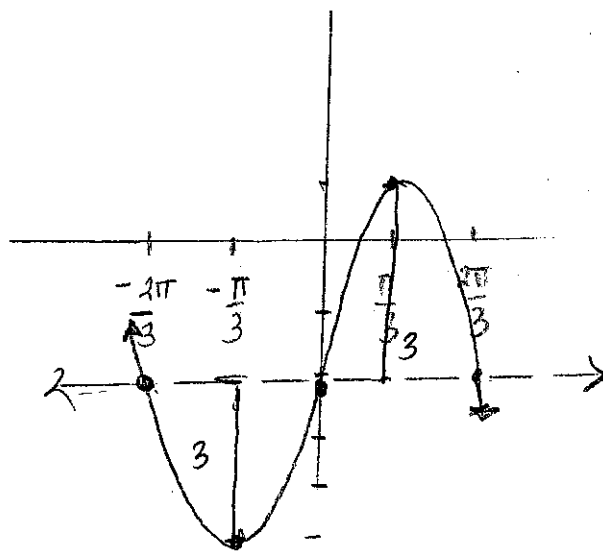
$\left(\frac{2}{3}\right) \frac{3}{2}\theta = -\pi \left(\frac{2}{3}\right)$

$\theta = -\frac{2\pi}{3}$

starting value for x/y table

θ	$\sin\theta$
$-\frac{2\pi}{3}$	INT -2
$-\frac{\pi}{3}$	MIN -5
0	INT -2
$\frac{\pi}{3}$	MAX 1
$\frac{2\pi}{3}$	INT -2

because of the negative leading coefficient



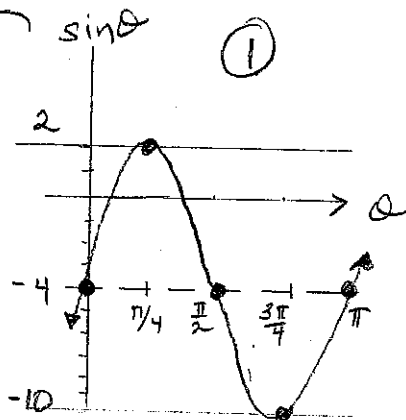
Sin/Cos Transformations Homework

For each function, state the amplitude, period, intervals, horizontal shift and vertical shift, and midline. Then graph one period.

no phase shift

1. $y = 6\sin 2\theta - 4$

- amplitude = 6
- period = $\frac{2\pi}{2} = \pi$
- intervals are $\frac{\pi}{4}$
- midline = -4
- MAX = $-4 + 6 = 2$
- MIN = $-4 - 6 = -10$



start at

θ	$y = 6\sin 2\theta - 4$
0	INT -4
$\pi/4$	MAX 2
$2\pi/4$	INT -4
$3\pi/4$	MIN -10
$4\pi/4$	INT -4

3. $y = 5\sin\left(\theta + \frac{\pi}{4}\right)$

- amplitude = 5
- period = 2π
- intervals: $\frac{2\pi}{4} = \frac{\pi}{2}$

phase shift!

Starting value:

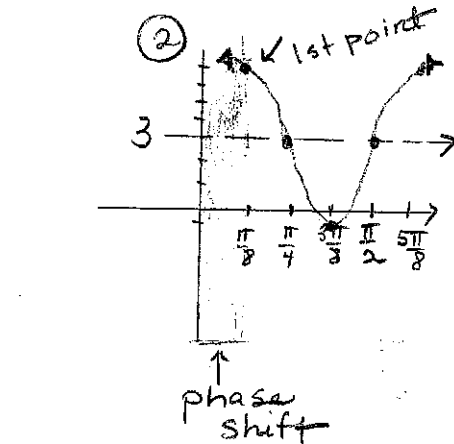
$\theta + \frac{\pi}{4} = 0$

$\theta = -\frac{\pi}{4}$

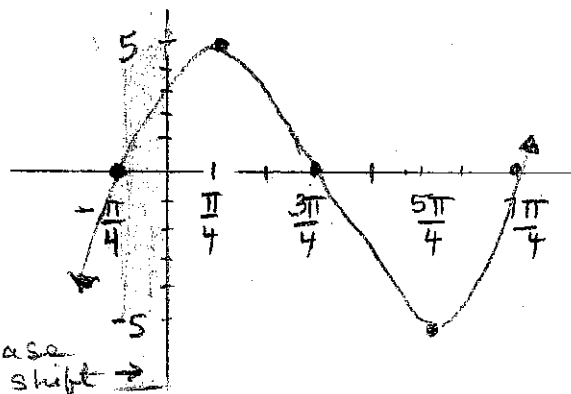
MIDLINE = 0

MAX = 5

MIN = -5



θ	$y = 5\sin\left(\theta + \frac{\pi}{4}\right)$
$-\frac{\pi}{4}$	INT 0
$\frac{\pi}{4}$	MAX 5
$\frac{3\pi}{4}$	INT 0
$\frac{5\pi}{4}$	MIN -5
$\frac{7\pi}{4}$	INT 0



Phase shift!

2. $y = 4\cos\left(4\theta - \frac{\pi}{2}\right) + 3$

DO NOT start at 0

- amplitude = 4
- period = $\frac{2\pi}{4} = \frac{\pi}{2}$
- intervals: $\frac{\pi}{2} \div 4 = \frac{\pi}{8}$
- midline = 3
- MAX = $3 + 4 = 7$
- MIN = $3 - 4 = -1$

Starting value: $4\theta - \frac{\pi}{2} = 0$

θ	$y = 4\cos\left(4\theta - \frac{\pi}{2}\right) + 3$
$\pi/8$	MAX 7
$2\pi/8$	INT 3
$3\pi/8$	MIN -1
$4\pi/8$	INT 3
$5\pi/8$	MAX 7

$4\theta = \frac{\pi}{2}$
 $\theta = \frac{\pi}{8}$

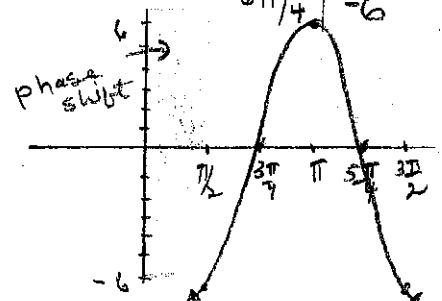
4. $y = -6\cos(2\theta - \pi)$

- amplitude = 6
- period = $\frac{2\pi}{2} = \pi$
- intervals: $\frac{\pi}{4}$
- phase shift: $2\theta - \pi = 0$

$2\theta = \pi$
 $\theta = \frac{\pi}{2}$

MIDLINE = 0
MAX = 6
MIN = -6

θ	$y = -6\cos(2\theta - \pi)$
$\pi/2$	-6
$3\pi/4$	0
$4\pi/4$	6
$5\pi/4$	0
$6\pi/4$	-6



5. $y = -2\sin\left(3\theta - \frac{\pi}{4}\right)$

amplitude = 2

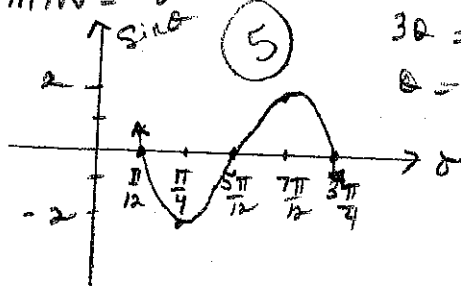
period = $\frac{2\pi}{3}$

intervals: $\frac{2\pi}{3} \div 4 = \frac{2\pi}{12} = \frac{\pi}{6}$

MIDLINE: 0

MAX = 2

MIN = -2



phase shift:

$3\theta - \frac{\pi}{4} = 0$

$3\theta = \frac{\pi}{4}$

$\theta = \frac{\pi}{12}$

θ	$\sin \theta$
$\frac{\pi}{12}$	INT 0
$\frac{3\pi}{12}$	MIN -2
$\frac{5\pi}{12}$	INT 0
$\frac{7\pi}{12}$	MAX 2
$\frac{9\pi}{12}$	INT 0

θ	$\cos \theta$
$-\frac{3\pi}{4}$	-4
0	-5
$\frac{3\pi}{4}$	-6
$\frac{6\pi}{4}$	-5
$\frac{9\pi}{4}$	-4

6. $y = \cos\left(\frac{2}{3}\theta + \frac{\pi}{2}\right) - 5$

amplitude = 1

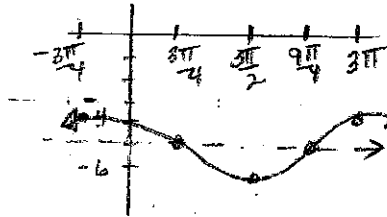
period = $\frac{2\pi}{\frac{2}{3}} = 2\pi \times \frac{3}{2} = 3\pi$

intervals: $\frac{3\pi}{4}$

MIDLINE: -5

MAX: $-5 + 1 = -4$

MIN: $-5 - 1 = -6$



phase shift:

$\frac{2}{3}\theta + \frac{\pi}{2} = 0$

$\frac{2}{3}\theta = -\frac{\pi}{2}$

$\theta = -\frac{\pi}{2} \times \frac{3}{2}$

$\theta = -\frac{3\pi}{4}$

7. $y = 3\sin\left(\frac{1}{4}\theta - \frac{2\pi}{3}\right) + 1$

amplitude = 3

midline = 1

MAX = $1 + 3 = 4$

MIN = $1 - 3 = -2$

period = $\frac{2\pi}{\frac{1}{4}} = 8\pi$

intervals: $\frac{8\pi}{4} = 2\pi$

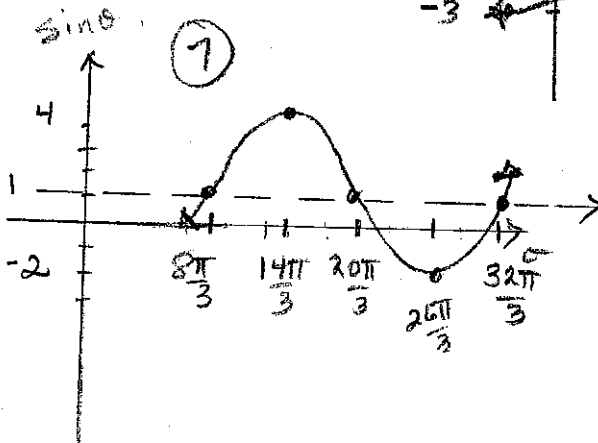
phase shift:

$\frac{1}{4}\theta - \frac{2\pi}{3} = 0$

$\frac{1}{4}\theta = \frac{2\pi}{3}$

$\theta = \frac{2\pi}{3} \times 4$

$\theta = \frac{8\pi}{3}$



8. $y = -3\cos\left(2\theta + \frac{\pi}{6}\right)$

amplitude = 3

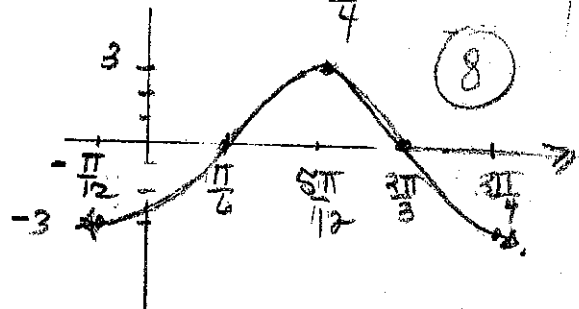
midline = 0

MAX = 3

MIN = -3

period = $\frac{2\pi}{2} = \pi$

intervals: $\frac{\pi}{4}$



phase shift:

$2\theta + \frac{\pi}{6} = 0$

$2\theta = -\frac{\pi}{6}$

$\theta = -\frac{\pi}{12}$

θ	$\sin \theta$
$\frac{8\pi}{3}$	INT 1
$\frac{14\pi}{3}$	MAX 4
$\frac{20\pi}{3}$	INT 1
$\frac{26\pi}{3}$	MIN -2
$\frac{32\pi}{3}$	INT 1

θ	$\cos \theta$
$-\frac{\pi}{12}$	MIN
$\frac{2\pi}{12}$	INT
$\frac{5\pi}{12}$	MAX
$\frac{8\pi}{12}$	INT
$\frac{9\pi}{12}$	MIN