How Do I Graph a Hyperbola?

$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ Horizontal Transverse Axis — Contains vertices + foci; like the major axis of an ellipse

$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$ Vertical Transverse Axis — Contains co-vertices

To find the foci, use $c^2 = a^2 + b^2$. The foci are always graphed on the transverse axis, inside each curve. The transverse axis is 2a units long; the conjugate axis is 2b units long.

The direction of opening depends on the variable of the leading term. The value of a^2 may not necessarily be larger than b^2!

Identify all the critical features, then graph each hyperbola.

- plot the center
- plot V + CV
- DRAW A REFERENCE BOX
- Sketch two diagonal asymptotes through opposite corners
- DRAW the curves (touch the vertex only (it's the only tangent point))
- add the foci

EX 1: $\frac{x^2}{4} - \frac{y^2}{9} = 1$ $c^2 = a^2 + b^2$, $c^2 = 4 + 9$, $c^2 = \sqrt{13}$

H or V? HORIZONTAL Center (0,0)

a = 2 b = 3 c = $\sqrt{13} \approx 3.6$

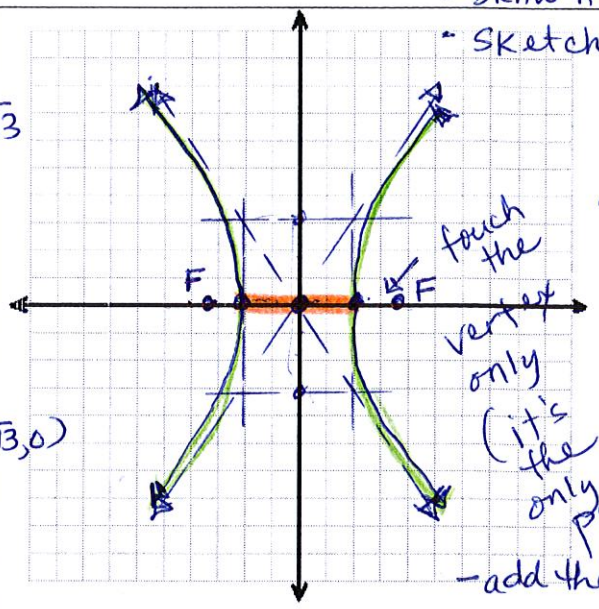
endpoints of transverse (Vertices) axis $(0 \pm 2, 0) \rightarrow (2,0), (-2,0)$

endpoints of conjugate axis $(0, 0 \pm 3) \rightarrow (0,3), (0,-3)$

coordinates of foci $(0 \pm \sqrt{13}, 0) \rightarrow (\sqrt{13}, 0), (-\sqrt{13}, 0)$

length of transverse axis $2 \cdot a = 2 \cdot 2 = 4$

length of conjugate axis $2 \cdot b = 2 \cdot 3 = 6$



EX 2: $\frac{y^2}{9} - \frac{x^2}{16} = 1$ $c^2 = 9 + 16 = \sqrt{25}$, $c = 5$

H or V? VERTICAL Center (0,0)

a = 3 b = 4 c = 5

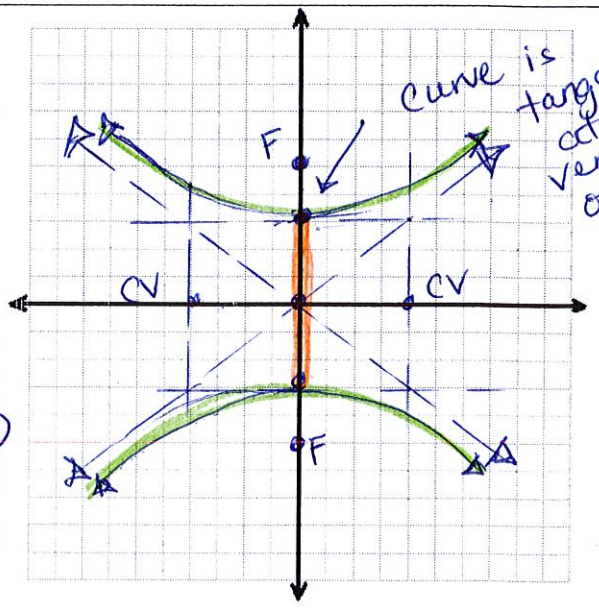
endpoints of transverse axis $(0, 0 \pm 3) \rightarrow (0,3), (0,-3)$

endpoints of conjugate axis $(0 \pm 4, 0) \rightarrow (4,0), (-4,0)$

coordinates of foci $(0, 0 \pm 5) \rightarrow (0,5), (0,-5)$

length of (vertical) transverse axis $2(3) = 6$

length of (horizontal) conjugate axis $2(4) = 8$



curve is tangent at vertex only

EX 3: $\frac{(x-1)^2}{\sqrt{25}} - \frac{(y+3)^2}{\sqrt{4}} = 1$ $c^2 = 25+4 = 29$

H or V? H Center $(1,-3)$

a = 5 b = ~~4~~ 2 c = $\sqrt{29} \approx 5.4$

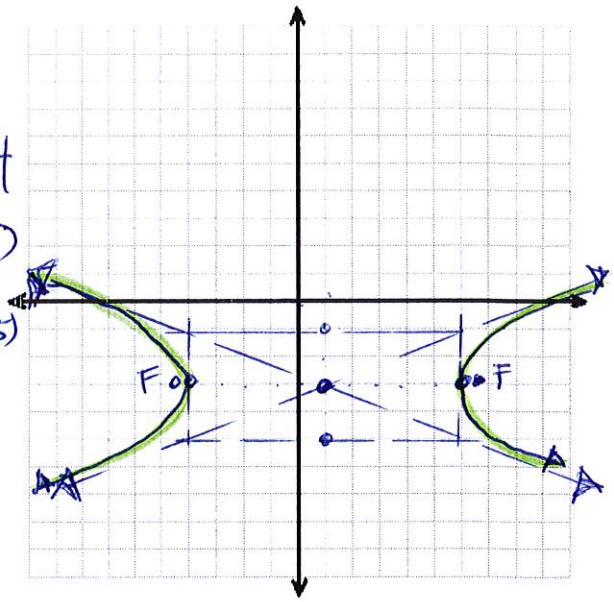
endpoints of transverse axis $(1 \pm 5, -3) \rightarrow (6,-3)\ (-4,-3)$

endpoints of conjugate axis $(1, -3 \pm 2) \rightarrow (1,-1), (1,-5)$

coordinates of foci $(1 \pm \sqrt{29}, -3)$

length of transverse axis $2(5) = 10$

length of conjugate axis $2(2) = 4$



EX 4: $y^2 - \frac{(x+4)^2}{\sqrt{9}} = 1$ $c^2 = 1+9 = 10$, $c = \sqrt{10}$

$\sqrt{1}$ $\sqrt{9}$

H or V? V Center $(-4,0)$

a = 1 b = 3 c = $\sqrt{10} \approx 3.2$

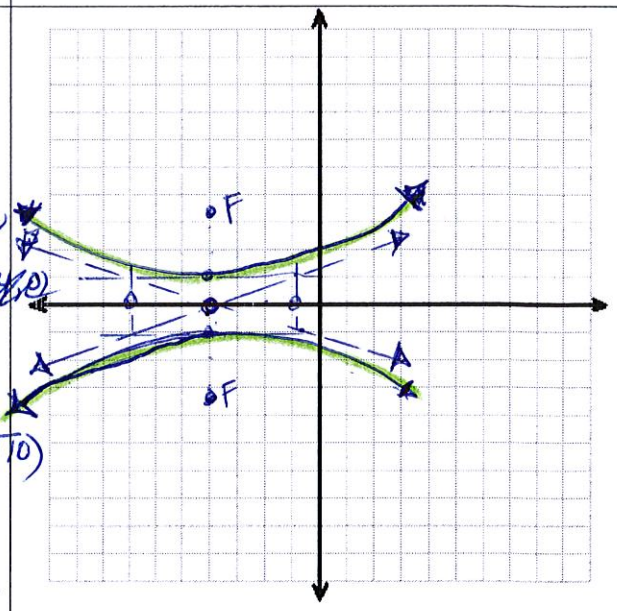
endpoints of transverse axis $(-4, 0 \pm 1) \rightarrow (-4,1)\ (-4,-1)$

endpoints of conjugate axis $(-4 \pm 3, 0)$ $(-1,0)$ $(-7,0)$

coordinates of foci $(-4, 0 \pm \sqrt{10}) \rightarrow (-4, \sqrt{10})\ (-4, -\sqrt{10})$

length of transverse axis $2(1) = 2$

length of conjugate axis $2(3) = 6$



You try:

5. $\frac{(x-1)^2}{36} - \frac{(y-3)^2}{4} = 1$

Horizontal; x^2 is the leading term

Horizontal or Vertical? (Explain how you know!)

$c^2 = 36+4$ $c^2 = 40$ $c = 2\sqrt{10}$ $c \approx 6.3$

Center $(1,3)$ a = 6 b = 2 c = ____

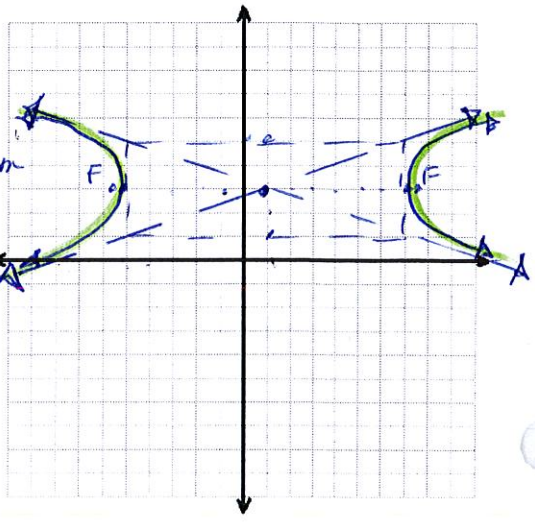
vertices $(1 \pm 6, 3) \rightarrow (7,3)\ (-5,3)$

co-vertices $(1, 3 \pm 2) \rightarrow (1,5), (1,1)$

coordinates of foci $(1 \pm 2\sqrt{10}, 3)$

length of transverse axis $2(6) = 12$

length of conjugate axis $2(2) = 4$



Another for good measure. . .

6. $\frac{(y-4)^2}{25} - \frac{(x+3)^2}{25} = 1$

Vertical; y^2 is the leading term

Horizontal or Vertical? (Explain how you know!)

Center $(-3, 4)$ a = 5 b = 5 c = $c^2 = 25+25$, $c^2 = 50$, $c = 5\sqrt{2}$, $c \approx 7.1$

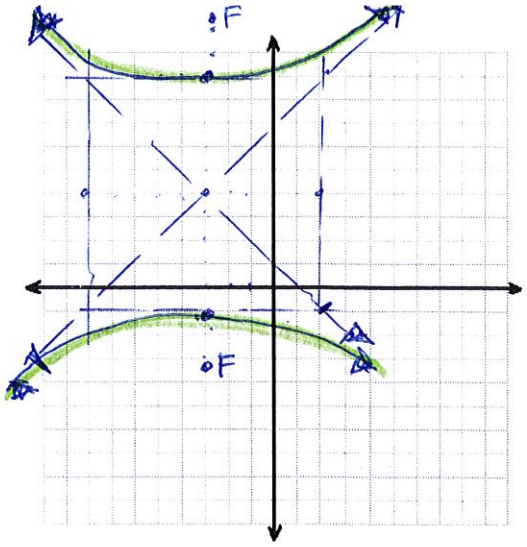
vertices $(-3, 4\pm5) \to (-3, 9)\ (-3, -1)$

co-vertices $(-3\pm5, 4) \to (2, 4), (-8, 4)$

coordinates of foci $(-3, 4 \pm 5\sqrt{2})$

length of transverse axis $2(5) = 10$

length of conjugate axis $2(5) = 10$



Use the "completing the square" strategy to rewrite these general form equations in standard form. Identify the center of the hyperbola, and tell whether the graph has a horizontal or vertical transverse axis. Determine the value of a, b and c.

7. $x^2 = y^2 + 8x + 12$

$x^2 - 8x - y^2 = 12$

$x^2 - 8x + 16 - y^2 = 12 + 16$

$(x-4)^2 - y^2 = 28$

$\boxed{\frac{(x-4)^2}{28} - \frac{y^2}{28} = 1}$

Caution!

8. $(x+6)^2 - 4(y+3)^2 = -36$

$-36 \quad -36 \quad -36$

y^2 is now the lead term

$-\frac{(x+6)^2}{36} + \frac{(y+3)^2}{9} = 1$ → REWRITE → $\boxed{\frac{(y+3)^2}{9} - \frac{(x+6)^2}{36} = 1}$

9. $-4y^2 + 9x^2 - 90x - 24y = -153$

$9x^2 - 90x - 4y^2 - 24y = -153$

$9(x^2 - 10x + 25) - 4(y^2 + 6y + 9) = -153 + 225 - 36$

$9(x-5)^2 - 4(y+3)^2 = 36$

$\boxed{\frac{(x-5)^2}{4} - \frac{(y+3)^2}{9} = 1}$

10. $49x^2 - 25y^2 + 294x + 200y = 1184$

$49x^2 + 294x - 25y^2 + 200y = 1184$

$49(x^2 + 6x + 9) - 25(y^2 - 8y + 16) = 1184 + 441 - 400$

$49(x+3)^2 - 25(y-4)^2 = 1225$

$\boxed{\frac{(x+3)^2}{25} - \frac{(y-4)^2}{49} = 1}$

11. $81x^2 - 36y^2 = 2916$

$2916 \quad 2916 \quad 2916$

$\boxed{\frac{x^2}{36} - \frac{y^2}{81} = 1}$

12. $25y^2 - 9x^2 - 100y - 72x - 269 = 0$

$25y^2 - 100y - 9x^2 - 72x = 269$

$25(y^2 - 4y + 4) - 9(x^2 + 8x + 16) = 269 + 100 - 144$

$25(y-2)^2 - 9(x+4)^2 = 225$

$\boxed{\frac{(y-2)^2}{9} - \frac{(x+4)^2}{25} = 1}$

$c^2 = 4 + 1$, $c^2 = 5$, $c \approx 2.$

factor out the -4 (Change the sign of the middle term)

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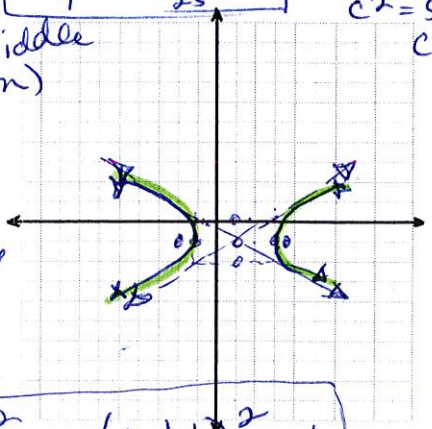
13. Graph the hyperbola given by the general form equation $x^2 - 4y^2 - 2x - 8y - 7 = 0$.

$x^2 - 2x - 4y^2 - 8y = 7$

$x^2 - 2x + 1 - 4(y^2 + 2y + 1) = 7 + 1 - 4$ } Keep balance

$\frac{(x-1)^2}{4} - \frac{4(y+1)^2}{4} = \frac{4}{4}$ → $\boxed{\frac{(x-1)^2}{4} - (y+1)^2 = 1}$



Advanced Algebra
Hyperbola Practice

Name _____

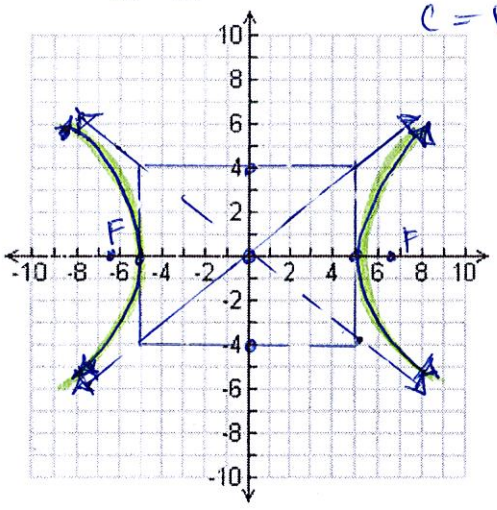
1. Complete the table for the equations of the hyperbola.

Equation	Center	Length of Transverse Axis	Length of Conjugate Axis
$x^2 - \frac{y^2}{4} = 1$	$(0,0)$	$2(1) = 2$	$2(2) = 4$
$\frac{(x+3)^2}{25} - \frac{(y-1)^2}{16} = 1$ sorry	$(-3,1)$	$2(5) = 10$	$2(4) = 8$
$16x^2 - 4y^2 = 256$ $\frac{256}{256} \quad \frac{256}{256} \quad \frac{256}{256}$ $\frac{x^2}{16} - \frac{y^2}{64} = 1$	$(0,0)$	$2(4) = 8$	$2(8) = 16$

Graph the hyperbola with the given equation. Plot the center, the vertices, the covertices, and the foci. Sketch the asymptotes.

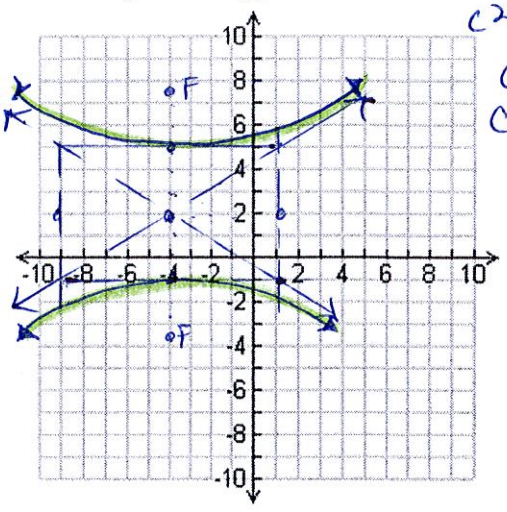
2. $\frac{x^2}{25} - \frac{y^2}{16} = 1$

$c^2 = 25 + 16$
$c^2 = 41$
$c = \sqrt{41} \approx 6.4$

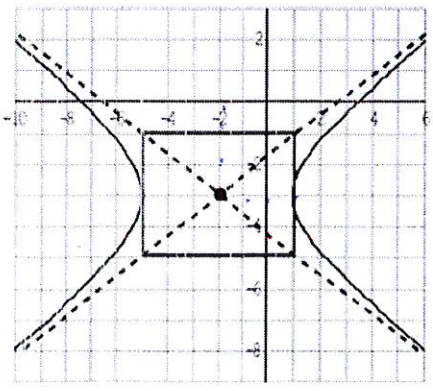


3. $\frac{(y-2)^2}{9} - \frac{(x+4)^2}{25} = 1$

$c^2 = 9 + 25$
$c^2 = 34$
$c = \sqrt{34}$
$c \approx 5.8$



5. Write the equation of the hyperbola.



C: $(-2,-3)$
HORIZONTAL TRANSVERSE AXIS
(x^2 is leading)

$\frac{(x+2)^2}{9} - \frac{(y+3)^2}{4} = 1$

6. Rewrite the equation in standard form.

$16x^2 - 9y^2 - 32x - 144y + 16 = 0$

$16x^2 - 32x - 9y^2 - 144y = -16$
$16(x^2 - 2x + 1) - 9(y^2 + 16y + 64) = -16 + 16$
$16(x-1)^2 - 9(y+8)^2 = -576$ -576

$\frac{16(x-1)^2}{-576} - \frac{9(y+8)^2}{-576} = 1$

$$\boxed{\frac{(y+8)^2}{9} - \frac{(x-1)^2}{36} = 1}$$