How Do I Graph a Hyperbola?

$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ Horizontal Transverse Axis

$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$ Vertical Transverse Axis

To find the foci, use $c^2 = a^2 + b^2$. The foci are always graphed on the transverse axis, inside each curve. The transverse axis is 2a units long; the conjugate axis is 2b units long.

The direction of opening depends on the variable of the leading term. The value of a^2 may not necessarily be larger than b^2!

↑ (positive)

Identify all the critical features, then graph each hyperbola.

EX 1: $\frac{x^2}{4} - \frac{y^2}{9} = 1$

The "lead term" is x^2

H or V? x^2 is positive! Center (0,0)

a = 2 b = 3 c = $c^2 = 4+9$, $c^2 = 13$, $c = \sqrt{13} \approx 3.6$

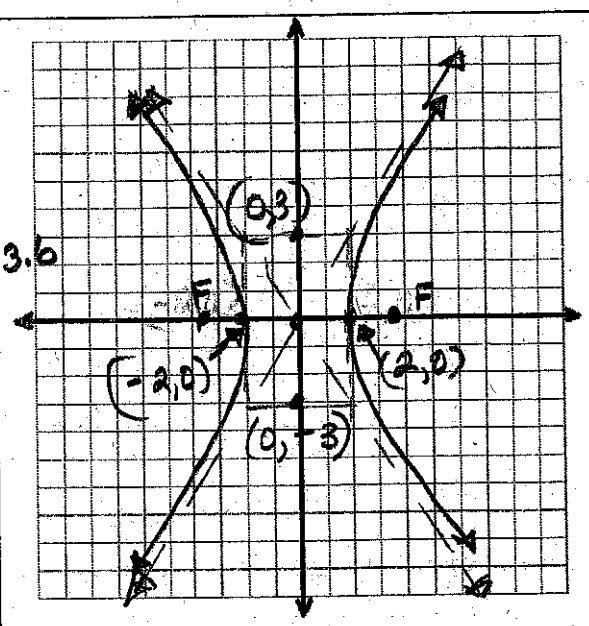
endpoints of transverse axis (0±2,0) → horizontal

endpoints of conjugate axis (0,0±3) → vertical

coordinates of foci $(0 \pm \sqrt{13}, 0)$

length of transverse axis 2a = 4

length of conjugate axis 2b = 6



EX 2: $\frac{y^2}{9} - \frac{x^2}{16} = 1$

The "lead term" is y^2

H or V? y^2 is positive! Center (0,0)

a = 3 b = 4 c = $c^2 = 9+16$, $c^2 = 25$, $c = 5$

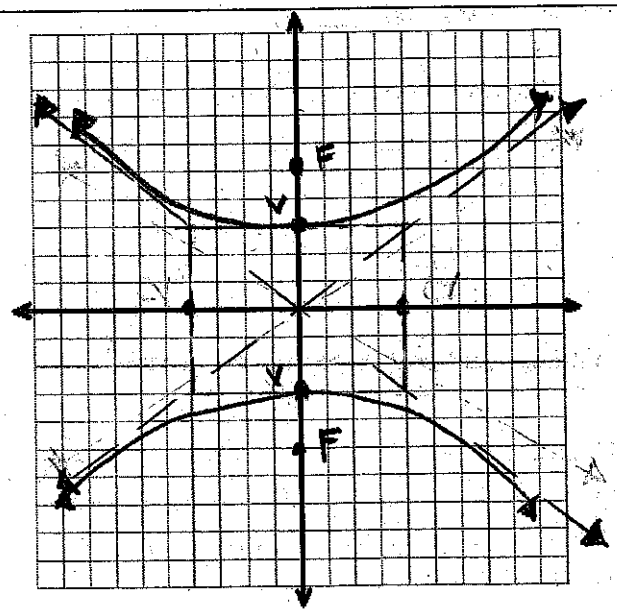
endpoints of transverse axis (0,3) (0,-3)

endpoints of conjugate axis (4,0) (-4,0)

coordinates of foci (0,5), (0,-5)

length of transverse axis 6

length of conjugate axis 8



lead term

EX 3: $\boxed{\frac{(x-1)^2}{25}} - \frac{(y+3)^2}{4} = 1$

H or V? H Center (1, -3)

a = 5 b = 2 c = $c^2 = 25 + 4$, $c^2 = 29$, $c = \sqrt{29} \approx 5.4$

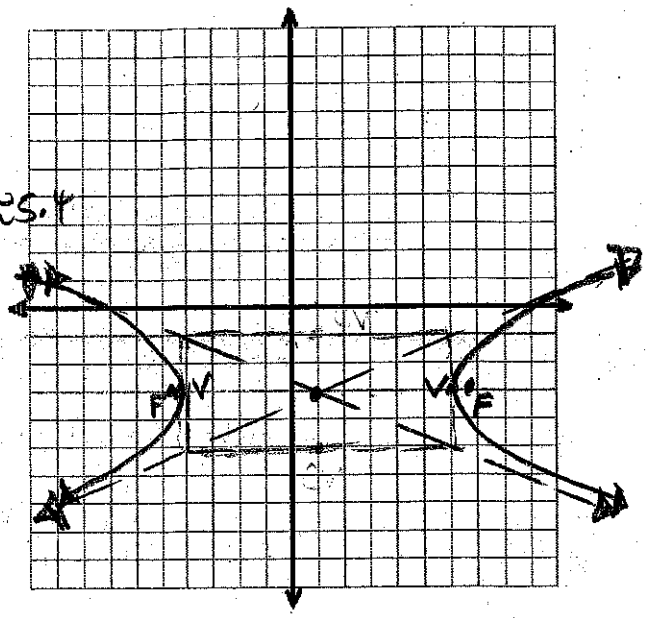
endpoints of transverse axis $(1 \pm 5, -3)$ $(6, -3), (-4, -3)$

endpoints of conjugate axis $(1, -3 \pm 2)$ $(1, -1)$ $(1, -5)$

coordinates of foci $(1 \pm \sqrt{29}, -3)$

length of transverse axis 10

length of conjugate axis 4



EX 4: $y^2 - \frac{(x+4)^2}{9} = 1$ CAREFUL!

H or V? V Center (-4, 0)

a = 1 b = 3 c = $c^2 = 1 + 9$, $c^2 = 10$, $c = \sqrt{10} \approx 3.2$

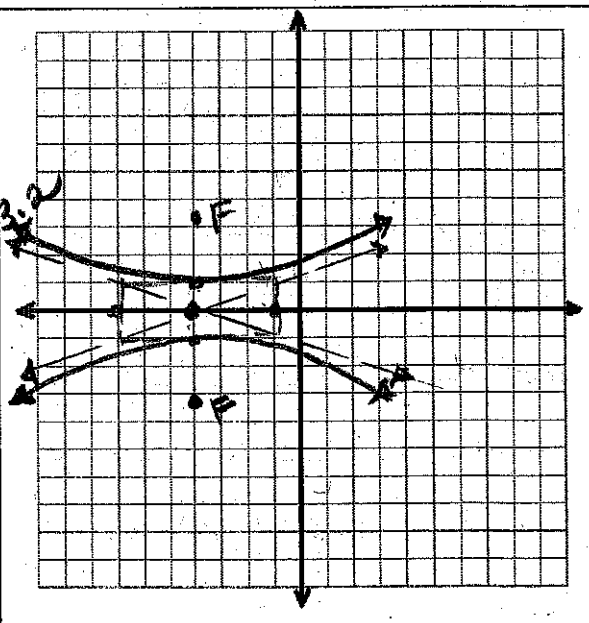
endpoints of transverse axis $(-4, 0 \pm 1)$ $(-4, 1), (-4, -1)$

endpoints of conjugate axis $(-4 \pm 3, 0)$ $(-1, 0)$ $(-7, 0)$

coordinates of foci $(-4, \pm\sqrt{10})$

length of transverse axis 2

length of conjugate axis 6



You try:

5. $\frac{(x-1)^2}{36} - \frac{(y-3)^2}{4} = 1$

Horizontal or Vertical? (Explain how you know!)

Center (1, 3) a = 6 b = 2 c = $c^2 = 40$, $c = \sqrt{40} = 2\sqrt{10} \approx 6.3$

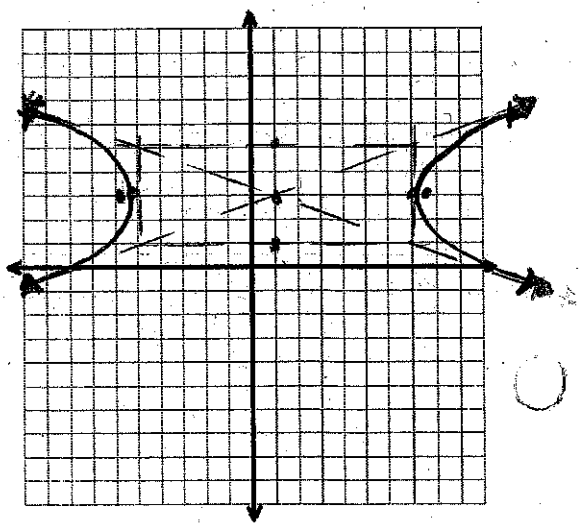
vertices $(1 \pm 6, 3) \rightarrow (7, 3), (-5, 3)$

co-vertices $(1, 3 \pm 2) \rightarrow (1, 5), (1, 1)$

coordinates of foci $(1 \pm 2\sqrt{10}, 3)$

length of transverse axis 12

length of conjugate axis 4

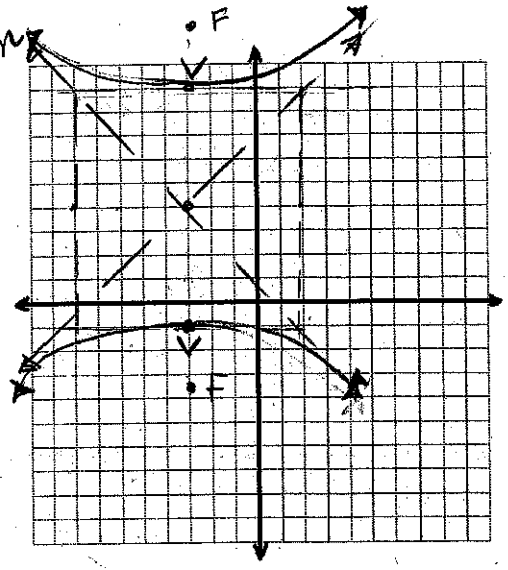


Another for good measure. . .

6. $\frac{(y-4)^2}{25} - \frac{(x+3)^2}{25} = 1$

Vertical, because the leading term is (y^2)

Horizontal or Vertical? (Explain how you know!)



Center $(-3, 4)$ a = 5 b = 5 c = $c^2 = 25+25$

$c^2 = 50$

$c = \sqrt{50}$ or $5\sqrt{2} \approx 7.1$

vertices $(-3, 4 \pm 5) \rightarrow (-3, 9), (-3, -1)$

co-vertices $(-3 \pm 5, 4) \rightarrow (2, 4), (-8, 4)$

coordinates of foci $(-3, 4 \pm 5\sqrt{2})$

length of transverse axis 10

length of conjugate axis 10

Use the "completing the square" strategy to rewrite these general form equations in standard form. Identify the center of the hyperbola, and tell whether the graph has a horizontal or vertical transverse axis. Determine the value of a, b and c.

7. $x^2 = y^2 + 8x + 12$

$x^2 - y^2 - 8x = 12$

$x^2 - 8x + 16 - y^2 = 12 + 16$

$\boxed{(x-4)^2 - y^2 = 24}$

$\frac{(x-4)^2}{24} - \frac{y^2}{24} = 1$

8. $(x+6)^2 - 4(y+3)^2 = -36$

$\frac{(x+6)^2}{-36} - \frac{4(y+3)^2}{-36} = \frac{-36}{-36}$

$\frac{(y+3)^2}{9} - \frac{(x+6)^2}{36} = 1$

9. $-4y^2 + 9x^2 - 90x - 24y = -153$

$9x^2 - 90x - 4y^2 - 24y = -153$

$9(x^2 - 10x + 25) - 4(y^2 + 6y + 9) = -153 + 225 - 36$

$9(x-5)^2 - 4(y+3)^2 = 36$

$\frac{(x-5)^2}{4} - \frac{(y+3)^2}{9} = 1$

10. $49x^2 - 25y^2 + 294x + 200y = 1184$

$49x^2 + 294x - 25y^2 + 200y = 1184$

$49(x^2 + 6x + 9) - 25(y^2 - 8y + 16) = 1184 + 441 - 400$

$49(x+3)^2 - 25(y-4)^2 = 1225$

$\frac{(x+3)^2}{25} - \frac{(y-4)^2}{49} = 1$

11. $81x^2 - 36y^2 = 2916$

$\frac{81x^2}{2916} - \frac{36y^2}{2916} = \frac{2916}{2916}$

$\frac{x^2}{36} - \frac{y^2}{81} = 1$

12. $25y^2 - 9x^2 - 100y - 72x - 269 = 0$

$25y^2 - 100y - 9x^2 - 72x = 269$

$25(y^2 - 4y + 4) - 9(x^2 + 8x + 16) = 269 + 100 - 144$

$25(y-2)^2 - 9(x+4)^2 = 225$

$\frac{(y-2)^2}{9} - \frac{(x+4)^2}{25} = 1$

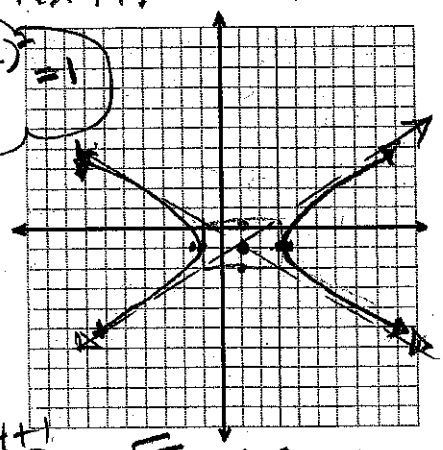
13. Graph the hyperbola given by the general form equation $x^2 - 4y^2 - 2x - 8y - 7 = 0$.

$x^2 - 2x - 4y^2 - 8y = 7$

$x^2 - 2x + 1 - 4(y^2 + 2y + 1) = 7 + 1 - 4$

$(x-1)^2 - 4(y+1)^2 = 4$

$\frac{(x-1)^2}{4} - (y+1)^2 = 1$



$c^2 = 4 + 1$

$c^2 = 5$ $c = \sqrt{5} \approx 2.2$

6. $x^2 - 18x - y^2 - 6y - 16 = 0$ H or V? standard form _____

$x^2 - 18x \quad - y^2 - 6y \quad = 16$

center $(9, -3)$ a $2\sqrt{22}$ b $2\sqrt{22}$ c $4\sqrt{11}$

$x^2 - 18x + 81 - (y^2 + 6y + 9) = 16 + 81 - 9$

$97 - 9$

$(x-9)^2 - (y+3)^2 = 88$

$\frac{(x-9)^2}{88} - \frac{(y+3)^2}{88} = 1$

$\sqrt{88}$

$\sqrt{4 \cdot 22}$

$2\sqrt{22}$

$c^2 = a^2 + b^2$

$c^2 = 88 + 88$

$c^2 = 176$

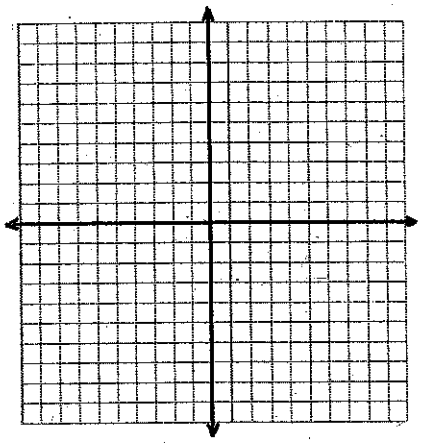
$c = \sqrt{176}$

$= 4\sqrt{11}$

7. $25y^2 - 9x^2 - 100y - 72x - 269 = 0$ H or V? standard form _____

a _____ b _____ c _____

8. Graph the hyperbola given by the general form equation $x^2 - 4y^2 - 2x - 8y - 7 = 0$.



Write a standard form equation of the hyperbola with the information given.

9. vertices (1, -1) and (1, -9) Vertical transverse axis
length of conjugate axis is 6

$\Delta y = 8 \therefore a = 4$

$b = 3 \leftarrow 2$

center $(1, -5)$

$\frac{(y+5)^2}{16} - \frac{(x-1)^2}{9} = 1$

10. Center (0, 5); vertex (-5, 5); focus (-7, 5) horizontal transverse axis

$a = 5 \quad c = 7$

$c^2 = a^2 + b^2$

$7^2 = 5^2 + b^2$

$49 = 25 + b^2$

$24 = b^2$

$\frac{x^2}{25} - \frac{(y-5)^2}{24} = 1$

11. length of major axis is 2; center is at (4, 1); eccentricity value is 3

HORIZONTAL TRANSVERSE

$a = 1$

same as c!

$3^2 = 1^2 + b^2$

$9 = 1 + b^2$

$8 = b^2$

$\frac{(x-4)^2}{1} - \frac{(y-1)^2}{8} = 1$

NOT ENOUGH INFO