Math 4, Unit 6: Trig Identities

Name: _____

Date: _____

Fundamental Trigonometric Identities

Reciprocal Identities

$\sin\theta = \frac{1}{\csc\theta}$ $\quad$ $\cos\theta = \frac{1}{\sec\theta}$ $\quad$ $\tan\theta = \frac{1}{\cot\theta}$

$\csc\theta = \frac{1}{\sin\theta}$ $\quad$ $\sec\theta = \frac{1}{\cos\theta}$ $\quad$ $\cot\theta = \frac{1}{\tan\theta}$

Tangent and Cotangent Identities

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\quad$ $\cot\theta = \frac{\cos\theta}{\sin\theta}$

Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$ $\quad$ $1 + \tan^2\theta = \sec^2\theta$ $\quad$ $1 + \cot^2\theta = \csc^2\theta$

Cofunction Identities

$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$ $\quad$ $\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$ $\quad$ $\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$

Negative Angle Identities

$\sin(-\theta) = -\sin\theta$ $\quad$ $\cos(-\theta) = \cos\theta$ $\quad$ $\tan(-\theta) = -\tan\theta$

1. What does the cofunction identity $\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$ tell you about the graphs of $y = \sin x$ and $y = \cos x$.

Find the values of the other five trigonometric functions of θ.

2. $\sin\theta = \frac{1}{3}, 0 < \theta < \frac{\pi}{2}$

3. $\tan\theta = \frac{3}{7}, 0 < \theta < \frac{\pi}{2}$

4. $\cos\theta = \frac{5}{6}, \frac{3\pi}{2} < \theta < 2\pi$

Simplify the expression.

5. $\sin x \cot x$

$\cancel{\sin x} \cdot \frac{\cos x}{\cancel{\sin x}}$

$\cos x$

6. $\frac{\sin(-\theta)}{\cos(-\theta)}$

$\frac{-\sin\theta}{\cos\theta} = -\tan\theta$

7. $\csc\theta \sin\theta + \cot^2\theta$

$\frac{1}{\cancel{\sin\theta}} \cdot \cancel{\sin\theta} + \cot^2\theta$

$1 + \cot^2\theta$

$\csc^2\theta$

8. $\cos\theta(1 + \tan^2\theta)$

$\cos\theta \cdot \sec^2\theta$

$\cos\theta \cdot \frac{1}{\cos^2\theta}$

$\frac{1}{\cos\theta} \rightarrow \sec\theta$

9. $1 + \tan^2\left(\frac{\pi}{2} - x\right)$

$1 + \cot^2 x$

$\csc^2 x$

10. $\frac{\cos\left(\frac{\pi}{2} - x\right)}{\csc x}$

$\frac{\sin x}{\frac{1}{\sin x}} \rightarrow \sin x \cdot \sin x$

$= \sin^2 x$

14. $\dfrac{\sec x \sin x + \sin x}{1+\sec x}$ ← Factor GCF from numerator $\dfrac{\sin x(\cancel{\sec x+1})}{\cancel{1+\sec x}} = \sin x$

11. $\dfrac{\cos\left(\frac{\pi}{2}-\theta\right)}{\csc\theta} + \cos^2\theta$

$\dfrac{\sin\theta}{\frac{1}{\sin\theta}} + \cos^2\theta$

$\sin^2\theta + \cos^2\theta = 1$

$\sin x$

12. $\sin\left(\frac{\pi}{2}-\theta\right)\sec\theta$

$\cos\theta \cdot \dfrac{1}{\cos\theta}$

$= 1$

13. $\dfrac{\cos^2 x}{\cot^2 x}$

$\dfrac{\cos^2\theta}{\frac{\cos^2\theta}{\sin^2\theta}} \rightarrow \cos^2\theta \cdot \dfrac{\sin^2\theta}{\cos^2\theta}$

$= \sin^2\theta$

14. $\dfrac{\sec x \sin x + \cos\left(\frac{\pi}{2}-x\right)}{1+\sec x}$

on top

$\boxed{\sin x}\dfrac{(\cancel{\sec x+1})}{\cancel{1+\sec x}}$

15. $\dfrac{\csc^2 x - \cot^2 x}{\sin(-x)\cot x}$

$\dfrac{1}{-\sin \cdot \frac{\cos}{\sin}}$

$-\dfrac{1}{\cos} = \boxed{-\sec x}$

16. $\dfrac{\cos^2 x \tan^2(-x) - 1}{\cos^2 x}$

$\dfrac{\cos^2 x \cdot -\tan^2 x - 1}{\cos^2 x}$

$\dfrac{-\cancel{\cos^2 x}\tan^2 x}{\cancel{\cos^2 x}} - \dfrac{1}{\cos^2 x}$

$-\tan^2 x - \sec^2 x$

OR $-(\tan^2 x + \sec^2 x)$

Verify the identity.

17. $\sin x \csc x = 1$

$\cancel{\sin x} \cdot \dfrac{1}{\cancel{\sin x}}$

$1 = 1$ //

18. $\tan\theta \csc\theta \cos\theta = 1$

$\dfrac{\cancel{\sin\theta}}{\cancel{\cos\theta}} \cdot \dfrac{1}{\cancel{\sin\theta}} \cdot \dfrac{\cancel{\cos\theta}}{1}$

$1 = 1$ //

19. $\dfrac{\cos\left(\frac{\pi}{2}-\theta\right)+1}{1-\sin(-\theta)} = 1$

$\dfrac{\sin\theta + 1}{1-(-\sin\theta)}$

$\dfrac{\cancel{\sin\theta+1}}{\cancel{1+\sin\theta}} = 1$

20. $\sin\left(\frac{\pi}{2}-x\right)\tan x = \sin x$

$\cos x \cdot \tan x$

$\cancel{\cos x} \cdot \dfrac{\sin x}{\cancel{\cos x}}$

$\sin x$ //

21. $\dfrac{\csc^2\theta - \cot^2\theta}{1-\sin^2\theta} = \sec^2\theta$

$\dfrac{1}{\cos^2\theta}$

$\sec^2\theta$ //

22. $2 - \cos^2\theta = 1 + \sin^2\theta$

$-1 \qquad -1$

$1 - \cos^2\theta = \sin^2\theta$

$\sin^2\theta = \sin^2\theta$ //

23. $\sin x + \cos x \cot x = \csc x$

$\frac{\sin x}{\sin x}$: $\sin x + \cos x \cdot \dfrac{\cos x}{\sin x}$

$\dfrac{\sin^2 x}{\sin x} + \dfrac{\cos^2 x}{\sin x}$

$\dfrac{\sin^2 x + \cos^2 x}{\sin x} = \dfrac{1}{\sin x} = \csc x$ //

24. $\dfrac{\sin^2(-x)}{\tan^2 x} = \cos^2 x$ DOES NOT EQUATE

$\dfrac{-\sin^2 x}{\frac{\sin^2 x}{\cos^2 x}} \rightarrow \dfrac{-\cancel{\sin^2 x} \cdot \cos^2 x}{\cancel{\sin^2 x}}$

$= -\cos^2 x$

$\dfrac{1+\cos x}{\sin x}$

$\dfrac{1}{\sin x} + \dfrac{\cos x}{\sin x}$

$\csc x + \cot x$ //

25. $\dfrac{1+\cos x}{\sin x} + \dfrac{\sin x}{1+\cos x} = 2\csc x$

$\dfrac{1+\cos x}{\sin x} + \dfrac{\sin x(1-\cos x)}{(1+\cos x)(1-\cos x)}$

Keep fraction

$\boxed{\dfrac{1+\cos x}{\sin x}}$

$\dfrac{\sin x(1-\cos x)}{1-\cos^2 x}$

$\dfrac{\sin x(1-\cos x)}{\sin^2 x} \rightarrow \boxed{\dfrac{1-\cos x}{\sin x}}$

Now fractions have common denominator

26. $\dfrac{\sin x}{1-\cos(-x)} = \csc x + \cot x$

$\dfrac{\sin x(1+\cos x)}{1-\cos x(1+\cos x)}$

$\dfrac{\sin x(1+\cos x)}{1-\cos^2 x}$

$\dfrac{\sin x(1+\cos x)}{\sin^2 x}$

$\dfrac{1+\cos x + 1 - \cos x}{\sin x} = \dfrac{2}{\sin x}$

OR $2\csc x$