Warm-up/Extra Practice

Inverse Trig Functions &
Composite Trig Functions Worksheet

Name_____

Directions: Write the *exact* trigonometric value of the following problems.

1. $\cos^{-1}\frac{\sqrt{3}}{2}$

$30^\circ, \frac{\pi}{6}$

2. $\sin^{-1}\frac{\sqrt{2}}{2}$

$45^\circ, \frac{\pi}{4}$

3. $\arcsin(-1)$

$-90^\circ, -\frac{\pi}{2}$

4. $\cos^{-1}(-1)$

$180^\circ, \pi$

5. $\arctan(1)$

$45^\circ, \frac{\pi}{4}$

6. $\tan^{-1}(-1)$

$-45^\circ, -\frac{\pi}{4}$

7. $\arcsin\left(-\frac{\sqrt{2}}{2}\right)$

$-45^\circ, -\frac{\pi}{4}$

8. $\tan^{-1}\sqrt{3}$

$60^\circ, \frac{\pi}{3}$

9. $\arccos\frac{1}{2}$

$60^\circ, \frac{\pi}{3}$

10. $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$

$-60^\circ, -\frac{\pi}{3}$

11. $\arccos\left(-\frac{\sqrt{2}}{2}\right)$

$135^\circ, \frac{3\pi}{4}$

12. $\cos^{-1}0$

$90^\circ, \frac{\pi}{2}$

13. $\tan^{-1}(0)$

$0^\circ, 0$

14. $\cot^{-1}0$

$90^\circ, \frac{\pi}{2}$

15. $\cos^{-1}2$

UND

16. $\cos\left(\underbrace{\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)}\right)$

$\cos(60^\circ)$

$\frac{1}{2}$

17. $\sin\left(\underbrace{\cos^{-1}\left(-\frac{1}{2}\right)}\right)$

$\sin(120^\circ)$

$\frac{\sqrt{3}}{2}$

18. $\tan\left(\underbrace{\sin^{-1}0}\right)$

$\tan(0)$

0

19. $\cot\left(\cos^{-1} 0\right)$

$\cot(90^\circ)$

0

20. $\sin^{-1}\left(\cos\left(\frac{7\pi}{6}\right)\right)$

$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

$-60^\circ, -\pi/3$

21. $\cos^{-1}\left(\sin\left(\frac{5\pi}{4}\right)\right)$

$\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

$135^\circ, \frac{3\pi}{4}$

22. $\cos^{-1}\left(\sin\left(\frac{\pi}{6}\right)\right)$

$\cos^{-1}\left(\frac{1}{2}\right)$

$60^\circ, \frac{\pi}{3}$

23. $\sin^{-1}\left(\cos\left(\frac{5\pi}{3}\right)\right)$

$\sin^{-1}\left(\frac{1}{2}\right)$

$30^\circ, \frac{\pi}{6}$

24. $\tan^{-1}\left(\sin\left(\frac{\pi}{2}\right)\right)$

$\tan^{-1}(1)$

$45^\circ, \frac{\pi}{4}$

25. $\tan^{-1}\left(\cos\left(\frac{\pi}{2}\right)\right)$

$\tan^{-1}(0)$

0

26. $\sin^{-1}\left(\sin\left(\frac{3\pi}{4}\right)\right)$

$\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$

$45^\circ, \frac{\pi}{4}$

27. $\cos^{-1}\left(\sin\left(-\frac{\pi}{3}\right)\right)$

$\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

$150^\circ, \frac{5\pi}{6}$

use right Δ trig!

28. $\cos\left(\sin^{-1}\left(\frac{8}{17}\right)\right)$

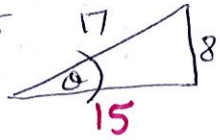
the angle whose sine is $\frac{8}{17}$

$8^2 + b^2 = 17^2$

$64 + b^2 = 289$

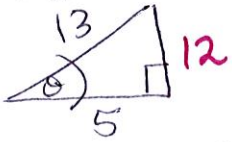
$b^2 = 225$

$b = 15$



$\boxed{\frac{15}{17}}$

29. $\sin\left(\cos^{-1}\left(\frac{5}{13}\right)\right)$



$5^2 + b^2 = 13^2$

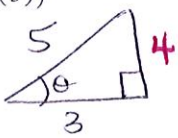
$25 + b^2 = 169$

$b^2 = 144$

$b = 12$

$\boxed{\frac{12}{13}}$

30. $\tan\left(\cos^{-1}\left(\frac{3}{5}\right)\right)$



$3^2 + b^2 = 5^2$

$9 + b^2 = 25$

$b^2 = 16$

$b = 4$

$\boxed{\frac{4}{3}}$

31. $\sin^{-1}\left(\cos\left(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)\right)$

$\cos(60^\circ)$

$\sin^{-1}\left(\frac{1}{2}\right)$

30° or $\frac{\pi}{6}$

32. $\tan\left(\sin^{-1}\left(\cos\left(\frac{\pi}{2}\right)\right)\right)$

$\sin^{-1}(0)$

$\tan(0)$

0

Notes: Right Triangle Trig

Important things to remember:

- The Pythagorean Theorem is $a^2 + b^2 = c^2$.
- The six trig ratios are:

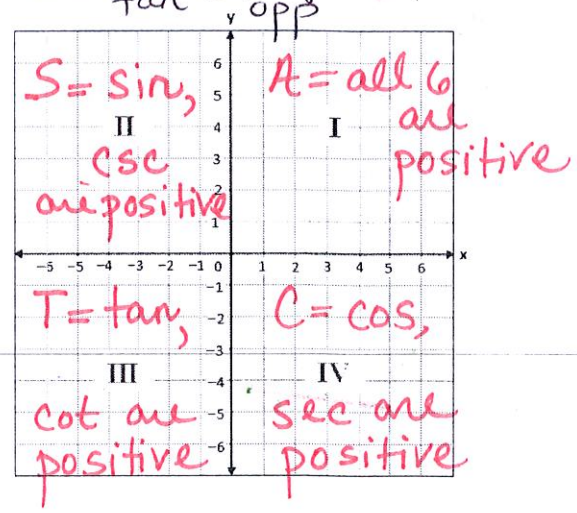
$\sin = \frac{opp}{hyp}$ $\cos = \frac{adj}{hyp}$ $\tan = \frac{opp}{adj}$

$\csc = \frac{1}{\sin} = \frac{hyp}{opp}$

$\sec = \frac{1}{\cos} = \frac{hyp}{adj}$

$\cot = \frac{1}{\tan} = \frac{adj}{opp}$

- Use the acronym "Always Study Trig Carefully" to remember which trig functions are positive in the quadrants:



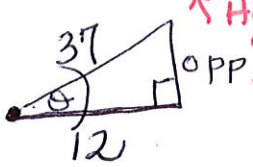
Problem type 1: Finding a trig value given a trig value (no constraints)

Given the $\cos\theta = \frac{12}{37}$, find $\csc\theta$

ADJACENT

HYPOTENUSE

Assume all positive



$\csc\theta = \frac{hyp}{opp}$

use pythagorean thm

$12^2 + b^2 = 37^2$

$144 + b^2 = 1369$

$\sqrt{b^2} = \sqrt{1225}$

$b = 35$

exact value.

$\csc\theta = \frac{37}{35}$

You try: Given $\cot\beta = \frac{11}{60}$, find $\sin\beta$

$\cot B = \frac{adj}{opp}$

$11^2 + 60^2 = c^2$

$121 + 3600 = c^2$

$3721 = c^2$

$61 = c$

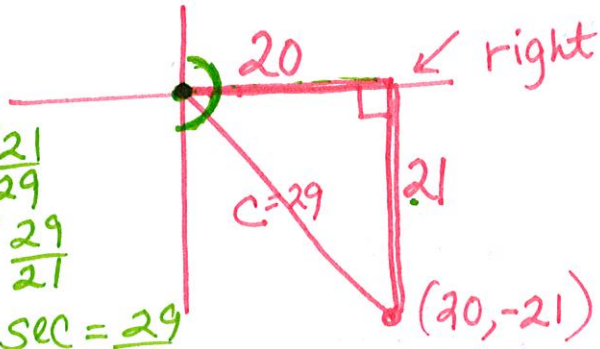
$\sin B = \frac{opp}{hyp}$

$\sin B = \frac{60}{61}$

Problem type 2: Finding trig values given a coordinate point (the constraint is the quadrant in which the point is located!)

EX 1: The point (20, -21) is on the terminal side of an angle in standard position. Determine the exact values of the six trigonometric functions of the angle.

Q4!



right angle forms at x-axis

$20^2 + 21^2 = c^2$

$400 + 441 = c^2$

$841 = c^2$

$c = 29$

$\tan = \frac{-21}{20}$

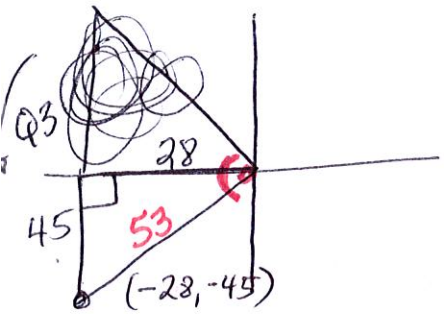
$\cot = \frac{-20}{21}$

$\sin\theta = \frac{-21}{29}$

$\csc\theta = \frac{-29}{21}$

$\cos = \frac{20}{29}$ $\sec = \frac{29}{20}$

You try: The point (-28, -45) is on the terminal side of an angle in standard position. Determine the exact values of the six trigonometric functions of the angle.



$28^2+45^2=c^2$

$784+2025=c^2$

$2809=c^2$

$c=53$

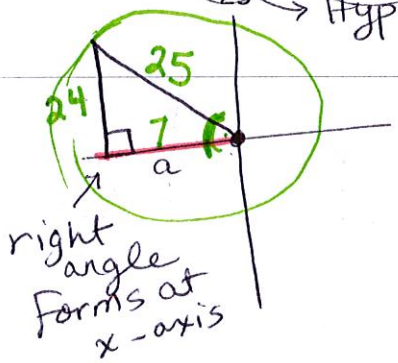
$\sin=\frac{-45}{53}$ $\csc=\frac{-53}{45}$

$\cos=\frac{-28}{53}$ $\sec=\frac{-53}{28}$

$\tan=\frac{45}{28}$ $\cot=\frac{28}{45}$

Problem type 3: Given a function value and an additional constraint

EX 1: Given $\sin\theta=\frac{24}{25}$ (24 → opp, 25 → hyp); constraint: θ lies in $\boxed{\text{Quadrant 2.}}$ Find the other five trig values.



$a^2+24^2=25^2$

$a^2+576=625$

$\sqrt{a^2}=\sqrt{49}$

$a=7$

$\sin\theta=\frac{24}{25}$ $\csc\theta=\frac{25}{24}$

$\cos\theta=\frac{-7}{25}$ $\sec=\frac{-25}{7}$

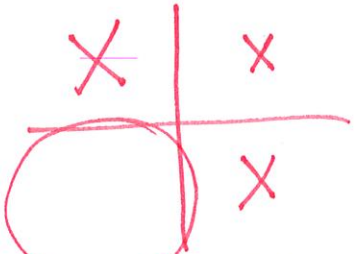
$\tan=\frac{-24}{7}$ $\cot=\frac{-7}{24}$

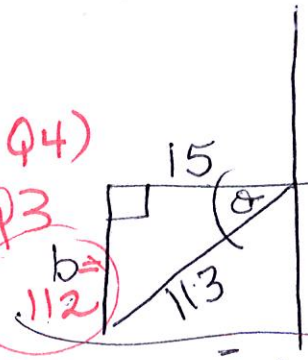
EX 2: Given $\sec\theta=-\frac{113}{15}$ and $\boxed{\tan\theta>0}$ (positive), find the other five trig values.

{ Cos is also negative

where can θ live? (not Q1, Q4)

Graph in Q3





$\frac{hyp}{adj}$

$15^2+b^2=113^2$

$225+b^2=12,769$

$b^2=12,544$

$b=112$

$\sin\theta=\frac{-112}{113}$ $\csc\theta=\frac{-113}{112}$

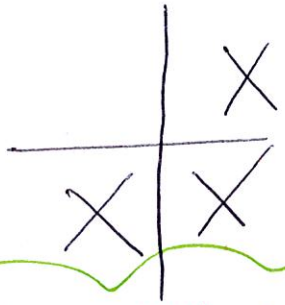
$\cos\theta=\frac{-15}{113}$

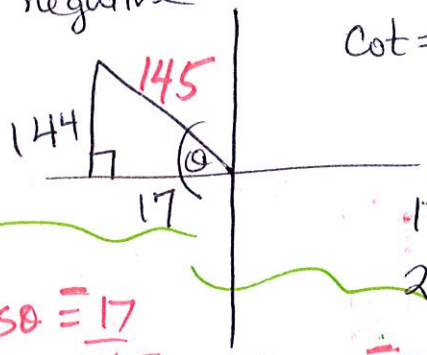
$\sec\theta=\frac{-113}{15}$

$\tan\theta=\frac{112}{15}$

$\cot=\frac{15}{112}$

You try: Given $\cot\theta=\frac{-17}{144}$ and $\cos\theta<0$ (negative), find the other five trig values.





$\cot=\frac{adj}{opp}$

$17^2+144^2=c^2$

$289+20,736=c^2$

$21,025=c^2$

$c=145$

$\sin\theta=\frac{144}{145}$ $\cos\theta=\frac{-17}{145}$

$\csc\theta=\frac{145}{144}$ $\sec\theta=\frac{-145}{17}$

$\tan\theta=\frac{-144}{17}$

$\cot\theta=\frac{-17}{144}$