How Can I Solve an Oblique Triangle? (Part Deux)

The trigonometric ratios: SOH- CAH –TOA for finding sides measures
The inverse trig ratios: $\sin^{-1}$, $\cos^{-1}$, and $\tan^{-1}$ for finding angle measures
The Triangle Sum Conjecture: The sum of the three interior angles of any triangle is 180°

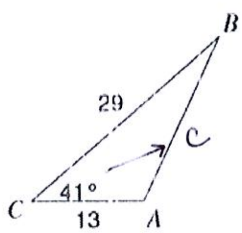
Recall that the Law of Sines allows us to solve when given ASA, AAS or SSA. The pesky SSA leads to the Ambiguous Case which can produce 0, 1 or 2 solutions. Here is the law for solving when given SAS or SSS.

$$a^2 = b^2 + c^2 - 2bc\cos A$$

The LAW OF COSINES: For any triangle ΔABC, $b^2 = a^2 + c^2 - 2ac\cos B$. Use this formula for SAS and SSS.

$$c^2 = a^2 + b^2 - 2ab\cos C$$

EX 1 Find AB

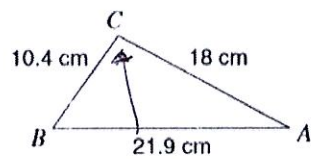


$c^2 = 29^2 + 13^2 - 2(29)(13)\cos 41°$

$c = \sqrt{29^2 + 13^2 - 2(29)(13)\cos 41°}$

$\boxed{c \approx 21.0}$

EX 2 Find the angle measures.



Always find largest angle first!

Since longest side is c, find m∠C first.

$21.9^2 = 10.4^2 + 18^2 - 2(10.4)(18)\cos C$

$\cos^{-1}\left(\dfrac{21.9^2 - 10.4^2 - 18^2}{-2(10.4)(18)}\right) = \cos C$

$C \approx 97.3°$

Next, find m∠B

$18^2 = 10.4^2 + 21.9^2 - 2(10.4)(21.9)\cos B$

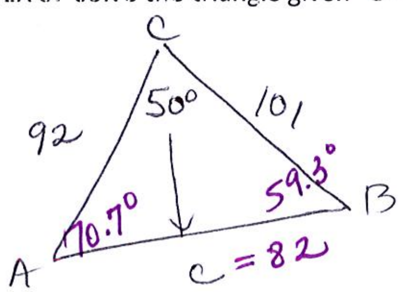
$\cos^{-1}\left(\dfrac{18^2 - 10.4^2 - 21.9^2}{-2(10.4)(21.9)}\right) = \cos B$

$B \approx 54.6$

$\therefore A = 180° - 54.6 - 97.3$

$\boxed{28.1°}$

EX 3: Solve the triangle given $C = 50°, a = 101, b = 92$



$c^2 = 92^2 + 101^2 - 2(92)(101)\cos 50°$

$c = \sqrt{92^2 + 101^2 - 2(92)(101)\cos 50°}$

$c \approx 82.0$

Find m∠A

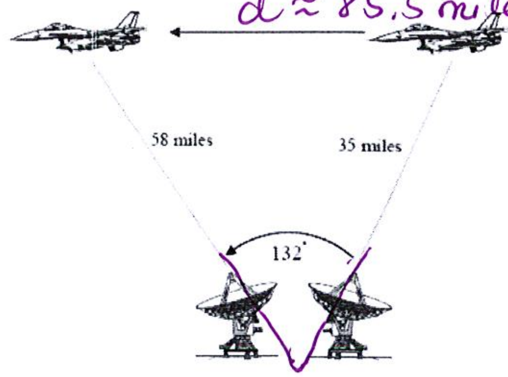
$101^2 = 92^2 + 82^2 - 2(92)(82)\cos A$

$\cos^{-1}\left(\dfrac{101^2 - 92^2 - 82^2}{-2(92)(82)}\right) = \cos A$

$A \approx 70.7°$

EX 4 A satellite dish can track the speed of a plane by recording the distance to the plane at two points in time and the angle through which the dish rotates. In the picture below, a satellite measured the distance to a jet at 35 miles. After 0.25 hours (15 minutes), it measured the distance to the jet at 58 miles. If the satellite rotated through an angle of 132°, determine the average speed of the plane to the nearest mile per hour.

$d \approx 85.5$ miles



$d^2 = 58^2 + 35^2 - 2(58)(35)\cos 132°$

$d = \sqrt{58^2 + 35^2 - 2(58)(35)\cos 132°}$

$d \approx 85.5$

$\frac{85.5 \text{ miles}}{.25 \text{ hours}} \approx 342 \text{ mph}$

We know how to find the area of a triangle if we can establish SAS (this relates to Law of Sines problems). Let's learn how to find the area of a triangle given SSS (Heron's Formula).

Given SSS use Heron's Formula: $Area = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \frac{1}{2}(a+b+c)$

this is the "semi-perimeter"

EX 1 Find the area of a triangle whose side measures are 24 cm, 53 cm, and 39 cm.

$s = \frac{1}{2}(24+53+39) = 58$

$A = \sqrt{58(58-24)(58-53)(58-39)}$

OR $A = \sqrt{58(34)(5)(19)}$

$A \approx 297.9 \text{ cm}^2$

EX 2 A triangular shaped yard has sides measuring 24.1 ft., 35.0 ft., and 40.3 ft. The homeowner wants to lay sod which costs $0.69 per square foot. How much would it cost to sod the yard?

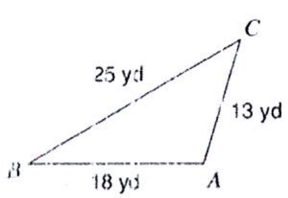
Find the semi-perimeter:

$s = \frac{1}{2}(24.1 + 35.0 + 40.3) = 49.7$

$A = \sqrt{49.7(49.7-24.1)(49.7-35.0)(49.7-40.3)}$

$A \approx 419.3 \text{ ft}^2 \times .69 = \boxed{\$289.31}$

You try: Find the area of the triangle shown



$s = \frac{1}{2}(25+13+18)$

$s = 28$

$A = \sqrt{28(28-25)(28-13)(28-18)}$

$A \approx 144.9 \text{ yds}^2$

Summarize:

For which cases do you use the Law of Sines?

AAS, ASA, SSA

For which cases do you use the Law of Cosines?

SSS, SAS

Which situation leads to the Ambiguous Case?

SSA