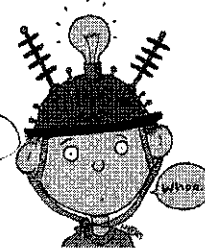


HOW CAN I SOLVE A WORD PROBLEM USING SYSTEMS OF EQUATIONS?



Problem 1: At a local grocery store, three bags of peanuts and four bags of almonds cost \$27.45. At the same store, five bags of peanuts and two bags of almonds cost \$24.05. Which are more expensive, peanuts or almonds? How much more?

Define variables:
 P = cost peanuts
 A = cost almonds

System of equations:
 Look for totals! Total
 $3P + 4A = 27.45$
 $5P + 2A = 24.05$

State your solution:
 Almonds are more expensive. \$4.65
 - 2.95
 \$1.70 more than peanuts

transfer system into a matrix equation

Solve the system showing all steps

$$\begin{bmatrix} 3 & 4 \\ 5 & 2 \end{bmatrix} \begin{bmatrix} P \\ A \end{bmatrix} = \begin{bmatrix} 27.45 \\ 24.05 \end{bmatrix}$$

matrix of coefficients

matrix of constants

matrix of variables

$$\begin{bmatrix} P \\ A \end{bmatrix} = \begin{bmatrix} 2.95 \\ 4.65 \end{bmatrix}$$

Problem 2: The owner of Circus Maximus was counting the money from one day's ticket sales. He knew that a total of 522 tickets were sold. Adult tickets cost \$15 each and children's tickets cost \$8 each. If the total receipts for the day were \$5086.00, how many of each kind of ticket were sold?

Define variables:
 A = # adult tickets
 C = # child tickets

System of equations:
 totals
 $1A + 1C = 522$
 $15A + 8C = 5086$

State your solution:
 130 adult tickets and 392 child tickets were sold

Solve the system showing all steps

$$\begin{bmatrix} 1 & 1 \\ 15 & 8 \end{bmatrix} \begin{bmatrix} A \\ C \end{bmatrix} = \begin{bmatrix} 522 \\ 5086 \end{bmatrix}$$

$$\begin{bmatrix} A \\ C \end{bmatrix} = \begin{bmatrix} 130 \\ 392 \end{bmatrix}$$

Problem 3: Qadir, Raj, and Sabrina go together to a local video store. Qadir rents two movies and three games for a total cost of \$24.30. Raj rents three movies and one game for a total cost of \$18.25. How much money does Sabrina need to rent one movie and two games?

Define variables:
 M = movie rental cost
 G = game rental cost

System of equations:
 $2M + 3G = 24.30$
 $3M + 1G = 18.25$

State your solution:
 Sabrina needs \$14.95 to rent one movie and 4 games

Solve the system showing all steps

$$\begin{bmatrix} 2 & 3 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} M \\ G \end{bmatrix} = \begin{bmatrix} 24.30 \\ 18.25 \end{bmatrix}$$

$$\begin{bmatrix} M \\ G \end{bmatrix} = \begin{bmatrix} 4.35 \\ 5.20 \end{bmatrix}$$

$$4.35 + 2(5.20)$$

Precalculus Worksheet: Solving Systems of Equations Using Matrices

Find the matrix inverse, if it exists. Write all entries in fraction form.

1. $\begin{bmatrix} 4 & 9 \\ -3 & 5 \end{bmatrix}$ $\begin{bmatrix} \frac{5}{47} & -\frac{9}{47} \\ \frac{3}{47} & \frac{4}{47} \end{bmatrix}$

$\frac{1}{|A|} \begin{bmatrix} 5 & -9 \\ 3 & 4 \end{bmatrix}$

3. $\begin{bmatrix} 0 & -4 & 5 \\ 6 & 1 & 1 \\ 3 & 2 & -3 \end{bmatrix}$

$\begin{bmatrix} \frac{5}{39} & \frac{2}{39} & \frac{3}{13} \\ -\frac{7}{13} & \frac{5}{13} & -\frac{10}{13} \\ -\frac{3}{13} & \frac{4}{13} & -\frac{8}{13} \end{bmatrix}$

2. $\begin{bmatrix} 6 & -9 \\ -8 & 12 \end{bmatrix}$

Square?

UNDEFINED

$6(12) - (-8)(-9)$
 $72 - 72 = 0$

4. $\begin{bmatrix} 1 & 2 & -2 \\ 2 & -1 & 1 \\ 2 & 4 & 1 \end{bmatrix}$

$\begin{bmatrix} \frac{1}{5} & \frac{2}{5} & 0 \\ 0 & -\frac{1}{5} & \frac{1}{5} \\ -\frac{2}{5} & 0 & \frac{1}{5} \end{bmatrix}$

\uparrow
 $\det = 0$

Use a matrix inverse to solve the following matrix equations.

5. $\begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 0 \\ -2 & 0 & 3 \end{bmatrix} X = \begin{bmatrix} 19 & 28 & 10 \\ 27 & 30 & 8 \\ -4 & 4 & 4 \end{bmatrix}$

$(3 \times 3)(3 \times 3) = 3 \times 3$
an unknown

$\begin{bmatrix} 8 & 7 & 1 \\ 3 & 9 & 5 \\ 4 & 6 & 2 \end{bmatrix}$

LEFT-MULTIPLY
 $X = A^{-1} \cdot B$

6. $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} X = \begin{bmatrix} 3 & 4 & -2 & 8 \\ 4 & 1 & 3 & -7 \end{bmatrix}$

$X = \begin{bmatrix} -1 & 3 & -5 & 15 \\ 5 & -2 & 8 & -22 \end{bmatrix}$

Rewrite the system as a matrix equation, then use an inverse to solve.

$\begin{bmatrix} 35 \\ 204 \\ 129 \end{bmatrix}$

7. $x - 2y + 3z = 14$
 $2x + y - 2z = 16$
 $-3x - 5y + 9z = 36$

$\begin{bmatrix} 1 & -2 & 3 \\ 2 & 1 & -2 \\ -3 & -5 & 9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 14 \\ 16 \\ 36 \end{bmatrix}$

matrix of coefficients

matrix of variables

FIND $A^{-1} \cdot B$

matrix of constants

$\begin{bmatrix} x \\ y \\ z \end{bmatrix}$

ANSWER

Write a system of equations to model the following word problems, then solve using an inverse matrix. Show your matrix representation in the space provided.

8. Ty bought four candy bars and three packs of gum for a total of \$5.73. Zack bought one candy bar and one pack of gum for \$1.68. What is the price of each item?

$C = \text{cost of candy bar}$
 $G = \text{cost of gum}$
 $4C + 3G = 5.73$
 $1C + 1G = 1.68$

candy bars
cost \$.69
gum cost \$.99

$$\begin{bmatrix} 4 & 3 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} C \\ G \end{bmatrix} = \begin{bmatrix} 5.73 \\ 1.68 \end{bmatrix}$$

$$\begin{bmatrix} C \\ G \end{bmatrix} = \begin{bmatrix} .69 \\ .99 \end{bmatrix}$$

9. The Laredo Sports Shop sold 10 balls, 3 bats, and 2 bases on Monday for \$99. On Tuesday, it sold 4 balls, 8 bats and 2 bases for \$78. On Wednesday, 2 balls, 3 bats and one base were sold for \$33.60. Find the price of each item.

$b = \text{balls}$
 $T = \text{bats}$
 $S = \text{bases}$

$$10B + 3T + 2S = 99$$

$$4B + 8T + 2S = 78$$

$$2B + 3T + 1S = 33.60$$

$$\begin{bmatrix} 10 & 3 & 2 \\ 4 & 8 & 2 \\ 2 & 3 & 1 \end{bmatrix} \begin{bmatrix} B \\ T \\ S \end{bmatrix} = \begin{bmatrix} 99 \\ 78 \\ 33.60 \end{bmatrix}$$

Bats cost \$8, balls cost \$5.40 and
bases cost \$1.40

10. The "Perpetually 21" store is having a winter clearance sale. Margot bought 4 sweaters, a pair of gloves and a scarf for \$34.95. Judith bought 3 sweaters, 2 pairs of gloves and 5 scarves for \$46.95. Amy bought a sweater and 3 scarves for \$18.99. Find the cost of each item.

$S = \text{sweater}$
 $G = \text{glove}$
 $F = \text{scarf}$

$$4S + 1G + 1F = 34.95$$

$$3S + 2G + 5F = 46.95$$

$$1S + 0G + 3F = 18.99$$

$$\begin{bmatrix} 4 & 1 & 1 \\ 3 & 2 & 5 \\ 1 & 0 & 3 \end{bmatrix} X = \begin{bmatrix} 34.95 \\ 46.95 \\ 18.99 \end{bmatrix}$$

Sweaters cost \$6.99, gloves cost \$2.99

Amy bought no gloves
scarves cost \$4.00

11. A beach resort has two vacations specials. One includes a 2 night stay and 3 meals for \$320. The other deal includes a 5 night stay and 8 meals for \$807. Find the cost per night and the cost per meal.

$$2N + 3M = 320$$

$$5N + 8M = 807$$

$$\begin{bmatrix} 2 & 3 \\ 5 & 8 \end{bmatrix} \begin{bmatrix} N \\ M \end{bmatrix} = \begin{bmatrix} 320 \\ 807 \end{bmatrix}$$

$$3. \begin{bmatrix} 8 & -3 \\ -5 & 2 \end{bmatrix} \begin{bmatrix} 320 \\ 807 \end{bmatrix} = \begin{bmatrix} 139 \\ 14 \end{bmatrix}$$

By HAND

1. FIND $|A|$
 $2(8) - 5(3) = 16 - 15 = 1$

2. FIND A^{-1}

$$\frac{1}{1} \begin{bmatrix} 8 & -3 \\ -5 & 2 \end{bmatrix} = \begin{bmatrix} 8 & -3 \\ -5 & 2 \end{bmatrix}$$

Cost is \$139 per night / \$14 per meal

12. Melinda needed to mail a package. She used \$.02 stamps and \$.10 stamps to mail the package. If she used 15 stamps worth \$.78, how many of each type of stamp did she use?

$T = \text{two cent stamps}$
 $N = \text{ten cent stamps}$

$$.02T + .10N = $.78$$

$$T + N = 15$$

Totals

$$\begin{bmatrix} .02 & .10 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} T \\ N \end{bmatrix} = \begin{bmatrix} .78 \\ 15 \end{bmatrix}$$

$$\begin{bmatrix} T \\ N \end{bmatrix} = \begin{bmatrix} 9 \\ 6 \end{bmatrix}$$

Melinda used
9 two-cent stamps and 6 ten-cent stamps