Precalculus **Name**_____

Matrices Review

Determine the dimensions of the matrices:

1. $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \\ 0 & 1 & 2 \\ 3 & 5 & 4 \end{bmatrix}$

5x3

2. $\begin{bmatrix} 5 & 3 & 0 \end{bmatrix}$

1x3

3. Find x & y $\begin{bmatrix} 4 & 9 & 4 \\ 13 & 15 & 6 \\ 2 & 7 & 0 \end{bmatrix} = \begin{bmatrix} 4 & 2x+1 & 4 \\ 13 & 15 & 6 \\ 2 & 3y-5 & 0 \end{bmatrix}$

$9 = 2x + 1$
$8 = 2x$
$4 = x$

$7 = 3y - 5$
$12 = 3y$
$4 = y$

Perform the indicated operation if possible:

4. $6\begin{bmatrix} 4 & 6 \\ 2 & 0 \end{bmatrix}$ $\begin{bmatrix} 24 & 36 \\ 12 & 0 \end{bmatrix}$

5. $\begin{bmatrix} 3 & 4 & -1 \\ 0 & 5 & 2 \end{bmatrix} - \frac{3}{2}\begin{bmatrix} 1 & 10 & 5 \\ -1 & 2 & 7 \end{bmatrix}$

6. $\begin{bmatrix} -2 & 0 & 2 \\ -4 & 0 & 4 \end{bmatrix} * \begin{bmatrix} 1 & 7 \\ 4 & 5 \\ 10 & -1 \end{bmatrix} = \begin{bmatrix} 18 & -16 \\ 36 & -32 \end{bmatrix}$

2x3 3x2

7. Find A^{-1} if A = $\begin{bmatrix} -5 & 1 \\ -2 & -3 \end{bmatrix}$

$|A| = (-5 \cdot -3) - (-2 \cdot 1)$
$15 - -2$
17

$A^{-1} = \frac{1}{17}\begin{bmatrix} -3 & -1 \\ 2 & -5 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} -\frac{3}{17} & -\frac{1}{17} \\ \frac{2}{17} & -\frac{5}{17} \end{bmatrix}$

8. Find A^{-1}, A = $\begin{bmatrix} 2 & 4 \\ 0 & -7 \end{bmatrix}$

$|A| = (2 \cdot -7) - (0 \cdot 4)$
$= -14$

$A^{-1} = \frac{1}{-14}\begin{bmatrix} -7 & -4 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{2}{7} \\ 0 & -\frac{1}{7} \end{bmatrix}$

They are inverses if their product = I

9. Show that $A = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$ are Inverses

$$AB = \begin{bmatrix} 2-1 & 1-1 \\ -2+2 & -1+2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \checkmark$$

yes; they are inverses

10. When would a matrix not have an inverse?

- if it is not a square matrix
- if the determinant = 0

11. Set up a Matrix equation for the system of equations and solve:

$7x - 2y = 4$

$2x - 3y = 12$

$$\underset{A}{\begin{bmatrix} 7 & -2 \\ 2 & -3 \end{bmatrix}} \underset{X}{\begin{bmatrix} x \\ y \end{bmatrix}} = \underset{B}{\begin{bmatrix} 4 \\ 12 \end{bmatrix}}$$

$|A| = (7 \cdot -3) - (2 \cdot -2)$
$= -21 - -4$
$= -17$

$$X = \frac{1}{-17} \begin{bmatrix} -3 & 2 \\ -2 & 7 \end{bmatrix} \begin{bmatrix} 4 \\ 12 \end{bmatrix}$$

$$X = -\frac{1}{17} \begin{bmatrix} -12 + 24 \\ -8 + 84 \end{bmatrix}$$

$$X = \frac{-1}{17} \begin{bmatrix} 12 \\ 76 \end{bmatrix}$$

$$X = \begin{bmatrix} -\frac{12}{17} \\ -\frac{76}{17} \end{bmatrix}$$

12. Evaluate $\begin{vmatrix} 4 & 7 \\ 2 & 0 \end{vmatrix}$

$(4 \cdot 0) - (2 \cdot 7)$
$0 - 14 = -14$

13. Evaluate det $\begin{bmatrix} 2 & -3 & 1 \\ 0 & 6 & 0 \\ -1 & 1 & 2 \end{bmatrix} \begin{matrix} 2 & -3 \\ 0 & 6 \\ -1 & 1 \end{matrix}$

$[(2 \cdot 6 \cdot 2) + (-3 \cdot 0 \cdot -1) + (1 \cdot 0 \cdot 1)] - [(-1 \cdot 6 \cdot 1) + (1 \cdot 0 \cdot 2) + (2 \cdot 0 \cdot -3)]$
$[24 + 0 + 0] - [-6 + 0 + 0]$
$24 - -6$
30

14. Find the Area of the region bounded by the given coordinates:
(-13, 3) (5, 20) (19, -4)

$$A = \pm \frac{1}{2} \left| \begin{matrix} -13 & 3 & 1 \\ 5 & 20 & 1 \\ 19 & -4 & 1 \end{matrix} \right. \begin{matrix} -13 & 3 \\ 5 & 20 \\ 19 & -4 \end{matrix}$$

$[(-260 + 57 - 20)] - [(380 + 52 + 15)]$
$[-223] - [447]$
$-\frac{1}{2}(-670)$
$= 335$ units^2

15. Determine if the points are collinear (-3,-5) (6,1) (10, 2)

$$\left| \begin{matrix} -3 & -5 & 1 \\ 6 & 1 & 1 \\ 10 & 2 & 1 \end{matrix} \right. \begin{matrix} -3 & -5 \\ 6 & 1 \\ 10 & 2 \end{matrix}$$

check to see if determinant = 0

$[(-3 - 50 + 12)] - [(10 - 6 - 30)]$

$[-41] - [-26]$
-15

No; they are NOT COLLINEAR

16. The cities of San Francisco (-30, 32), Oakland (-16, 40), and San Jose (0, 0) form a triangular shaped area. Use a determinant to estimate the area of the region formed by the three cities.

$$A = \pm\frac{1}{2}\begin{vmatrix} -30 & 32 & 1 \\ -16 & 40 & 1 \\ 0 & 0 & 1 \end{vmatrix}$$

$$A = \pm\frac{1}{2}(-688)$$

$$\left(-\frac{1}{2}\right)(-688)$$

344 miles^2

17. A fruit grower raises two crops, which are shipped to three outlets. This is represented by the matrix A $\begin{bmatrix} 125 & 100 & 75 \\ 100 & 175 & 125 \end{bmatrix}$. The profit per unit is represented by the matrix B $\begin{bmatrix} 3.50 & 6.50 \end{bmatrix}$. Find BA and state what the answer matrix represents.

The answer matrix ~~mat~~ represents the profit earned at each of the 3 stores on those 2 produce items $\begin{bmatrix} \$1087.50 & \$1487.50 & \$1075 \end{bmatrix}$

18. The state fair is a popular field trip destination. This year the senior class at High School A and the senior class at High School B both planned trips there. The senior class at High School A rented and filled 8 vans and 8 buses with 240 students. High School B rented and filled 4 vans and 1 bus with 54 students. Every van had the same number of students in it as did the buses. Find the number of students in each van and in each bus by using matrices to solve a system of equations.

$$8V + 8B = 240$$
$$4V + 1B = 54$$

$$\underbrace{\begin{bmatrix} 8 & 8 \\ 4 & 1 \end{bmatrix}}_{A}\underbrace{\begin{bmatrix} V \\ B \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 240 \\ 54 \end{bmatrix}}_{B}$$

$$|A| = (8\cdot 1) - (4\cdot 8)$$
$$8 - 32$$
$$-24$$

$$X = \frac{1}{-24}\begin{bmatrix} 1 & -8 \\ -4 & 8 \end{bmatrix}\begin{bmatrix} 240 \\ 54 \end{bmatrix}$$

$$X = -\frac{1}{24}\begin{bmatrix} 240 - 432 \\ -960 + 432 \end{bmatrix}$$

$$X = \frac{-1}{24}\begin{bmatrix} -192 \\ -528 \end{bmatrix}$$

$$X = \begin{bmatrix} 8 \\ 22 \end{bmatrix}$$

There were 8 students in each van; there were 22 students in each bus

Using the given matrices, evaluate the expression.

$A = \begin{bmatrix} -1 & 3 \\ -4 & 5 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}$ $C = \begin{bmatrix} -7 & -2 \\ 0 & -3 \end{bmatrix}$ $D = \begin{bmatrix} 3 & -1 & 2 \\ 4 & -2 & 0 \\ 0 & -3 & -4 \end{bmatrix}$ $E = \begin{bmatrix} 2 & 5 & -3 \\ 7 & 2 & -6 \\ 1 & -5 & -1 \end{bmatrix}$

19. A + 2B

$\begin{bmatrix} -1 & 7 \\ 2 & -3 \end{bmatrix}$

20. 4D – E

$\begin{bmatrix} 10 & -9 & 11 \\ 9 & -10 & 6 \\ -1 & -7 & -15 \end{bmatrix}$

21. A(B + C)

$\begin{bmatrix} 16 & -21 \\ 43 & -35 \end{bmatrix}$

22. 2AC + 3AB

$\begin{bmatrix} 41 & -56 \\ 101 & -98 \end{bmatrix}$

In standard form, lead. term should be positive

$-10x - 7y = -71$

better written $\boxed{10x + 7y = 71}$

$$\begin{aligned} 36 - 7x + 5y &= 0 \\ +\,35 - 3x - 12y &= 0 \\ \hline 71 - 10x - 7y &= 0 \end{aligned}$$

23. Use a matrix determinant to write the standard form equation of the line passing through the points (12, -7) and (5, 3).

$$\begin{vmatrix} 12 & -7 & 1 \\ 5 & 3 & 1 \\ x & y & 1 \end{vmatrix} = 0$$

$$\left(\begin{vmatrix} 12 & -7 & 1 \\ 5 & 3 & 1 \\ x & y & 1 \end{vmatrix}\begin{matrix} 12 & -7 \\ 5 & 3 \\ x & y \end{matrix}\right) = 0$$

$$(36 - 7x + 5y) - (3x + 12y - 35) = 0$$