Midterm Review
Pre-calculus

1. Convert 135° to radian measure. $\dfrac{3\pi}{4}$

2. Convert $\dfrac{4\pi}{5}$ to degrees. 144°

3. What is the reference angle for 240°? 60°

4. What is the least positive angle that is coterminal with 595°? 235°

5. What is the value of $\sin 225°$? $-\dfrac{\sqrt{2}}{2}$

6. What is the value of $\cos(\dfrac{\pi}{4})$? $\sqrt{\dfrac{2}{2}}$

7. What is the period of $y = 2\sin 2x$? π

8. Graph $y = \dfrac{1}{3}\cos(2x)$

9. What is the period of $y = -\dfrac{1}{2}\cot\dfrac{2}{3}\theta$? $\dfrac{3\pi}{2}$

10. What is the value of $\sec(-60°)$? 2

11. Given the point (-1, 6) is a point on the terminal side of an angle, find $\csc\theta$ $\dfrac{\sqrt{37}}{6}$

12. Graph $y = 3\sec\dfrac{1}{2}\theta$

13. What is the phase shift of $y = -7\sin(2\theta - \pi) + 3$ $\pi/2$

14. What is the value of $Tan^{-1}(1)$? 45° ($\pi/4$) and 225° ($\dfrac{5\pi}{4}$)

15. What is the amplitude of $y = -7\sin(2\theta - \pi) + 3$ 7

16. Find the solution of the system using matrices:
$$x - 2y = 4$$
$$2x - 3y = 2$$
$\begin{bmatrix} -8 \\ -6 \end{bmatrix}$

17. Find the determinant of:
$$\begin{bmatrix} 2 & -3 & 1 \\ 0 & 6 & 0 \\ -1 & 1 & 2 \end{bmatrix}$$ 30

18. $\begin{vmatrix} 4 & 7 \\ 2 & 0 \end{vmatrix}$ -14

19. Find the inverse of:
$$\begin{bmatrix} 3 & -1 & 2 \\ 4 & -2 & 0 \\ 0 & -3 & -4 \end{bmatrix}$$
$\begin{bmatrix} -\dfrac{1}{2} & \dfrac{5}{8} & -\dfrac{1}{4} \\ -1 & \dfrac{3}{4} & -\dfrac{1}{2} \\ \dfrac{3}{4} & -\dfrac{9}{16} & \dfrac{1}{8} \end{bmatrix}$

20. Find the Area of the region bounded by the given coordinates:

(-1 , 3) (5 , 2) (9 , 4) 8 units^2

21. Multiply:

$$\begin{bmatrix} . & 5 & 3 \\ 0 & 2 & 4 \end{bmatrix} \cdot \begin{bmatrix} -3 & 4 \\ -4 & -1 \\ 6 & -6 \end{bmatrix}$$ $\begin{bmatrix} -5 & -19 \\ 16 & -26 \end{bmatrix}$

22. Write an equation of a parabola with a vertex at the origin and a focus at (5,0) $x = \frac{1}{20}y^2$

23. Identify the vertex, focus, and directrix, and direction of the graph of $(x+3)^2 = 16(y+4)$

24. Graph: $x = \frac{-1}{8}y^2$

25. Write an equation of an ellipse with a center (-3, -1), a = 3, and b = 2. Graph the ellipse.

26. Write an equation of a circle with center (12, -9) and radius 4. $(x-12)^2 + (y+9)^2 = 16$

27. Find the center of and graph the hyperbola with the equation $\dfrac{(x-1)^2}{16} - \dfrac{(y-5)^2}{25} = 1$ center (1,5)

28. Identify the conic section and find the center. $9x^2 + 25y^2 - 36x + 50y - 164 = 0$ Hyp Ellipse ; center (2,-1)

29. Find the vertex of $y^2 + 12x - 2y - 35 = 0$ (3,1)

30. Find the center of the hyperbola with equation $16x^2 - y^2 + 32x + 6y + 39 = 0$ (-1,3)

31. What is the graph of $4x^2 = 4y^2 + 16y - 15$ Hyperbola

32. Find the center and radius for the equation $(x-8)^2 + y^2 = 625$ (8,0) ; r=25

33. Find the length of the major axis of the equation $\dfrac{(x-7)^2}{81} + \dfrac{y^2}{25} = 1$ 2(9) = 18

1. $135° \times \dfrac{\pi}{180°} = \dfrac{135\pi}{180} = \dfrac{3\pi}{4}$

2. $\dfrac{4\pi}{5} \times \dfrac{180°}{\pi} = 144°$

3. $240°$

the reference angle is $60°$
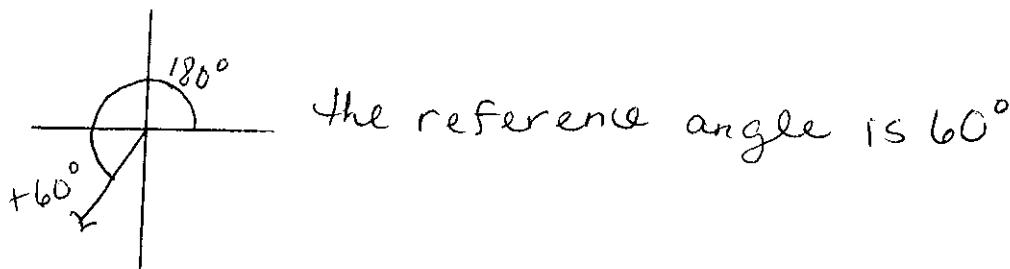
4. $595°$
   $\underline{-360}$
   $235°$ = the "least positive" (principle) angle

5. $\sin(225°) =$

   $\dfrac{-\sqrt{2}}{2}$

6. $\cos\dfrac{\pi}{4} = \dfrac{\sqrt{2}}{2}$

7. $y = 2\sin 2x$
   $\uparrow$
   $B = 2$

   $period = \dfrac{2\pi}{2} = \pi$

8. $y = \dfrac{1}{3}\cos(2x)$ no phase shift

   $period = \dfrac{2\pi}{2} = \pi$

   intervals: $\pi/4$
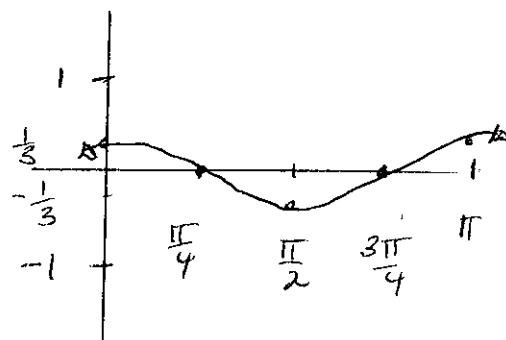
9. $y = -\frac{1}{2} \cot \frac{2}{3}\theta$

   $\text{period} = \dfrac{\pi}{\frac{2}{3}}$ or $\pi \cdot \frac{3}{2} = \frac{3\pi}{2}$

10. $\sec(-60°)$

    $\sec = \dfrac{1}{\cos\theta} = 2$

    $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

11.

    $(-1, 6)$
    6
    $\sqrt{37}$
    1
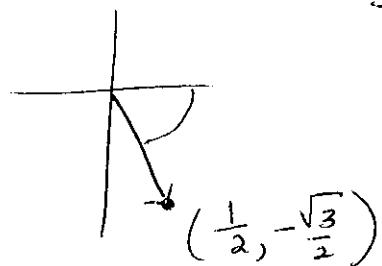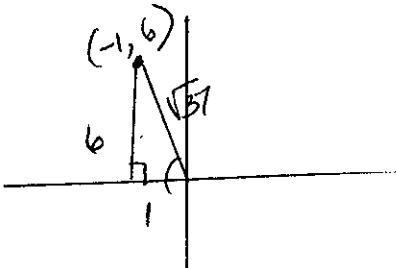
    $1^2 + 6^2 = c^2$
    $1 + 36 = c^2$
    $37 = c^2$
    $c = \sqrt{37}$

    $\csc\theta = \dfrac{1}{\sin\theta} = \dfrac{hyp}{opp} = \dfrac{\sqrt{37}}{6}$

    $\dfrac{6}{\sqrt{37}} \quad \dfrac{\sqrt{37}}{\sqrt{}}$
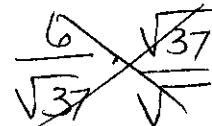
12. $y = 3\sec\frac{1}{2}\theta$

    $\text{period} = \dfrac{2\pi}{\frac{1}{2}} = 4\pi$

    intervals: π
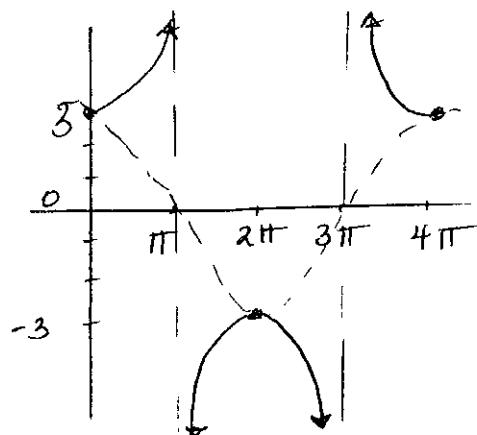
    no phase shift

13. $y = -7\sin(2\theta - \pi) + 3$

    phase shift: $2\theta - \pi = 0$

    $2\theta = \pi$

    $\theta = \dfrac{\pi}{2}$

14. $\tan^{-1}(1)$ What angle(s) have a tangent value of 1?
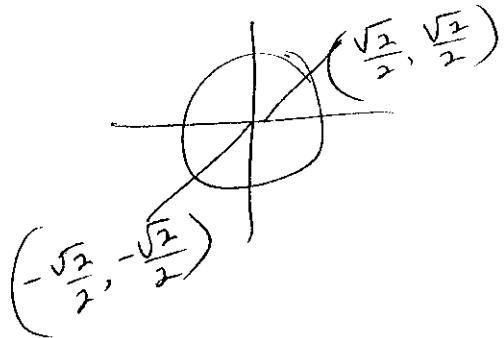
$45°$ and $225°$

$\left(\dfrac{\pi}{4} \text{ and } \dfrac{5\pi}{4}\right)$

15. $y = -7\sin(2\theta - \pi) + 3$

amplitude $= 7$

16. $x - 2y = 4$

$2x - 3y = 2$

Find $A^{-1} \cdot B$

You may use a calculator!

$$\begin{bmatrix} 1 & -2 \\ 2 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

$\nearrow A$ $\nearrow B$

$|A| = (1 \cdot -3) - (2 \cdot -2)$

$-3 + 4$

1

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{1}\begin{bmatrix} -3 & 2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = 1\begin{bmatrix} -12 + 4 \\ -8 + 2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -8 \\ -6 \end{bmatrix}$$

17.

$$\begin{vmatrix} 2 & -3 & 1 \\ 0 & 6 & 0 \\ -1 & 1 & 2 \end{vmatrix} \begin{matrix} 2 & -3 \\ 0 & 6 \\ -1 & 1 \end{matrix}$$

$$[(2 \cdot 6 \cdot 2) + (-3 \cdot 0 \cdot -1) + (1 \cdot 0 \cdot 1)] - [(-1 \cdot 6 \cdot 1) + (1 \cdot 0 \cdot 2) + (2 \cdot 0 \cdot -3)]$$

$$[24 + 0 + 0] - [-6 + 0 + 0]$$

$$24 + 6$$

$$30$$

18.

$$\begin{vmatrix} 4 & 7 \\ 2 & 0 \end{vmatrix} = (4 \cdot 0) - (2 \cdot 7)$$

$$0 - 14$$

$$-14$$

19.

$$\begin{bmatrix} 3 & -1 & 2 \\ 4 & -2 & 0 \\ 0 & -3 & -4 \end{bmatrix}^{-1} = \begin{bmatrix} -\frac{1}{2} & \frac{5}{8} & -\frac{1}{4} \\ -1 & \frac{3}{4} & -\frac{1}{2} \\ \frac{3}{4} & -\frac{9}{16} & \frac{1}{8} \end{bmatrix}$$

20.

$$A = \pm \frac{1}{2} \begin{vmatrix} -1 & 3 & 1 \\ 5 & 2 & 1 \\ 9 & 4 & 1 \end{vmatrix} \begin{matrix} -1 & 3 \\ 5 & 2 \\ 9 & 4 \end{matrix}$$

$$[(-1 \cdot 2 \cdot 1) + (3 \cdot 1 \cdot 9) + (1 \cdot 5 \cdot 4)] - [(9 \cdot 2 \cdot 1) + (4 \cdot 1 \cdot -1) + (1 \cdot 5 \cdot 3)]$$

$$[-2 + 27 + 20] - [18 + 4 + 15]$$

$$45 - (29)$$

$$16 \longrightarrow \frac{1}{2}(16) = 8 \text{ units}^2$$

21. $\begin{bmatrix} 1 & 5 & 3 \\ 0 & 2 & 4 \end{bmatrix} \begin{bmatrix} -3 & 4 \\ -4 & -1 \\ 6 & -6 \end{bmatrix} = \begin{bmatrix} -5 & -19 \\ 16 & -26 \end{bmatrix}$

$$2 \times 3 \qquad 3 \times 2$$

22. $x = y^2$

$x = \frac{1}{20} y^2$

23. $(x+3)^2 = 16(y+4)$

$\frac{1}{16}(x+3)^2 = y + 4$

$\frac{1}{16}(x+3)^2 - 4 = y$

or

$y = \frac{1}{16}(x+3)^2 - 4$

$p = 4$

Graph is vertical
opening upward

This is an alternate
form of equation

Vertex $(-3, \overset{+4}{-4})$
-4

focus $(-3, 0)$
directrix: $y = -8$

24. $x = -\frac{1}{8} y^2$ Graph opens to left

$4p = 8$

$p = 2$

25.
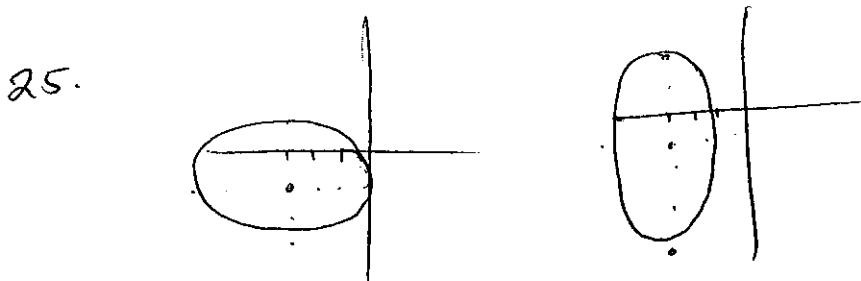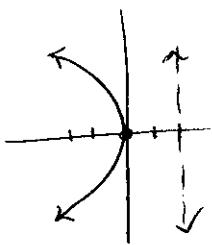
We don't know whether horizontal or vertical

Could be $\dfrac{(x+3)^2}{9} + \dfrac{(y+1)^2}{4} = 1$

or $\dfrac{(x+3)^2}{4} + \dfrac{(y+1)^2}{9} = 1$

26. $(x-12)^2 + (y+9)^2 = 16$

27. $\dfrac{(x-1)^2}{16} - \dfrac{(y-5)^2}{25} = 1$

Center: $(1,5)$

horizontal opening

$c^2 = a^2 + b^2$

$c^2 = 16 + 25$

$c^2 = 41$

$c = \sqrt{41}$ or ≈ 6.4
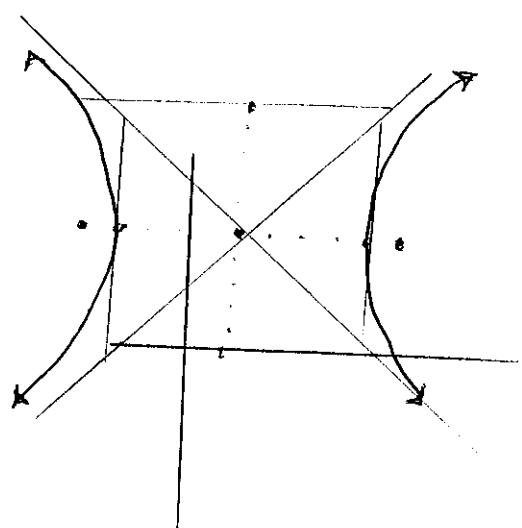
28. $9x^2 + 25y^2 - 36x + 50y - 164 = 0$

$9x^2 - 36x + 25y^2 + 50y = 164$

$9(x^2 - 4x + 4) + 25(y^2 + 2y + 1) = 164$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad + 36$
$9(x-2)^2 + 25(y+1)^2 = 225 \qquad\qquad\quad +25$

$$\frac{(x-2)^2}{25} + \frac{(y+1)^2}{9} = 1 \qquad \text{Ellipse;}$$
$$\text{center } (2,-1)$$

29. $y^2 + 12x - 2y - 35 = 0$

$12x = -y^2 + 2y + 35$

$12y = -(y^2 - 2y + 1) \qquad\qquad + 35$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad + 1$

$12x = -(y-1)^2 + 36$

$x = -\frac{1}{12}(y-1)^2 + \frac{36}{12}$

$x = -\frac{1}{12}(y-1)^2 + 3 \qquad\qquad \text{Vertex } (3,1)$

30. $16x^2 - y^2 + 32x + 6y + 39 = 0$

$16x^2 + 32x - y^2 + 6y = -39$

$16(x^2 + 2x + 1) - (y^2 - 6y + 9) = -39$
$ +16$
$ -9$

$16(x+1)^2 - (y-3)^2 = -32$

$-\dfrac{(x+1)^2}{2} + \dfrac{(y-3)^2}{32} = 1$

$\dfrac{(y-3)^2}{32} - \dfrac{(x+1)^2}{2} = 1$
$\qquad\qquad\qquad\qquad\qquad$ center $(-1, 3)$

31. $\qquad 4x^2 = 4y^2 + 16y - 15$

$\qquad 4x^2 - 4y^2 - 16y + 15 = 0 \qquad$ Hyperbola

32. $\qquad (x-8)^2 + y^2 = 625$

$\qquad$ Center $(8, 0)$; $r = 25$

33. $\qquad \dfrac{(x-7)^2}{81} + \dfrac{y^2}{25} = 1$
$\qquad\qquad\qquad\qquad\qquad$ Major axis = 18 units

$\qquad$ Horizontal Ellipse
$\qquad\qquad$ Center: $(7, 0)$
$\qquad\qquad\qquad\qquad\qquad$ minor axis = 10 units