

MIDTERM REVIEW

$$1. \quad 135^\circ \times \frac{\pi}{180^\circ}$$

$$\frac{135^\circ \pi}{180^\circ}$$

$$\frac{3\pi}{4}$$

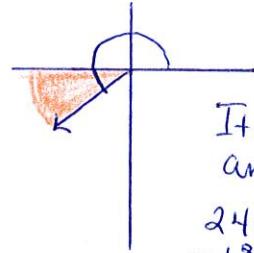
4. 595°
 -360°
 $\underline{235^\circ}$ is
 the least
 positive
 coterminal
 angle

$$2. \quad \frac{4\pi}{5} \times \frac{180^\circ}{\pi}$$

$$\frac{720^\circ}{5}$$

$$144^\circ$$

3. 240° lies in Q3

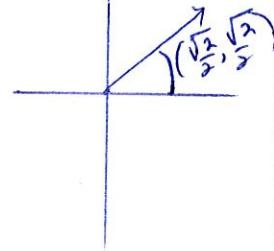


Its reference angle is
 $240^\circ - 180^\circ = 60^\circ$

$$6. \quad \cos \frac{\pi}{4}$$

(Same as $\cos 45^\circ$)

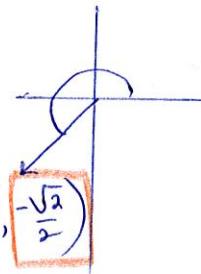
$$= \frac{\sqrt{2}}{2}$$



$$5. \quad \sin 225^\circ$$

$$= -\frac{\sqrt{2}}{2}$$

You may use
 unit circle
 or
 calculator



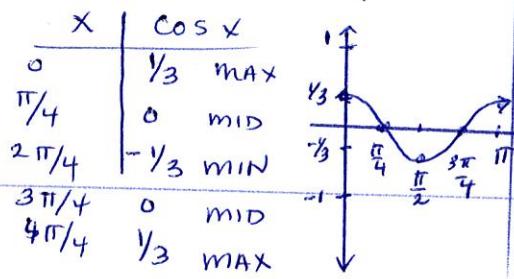
$$7. \quad y = 2 \sin 2x$$

value of "b"
 indicates change
 of period.

amplitude = $\frac{1}{3}$

period = $\frac{2\pi}{2} = \pi$

intervals occur at $\frac{\pi}{4}$ units



$$9. \quad y = -\frac{1}{2} \cot \frac{2}{3}\theta$$

period = $\frac{2\pi}{\frac{2}{3}} = 3\pi$

$$\text{or } \frac{2\pi}{1} \times \frac{3}{2} = \frac{6\pi}{2} = 3\pi$$

13. Given

$$y = -7 \sin(2\theta - \pi) + 3$$

find the phase shift

Set equal to zero

$$2\theta - \pi = 0$$

$$2\theta = \pi$$

$$\theta = \frac{\pi}{2}$$

The phase shift
 is $\frac{\pi}{2}$ to the right

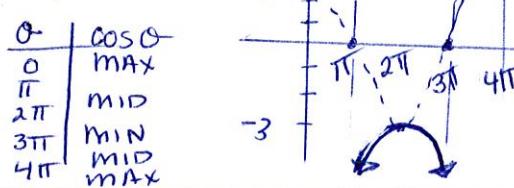
$$12. \quad y = 3 \sec \frac{1}{2}\theta$$

use graph of $y = 3 \cos \frac{1}{2}\theta$
 as skeleton

period = $\frac{2\pi}{\frac{1}{2}} = 4\pi$

intervals: $\frac{4\pi}{4} = \pi$

amplitude = 3



SIN + CSC are positive
 in Q2

$$(-1, 6)$$

$$1^2 + 6^2 = c^2$$

$$1 + 36 = c^2$$

$$37 = c^2$$

$$c = \sqrt{37}$$

$$\sin \theta = \frac{6}{\sqrt{37}}$$

$$\therefore \csc \theta = \frac{\sqrt{37}}{6}$$

EVALUATE

$$\tan^{-1}(1) = 45^\circ \text{ or } \frac{\pi}{4}$$

"At what angle does the tangent equal 1?"
Don't forget restrictions:

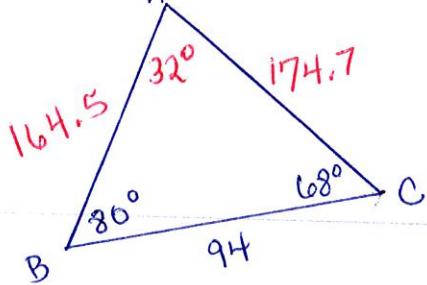
$$\begin{array}{ll} \sin: Q1, Q4 & \tan: Q1, Q4 \\ \cos: Q1, Q2 & \end{array}$$

17. Solve the triangle

$$B = 80^\circ, C = 68^\circ, a = 94$$

ASA

use LAW OF SINES



You must find an angle/side combination.

$$\frac{\sin 32^\circ}{94} = \frac{\sin 80^\circ}{b}$$

$$b = \frac{94 \sin 80^\circ}{\sin 32^\circ}$$

$$b \approx 174.7$$

$$\frac{\sin 32^\circ}{94} = \frac{\sin 68^\circ}{c}$$

$$c = \frac{94 \sin 68^\circ}{\sin 32^\circ}$$

$$c \approx 164.5$$

$$15. y = |-7| \sin(2\theta - \pi) + 3 \quad \text{take absolute value}$$

the amplitude is 7 (not -7!). If you miss this question, I'm sending you back to Alg I.

$$\begin{array}{l} a = 18 \\ b = 24 \\ c = 80^\circ \end{array} \quad \text{SAS}$$

$$A = \frac{1}{2}(18)(24)\sin 80^\circ$$

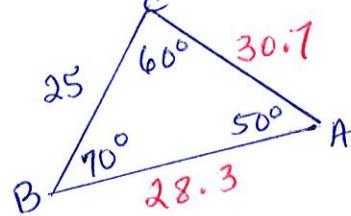
$$A \approx 212.7 \text{ units}^2$$

18. Solve the triangle

$$A = 50^\circ, B = 70^\circ, a = 25$$

AAS

Law of sines



$$\frac{\sin 50^\circ}{25} = \frac{\sin 70^\circ}{b}$$

$$b = 25 \frac{\sin 70^\circ}{\sin 50^\circ}$$

$$b \approx 30.7$$

$$\frac{\sin 50^\circ}{25} = \frac{\sin 60^\circ}{c}$$

$$c = 25 \frac{\sin 60^\circ}{\sin 50^\circ}$$

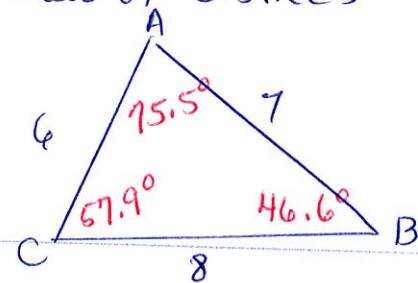
$$c \approx 28.3$$

19. Solve the triangle

$$b = 8, c = 7, a = 8$$

SSS

Law of cosines



$$\text{mLA} =$$

$$8^2 = 6^2 + 7^2 - 2(6)(7)\cos A$$

$$\cos^{-1} \left(\frac{8^2 - 6^2 - 7^2}{-2(6)(7)} \right)$$

$$\text{mLA} \approx 75.5^\circ$$

$$\text{mLB} =$$

$$6^2 = 7^2 + 8^2 - 2(7)(8)\cos B$$

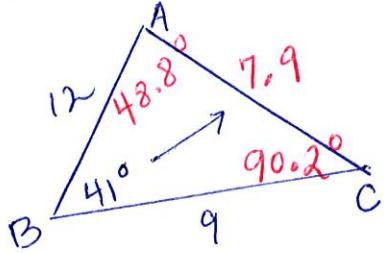
$$\cos^{-1} \left(\frac{6^2 - 7^2 - 8^2}{-2(7)(8)} \right)$$

$$\text{mLB} \approx 46.6^\circ$$

20. Solve the triangle

$$a = 9, c = 12, B = 41^\circ$$

SAS
use Law of cosines



$$b^2 = 9^2 + 12^2 - 2(9)(12)\cos 41^\circ$$

$$b = \sqrt{9^2 + 12^2 - 2(9)(12)\cos 41^\circ}$$

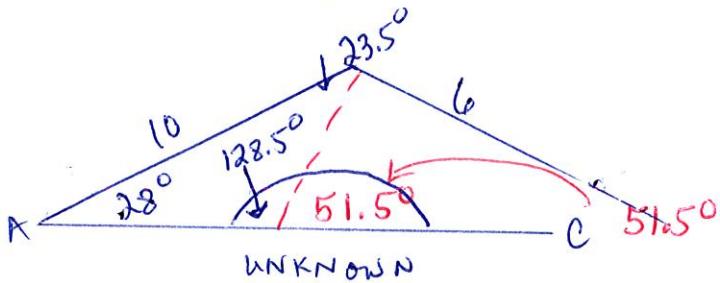
$$b \approx 7.9$$

$$\text{m}\angle C =$$

$$\cos^{-1} \left(\frac{12^2 - 7.9^2 - 9^2}{-2(7.9)(9)} \right)$$

$$\text{m}\angle C \approx 90.2^\circ$$

23. $A = 28^\circ, a = 6, c = 10$



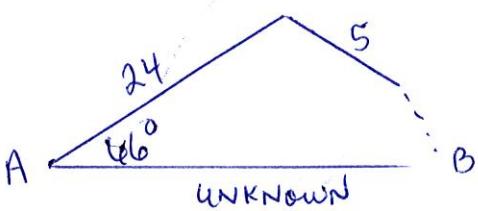
$$\frac{\sin 28^\circ}{6} = \frac{\sin C}{10}$$

$$\sin^{-1} \left(\frac{10 \sin 28^\circ}{6} \right) = \sin C$$

$$\text{m}\angle C \approx 51.5$$

21. How many triangles?

$$A = 66^\circ, a = 5, b = 24$$



$$\frac{\sin 66^\circ}{5} = \frac{\sin B}{24}$$

$$\sin \left(\frac{24 \sin 66^\circ}{5} \right) = \sin B$$

DOMAIN ERROR

MEANS NO

TRIANGLE EXISTS

NO SOLUTION

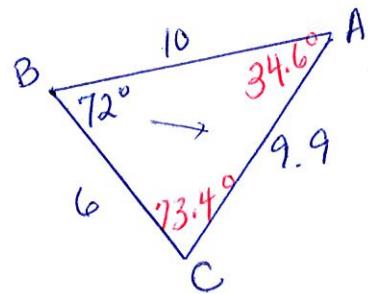
$$22. B = 72^\circ$$

$$a = 6$$

$$c = 10$$

This not SSA.

Do we need to check to make sure the triangle is valid?



$$b^2 = 6^2 + 10^2 - 2(6)(10)\cos 72^\circ$$

$$b \approx 9.9$$

Find $\text{m}\angle C$

$$\cos^{-1} \left(\frac{10^2 - 6^2 - 9.9^2}{-2(6)(9.9)} \right)$$

$$\text{m}\angle C \approx 73.4^\circ$$

1 triangle exists

2 triangles exist

24. $2\sin x + 1 = 0$

$$2\sin x = -1$$

$$\sin^{-1}(\sin x) = \left(-\frac{1}{2}\right)$$

use Q1+Q4
where is the
sin value = $-\frac{1}{2}$?

$$\theta = -\frac{\pi}{6}$$

SOLVE

27. $x - 2y = 4$

$$2x - 3y = 2$$

Change to matrix
form

$$\begin{bmatrix} A & X \\ 1 & -2 \\ 2 & -3 \end{bmatrix} = \begin{bmatrix} B \\ 4 \\ 2 \end{bmatrix}$$

- enter matrix of coefficients as matrix A
- enter matrix of constants as matrix B

- SOLVE FOR X by
finding $A^{-1}B$

Remember, order matters!

All of this can be done
in your calculator.

By hand:

$$\text{If } A = \begin{bmatrix} 1 & -2 \\ 2 & -3 \end{bmatrix}$$

$$\text{then } |A| = (1 \cdot -3) - (2 \cdot -2) \\ -3 - (-4) = 1$$

$$\text{so } A^{-1} = \frac{1}{|A|} \begin{bmatrix} -3 & 2 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

$$\text{or } 1 \begin{bmatrix} -3 & 2 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 2 \end{bmatrix} = \begin{bmatrix} -8 \\ -6 \end{bmatrix}$$

25. $4\sin^2 x - 3 = 0$

$$4\sin^2 x = 3$$

$$\sin^2 x = \frac{3}{4}$$

$$\sin x = \pm \frac{\sqrt{3}}{\sqrt{4}} = \pm \frac{\sqrt{3}}{2}$$

$$x = \frac{\pi}{3} \text{ and } x = -\frac{\pi}{3}$$

Find
 $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$
and
 $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

26. $2\cos^2 \theta - 3\cos \theta + 1 = 0$

$$-2 \cancel{\cos^2 \theta} - 3\cancel{\cos \theta} + 1 = 0$$

$$(\cos \theta - 2)(\cos \theta - 1) = 0$$

$$\cos \theta = 1 \quad \cos \theta = \frac{1}{2}$$

Find $\cos^{-1}(1)$ and $\cos^{-1}\left(\frac{1}{2}\right)$

$$\theta = 0 \text{ and } \theta = \frac{\pi}{3}$$

28.
$$\begin{bmatrix} 3 & -1 & 2 \\ 4 & -2 & 0 \\ 0 & -3 & -4 \end{bmatrix}$$

Enter this as a
3x3 into your
calculator. Choose
det from
matrix menu. →
OR →
BY HAND

$$\begin{array}{ccc|cc} 3 & -1 & 2 & 3 & -1 \\ 4 & -2 & 0 & 4 & -2 \\ 0 & -3 & -4 & 0 & -3 \end{array}$$

$$[(3 \cdot -2 \cdot 4) + (-1 \cdot 0 \cdot 0) + (2 \cdot 4 \cdot -3)] - [(0 \cdot -2 \cdot 2) + (-3 \cdot 0 \cdot 3) + (-4 \cdot 4 \cdot -1)]$$

$$[24 + 0 - 24] -$$

$$[0 + 0 + 16]$$

$$0 - 16 \\ -16$$

29.

$$\begin{vmatrix} 4 & 7 \\ 2 & 0 \end{vmatrix}$$

This
notation
indicates
DETERMINANT

$$(4 \cdot 0) - (2 \cdot 7)$$

$$0 - 14 \\ -14$$

30. $\begin{bmatrix} 3 & -1 & 2 \\ 4 & -2 & 0 \\ 0 & -3 & -4 \end{bmatrix}$

We did not learn to do
inverses of 3×3 s by hand.

Enter into calculator as 3×3 ,
then find A^{-1} .

Go to MATH \rightarrow FRAC \rightarrow ENTER, ENTER
TO CHANGE TO FRACTIONS

$$\begin{bmatrix} -\frac{1}{2} & \frac{5}{8} & -\frac{1}{4} \\ -1 & \frac{3}{4} & -\frac{1}{2} \\ \frac{3}{4} & -\frac{9}{16} & \frac{1}{8} \end{bmatrix}$$

31. Find the area of the triangle with vertices
 $(-1, 3)$, $(5, 2)$ and $(9, 4)$

Enter into 3×3 matrix; add a column of 1's

$$A = \pm \frac{1}{2} \begin{bmatrix} -1 & 3 & 1 \\ 5 & 2 & 1 \\ 9 & 4 & 1 \end{bmatrix}$$

choose
the sign
that guarantees
positive outcome.

- Find the determinant
- Take the absolute value
(i.e. make it positive!)
- Divide by 2

OR BY HAND

$$\begin{vmatrix} -1 & 3 & 1 & | & -1 & 3 \\ 5 & 2 & 1 & | & 5 & 2 \\ 9 & 4 & 1 & | & 9 & 4 \end{vmatrix}$$

$$[(-1 \cdot 2 \cdot 1) + (3 \cdot 1 \cdot 9) + (1 \cdot 5 \cdot 4)] - [(9 \cdot 2 \cdot 1) + (4 \cdot 1 \cdot -1) + (1 \cdot 5 \cdot 3)]$$

$$[-2 + 27 + 20] - [18 - 4 + 15]$$

$$45 - (29)$$

$$\frac{16}{2} = 8 \text{ units}^2$$

32. $R_1 \begin{bmatrix} 1 & 5 & 3 \end{bmatrix}$ $R_2 \begin{bmatrix} 0 & 2 & 4 \end{bmatrix}$
 $\begin{array}{c} C_1 \\ C_2 \end{array}$ $\begin{array}{c} C_1 \\ C_2 \end{array}$

2×3

$$R_1 \begin{bmatrix} -5 & -19 \\ 16 & -26 \end{bmatrix}$$

 R_2

- verify the product exists
- create template for 2×2 answer

OR use your calculator !!

You may use
your
calculator!