Law of Cosines

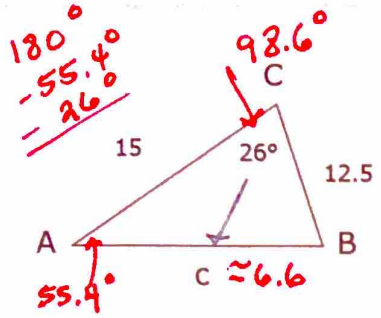
$a^2 = b^2 + c^2 - 2 \cdot c \cdot b \cdot \cos A$

$b^2 = a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos B$

$c^2 = b^2 + a^2 - 2 \cdot a \cdot b \cdot \cos C$

As always, use an inverse function when solving for an angle measure.

Example 1 Solve ΔABC given the diagram.



we must find side c first since we are given only ∠C.

$c^2 = 15^2 + 12.5^2 - 2(15)(12.5)\cos 26°$ ← set up equation

$c = \sqrt{15^2 + 12.5^2 - 2(15)(12.5)\cos 26°}$ ← square root property

$c \approx 6.6$

Solve for either angle A or B: (I pick A)

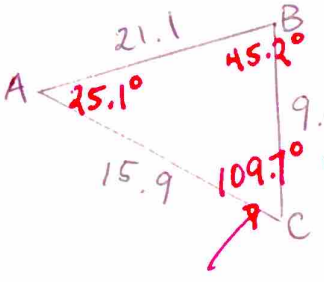
$12.5^2 = 15^2 + 6.6^2 - 2(15)(6.6)\cos A$

$12.5^2 - 15^2 - 6.6^2 = -2(15)(6.6)\cos A$

$\cos^{-1}\left(\frac{12.5^2 - 15^2 - 6.6^2}{-2(15)(6.6)}\right) = \cos^{-1}(\cos A)$ use cosine^{-1} to find m∠A

$m\angle A \approx 55.4°$

Example 2 (SSS): Solve ΔABC given a = ~~9.47~~ 9.5 ft., b = 15.9 ft. and c = 21.1 ft.



180° − 25.1° − 45.2° = 109.7°

Pick any angle to begin. (I pick A)

$9.5^2 = 21.1^2 + 15.9^2 - 2(21.1)(15.9)\cos A$

$\cos^{-1}\left(\frac{9.5^2 - 21.1^2 - 15.9^2}{-2(21.1)(15.9)}\right) = \cos^{-1}(\cos A)$

$m\angle A \approx 25.1°$

Solve for next angle (I pick B)

$15.9^2 = 21.1^2 + 9.5^2 - 2(21.1)(9.5)\cos B$

$\cos^{-1}\left(\frac{15.9^2 - 21.1^2 - 9.5^2}{-2(21.1)(9.5)}\right) = \cos^{-1}(\cos B)$

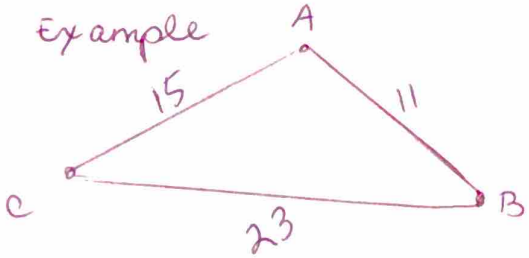
$m\angle B \approx 45.2°$

Use Heron's Formula to find the area of an oblique triangle given SSS

Step 1: Find the semi – perimeter of the triangle: $s = \frac{a+b+c}{2}$

Step 2: Apply *Heron's formula*: $A = \sqrt{s(s-a)(s-b)(s-c)}$

Example



Step 1: Find the semi-perimeter

$s = \frac{15 + 11 + 23}{2}$

$s = 24.5$

Step 2: Use Heron's Formula

$A = \sqrt{24.5(24.5 - 15)(24.5 - 11)(24.5 - 23)}$

$A \approx 68.7 \text{ units}^2$