Precalculus Notes: Resolving Vectors

$\tan^{-1}\left(\frac{y}{x}\right)$ $+180°$ {Q2 | Q1 / Q3 | Q4 $+360°$

Some problems will involve multiple vectors. We will simply combine all the steps that we have learned previously to solve them.

Step 1: Find the components of each individual vector $\mathbf{V_x = V\cos\theta}$ and $\mathbf{V_y = V\sin\theta}$

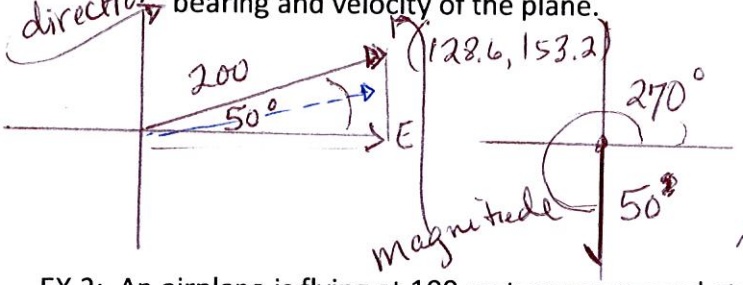
Step 2: Sum the x and y components of all the vectors

Step 3: Find the magnitude and direction of this component vector

magnitude $\approx \sqrt{128.6^2 + 103.2^2} \approx 164.9$ mph

Remember, $magnitude = \sqrt{x^2 + y^2}$ $direction(\theta) = \tan^{-1}\left(\frac{y}{x}\right)$ $\tan^{-1}\left(\frac{103.2}{128.6}\right) \approx 38.7°$

EX 1: An airplane is flying 200 mph at 50° north of east. The wind velocity is 50 mph due south. Find the bearing and velocity of the plane.

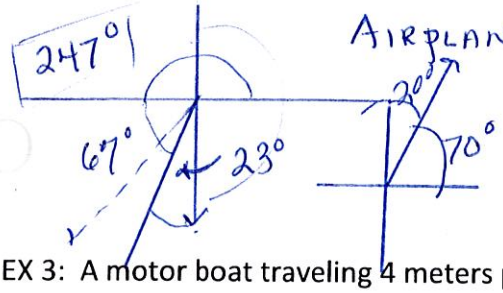


Airplane $\langle 200\cos 50°, 200\sin 50° \rangle$
$\langle 128.6, 153.2 \rangle$

Wind $\langle 50\cos 270°, 50\sin 270° \rangle$
$\langle 0, -50 \rangle$

Airplane + Wind $\langle 128.6, 103.2 \rangle$

EX 2: An airplane is flying at 100 meters per second at 23° west of south into a wind blowing 30 meters per second at 20° east of north. Find the resultant direction and velocity of the plane.



AIRPLANE $\langle 100\cos 247°, 100\sin 247° \rangle$
$\langle -39.1, -92.1 \rangle$

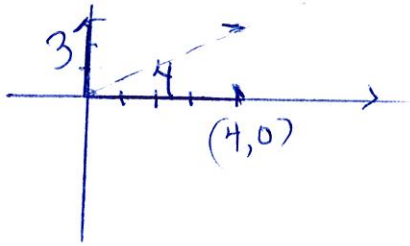
WIND $\langle 30\cos 70°, 30\sin 70° \rangle$
$\langle 10.3, 28.2 \rangle$

A+W $\langle -28.8, -63.9 \rangle$

mag $\sqrt{28.8^2 + 63.9^2} \approx 70.1$ m/sec

$\tan^{-1}\left(\frac{-63.9}{-28.8}\right) \approx 65.7° + 180° = 245.7°$

EX 3: A motor boat traveling 4 meters per second to the east encounters a current flowing 3 meters per second to the north. Find the impacted direction and speed of the motor boat.



Boat $\langle 4\cos 0°, 4\sin 0° \rangle$
$\langle 4, 0 \rangle$

Current $\langle 3\cos 90°, 3\sin 90° \rangle$
$\langle 0, 3 \rangle$

Boat current $\langle 4, 3 \rangle$

magnitude: $\sqrt{4^2 + 3^2} = 5$

direction: $\tan^{-1}\left(\frac{3}{4}\right) \approx 36.9°$

EX 4: Four trucks are pulling a car from a ditch. The first truck is pulling with 40 N of force at 20°. The second is pulling with 33 N of force at 37°. The third truck exerts a force of 25 N at 95°. Finally, the fourth truck is pulling with a force of 60 N at 135°. Find the resulting force and direction at which the car gets pulled from the ditch.

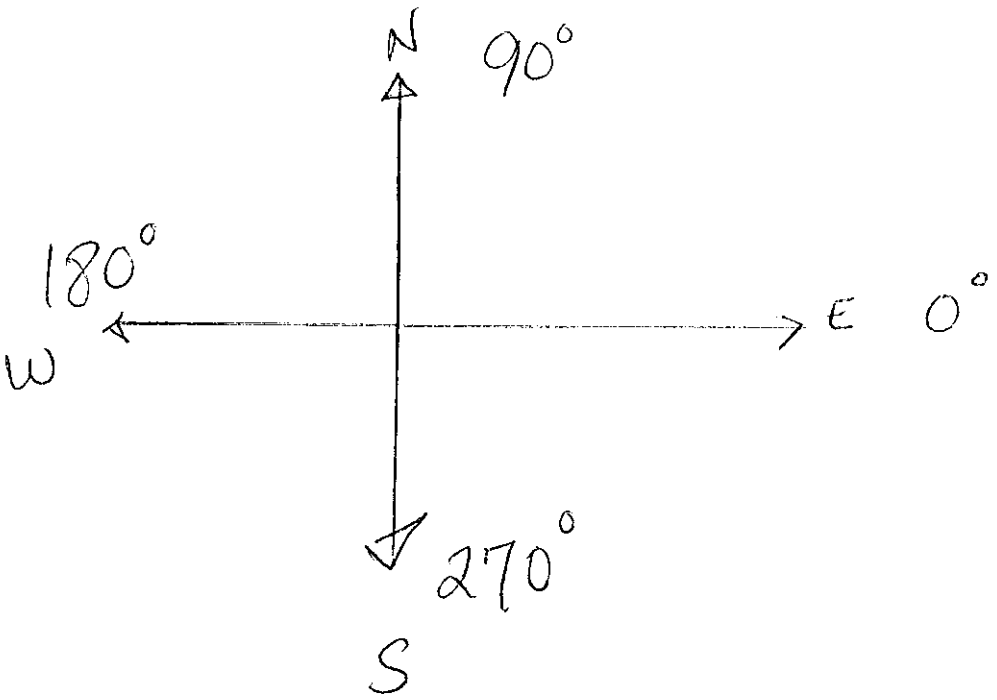
T1 $\langle 40\text{N}\cos 20°, 40\text{ N}\sin 20° \rangle$
T2 $\langle 33\cos 37°, 33\sin 37° \rangle$
T3 $\langle 25\cos 95°, 25\sin 95° \rangle$
T4 $\langle 60\cos 135°, 60\sin 135° \rangle$

x	y
37.6	13.7
26.4	19.9
-2.2	24.9
-42.4	42.4

total component $\langle 19.4, 100.9 \rangle$

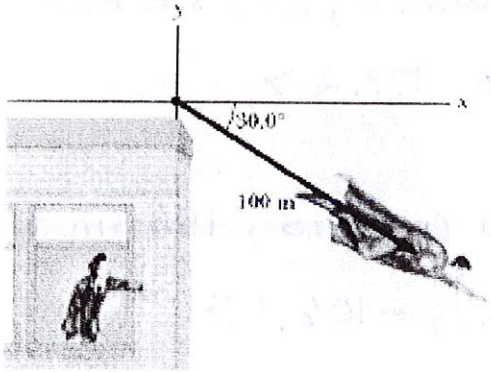
$\sqrt{19.4^2 + 100.9^2} \approx 102.7$ N

$\tan^{-1}\left(\frac{100.9}{19.4}\right) \approx 79.1°$


N
90°
180°
W
E
0°
270°
S

Resolving Vectors

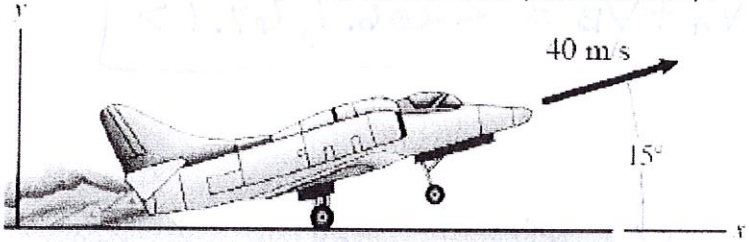
1. What are Superman's x and y components?



$\langle 100\cos 330°, 100\sin 330° \rangle$

$\langle 86.6, -50 \rangle$

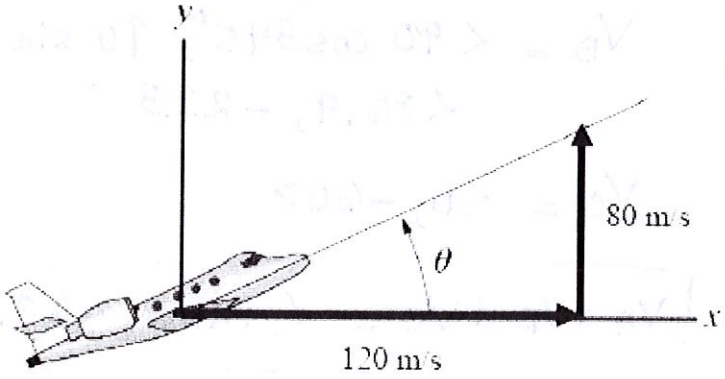
2. What are the x and y components of the plane's velocity?



$\langle 40\cos 15°, 40\sin 15° \rangle$

$\langle 38.6, 10.4 \rangle$

3. How fast and at what angle is the plane traveling?



Vector components:

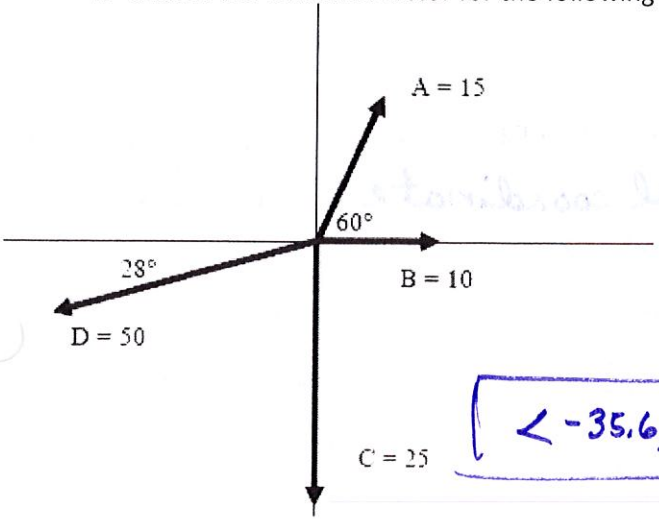
$\langle 120, 80 \rangle$

"How fast?" = magnitude

$\sqrt{120^2+80^2} \approx 144.2 \frac{m}{s}$

angle $= \tan^{-1}\left(\frac{80}{120}\right) \approx 33.7°$

4. What is the resultant vector for the following forces? (The table may help)



	X	Y
A	$15\cos 60°$	$15\sin 60°$
B	$10\cos 0°$	$10\sin 0°$
C	$25\cos 270°$	$25\sin 270°$
D	$50\cos 208°$	$50\sin 208°$

$\langle 7.5, 13.0 \rangle$

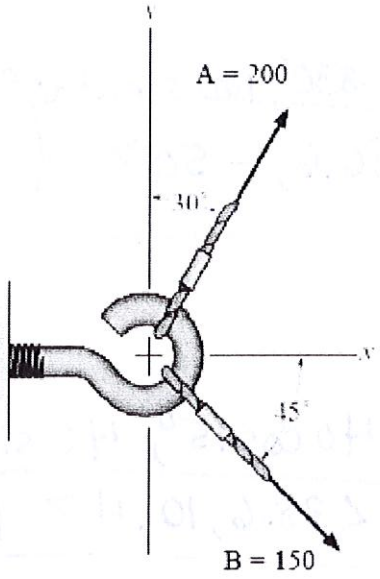
$\langle -35.6, -35.5 \rangle$ $\langle 10, 0 \rangle$

$\langle 0, -25 \rangle$

$\langle -44.1, -23.5 \rangle$

Add all $\langle -35.6, -35.5 \rangle$

5. Determine the resultant of the following



$V_A = \langle 200\cos 60^\circ, 200\sin 60^\circ \rangle$

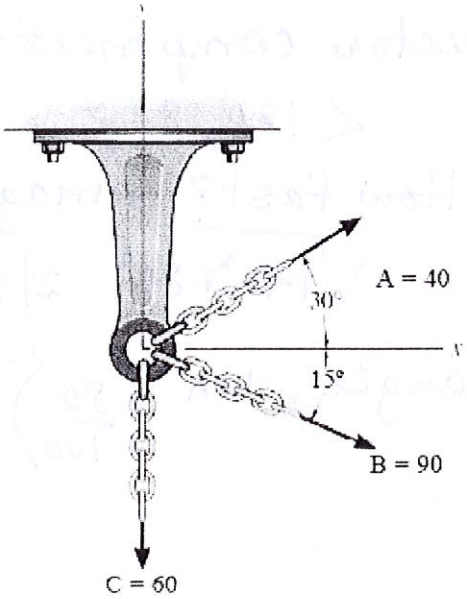
$\langle 100, 173.2 \rangle$

$V_B = \langle 150\cos 315^\circ, 150\sin 315^\circ \rangle$

$\langle 106.1, -106.1 \rangle$

$V_A + V_B = \langle 206.1, 67.1 \rangle$

6. Determine the resultant of the following



$V_A = \langle 40\cos 30^\circ, 40\sin 30^\circ \rangle$

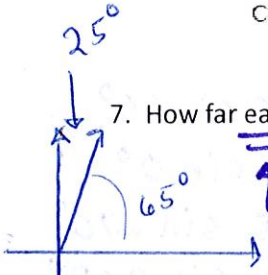
$\langle 34.6, 20 \rangle$

$V_B = \langle 90\cos 345^\circ, 90\sin 345^\circ \rangle$

$\langle 86.9, -23.3$

$V_C = \langle 0, -60 \rangle$

$V_A + V_B + V_C = \langle 121.5, -63.3 \rangle$



7. How far east has a person walked if he travels 350 m in a direction of 25° E of N?

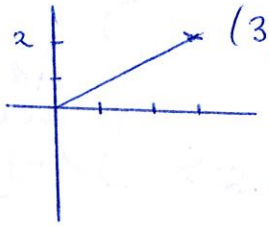
identify x (horizontal coordinate)

$\langle 350\cos 65^\circ, 350\sin 65^\circ \rangle$

$\langle 147.9, 317.2 \rangle$

The person has walked 147.9 m to the east.

8. What would be the resulting displacement and direction if a snail crawls 2.0 m north then 3.0 m east?



magnitude & direction

magnitude $= \sqrt{3^2+2^2}$

$\sqrt{9+4}$

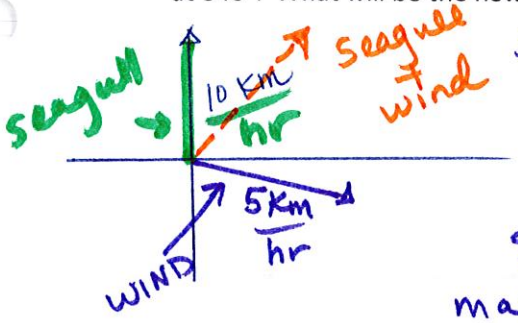
$\sqrt{13} \approx 3.6$ m

direction

$\tan^{-1}(2/3)$

$\approx 33.7^\circ$

9. A seagull flying with an airspeed of 10 km/hr is flying north but suddenly encounters a wind of 5 km/h at 340°. What will be the new direction and airspeed of the seagull?



Seagull $\langle 0, 10 \rangle$

WIND $\langle 5\cos 340°, 5\sin 340° \rangle$

$\langle 4.7, -1.7 \rangle$

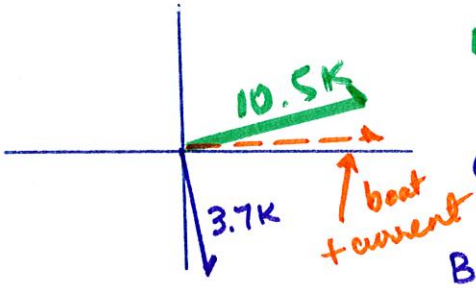
Seagull + wind $\langle 4.7, 8.3 \rangle$

mag: $\sqrt{4.7^2 + 8.3^2} \approx 9.5$ km/h

direction: $\tan^{-1}\left(\frac{8.3}{4.7}\right)$

$\approx 60.5°$

10. A boat crosses a river at 10.5 knots at a bearing of 23° of N of E. If the current in the river is 3.7 knots at 8° E of S. What is the new speed and bearing?



Boat $\langle 10.5\cos 23°, 10.5\sin 23° \rangle$

$\langle 9.7, 4.1 \rangle$

Current $\langle 3.7\cos 278°, 3.7\sin 278° \rangle$

$\langle 0.5, -3.7 \rangle$

Boat + Current $\langle 10.2, 0.4 \rangle$

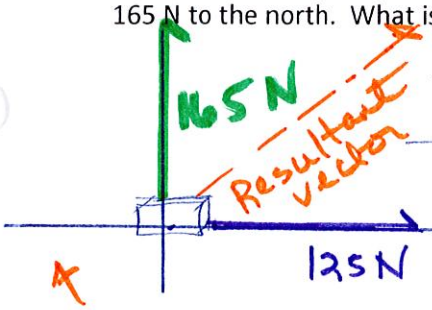
speed = $\sqrt{10.2^2 + .4^2}$

≈ 10.2K

direction $\tan^{-1}\left(\frac{.4}{10.2}\right)$

$\approx 2.2°$

11. Two boys push on a box. One pushes with a force of 125 N to the east. The other exerts a force of 165 N to the north. What is the size and direction of the resultant force on the box?



$\langle 0, 165 \rangle$

$\langle 125, 0 \rangle$

$\langle 125, 165 \rangle$

Resultant force:

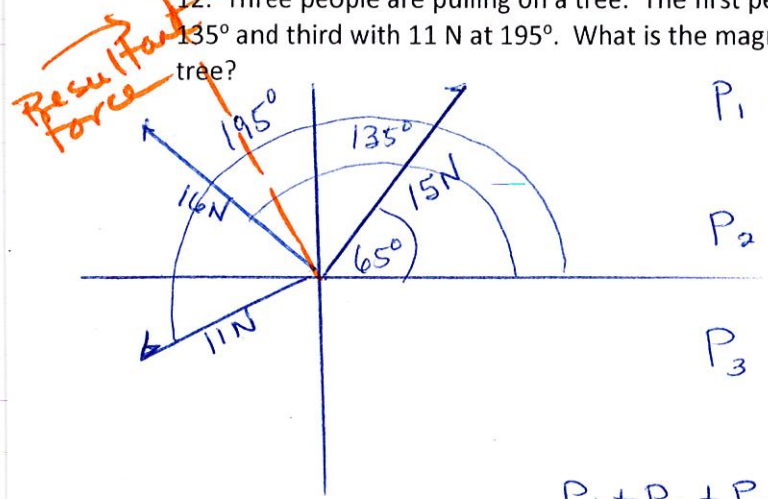
$\sqrt{125^2 + 165^2}$

≈ 207 N

direction: $\tan^{-1}\left(\frac{165}{125}\right)$

$\approx 52.9°$

12. Three people are pulling on a tree. The first person pulls with 15 N at 65°, the second with 16 N at 135° and third with 11 N at 195°. What is the magnitude and direction of the resultant force on the tree?



$P_1 \langle 15\cos 65°, 15\sin 65° \rangle$

$\langle 6.3, 13.6 \rangle$

$P_2 \langle 16\cos 135°, 16\sin 135° \rangle$

$\langle -11.3, 11.3 \rangle$

$P_3 \langle 11\cos 195°, 11\sin 195° \rangle$

$\langle -10.6, -2.8 \rangle$

$P_1 + P_2 + P_3$ combined forces $\langle -15.6, 22.1 \rangle$

Q2

magnitude: $\sqrt{15.6^2 + 22.1^2}$

≈ 27.1N

direction: $\tan^{-1}\left(\frac{22.1}{-15.6}\right) + 180°$

$\approx 125.2°$