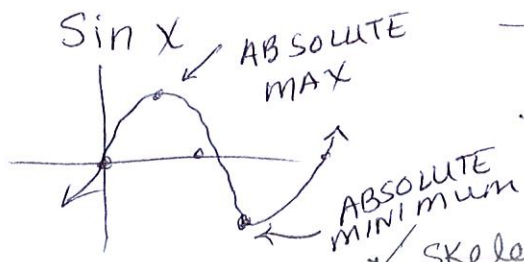
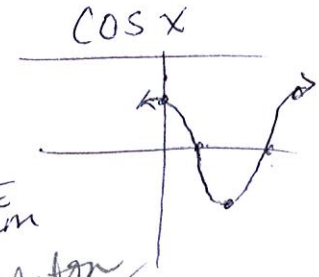
→ cos → sin skeleton

How can I Graph the Secant and Cosecant Functions?

Begin with the same analysis procedures as for graphing transformations on sine and cosine. Determine all the following based on the associated reciprocal function:

- period
- length of intervals
- phase shift (if applicable)
- midline
- maximum and minimum values





Skeleton

1. Create a table of values and LIGHTLY sketch the graph of the associated reciprocal function.
2. Lightly sketch VERTICAL asymptotes through the MIDLINE ~~Asymp~~ INTERCEPTS
3. The points that were absolute maximums on the graph become RELATIVE MINIMUM while the points that were ABSOLUTE MINIMUM become relative maximums.
4. One period of the secant graph will consist of two half-parabolas and a whole parabolas while the cosecant graph will consist of two whole parabolas.

EX 1: Graph $y = 3\csc 2\theta$ + 0

Graph $y = 3\sin 2\theta$

period : $\frac{2\pi}{2} = \pi$

intervals : $\frac{\pi}{4}$

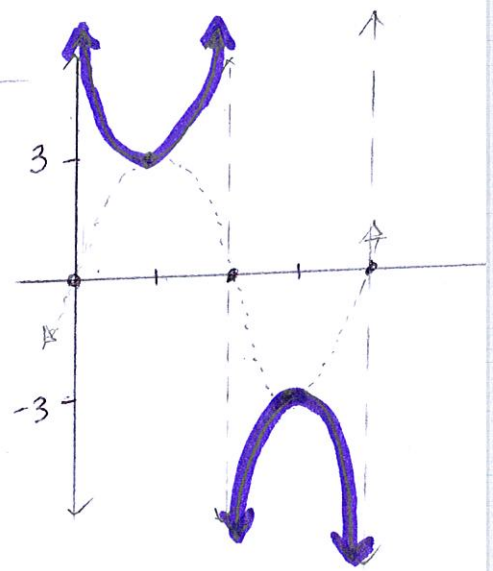
phase shift : NA (begin at 0)

MIDLINE : 0

MAX : 0 + 3 = 3

0 − 3 = −3

	θ	$\sin\theta$	
	0	MID	0
$+\frac{\pi}{4}$	$\frac{\pi}{4}$	MAX	3
$+\frac{\pi}{4}$	$\pi/2$	MID	0
$+\frac{\pi}{4}$	$\frac{3\pi}{4}$	MIN	−3
$+\frac{\pi}{4}$	π	MID	0



Reflection

EX 2: $y = -2\sec\theta + 4$

Graph $y = -2\cos\theta + 4$

Period : 2π

intervals : $\frac{2\pi}{4} = \frac{\pi}{2}$

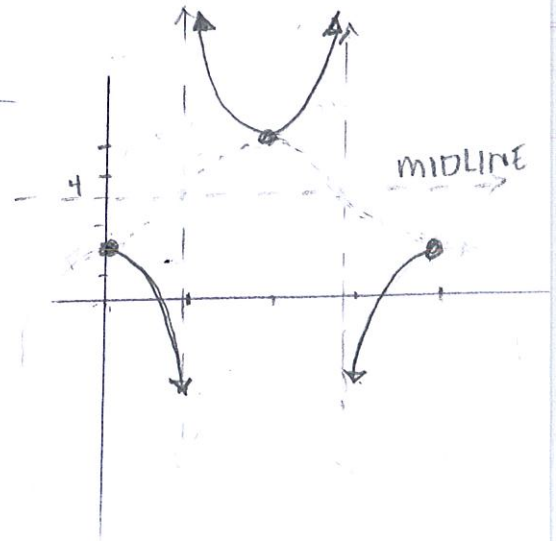
phase shift: NA
begin at 0

MIDLINE : 4

MAX : 4 + 2 = 6

MIN : 4 − 2 = 2

θ	$-\cos\theta$
0	MIN 2
$\pi/2$	MID 4
π	MAX 6
$3\pi/2$	MID 4
2π	MIN 2



✓ midline = -5

EX 3: $y = -5 + 2\csc(4\theta - \pi)$

Graph $y = 2\sin(4\theta - \pi) - 5$

period: $\frac{2\pi}{4} = \frac{\pi}{2}$

intervals: $\frac{\pi}{2 \cdot 4} = \frac{\pi}{8}$

phase shift: $4\theta - \pi = 0$

$4\theta = \pi$

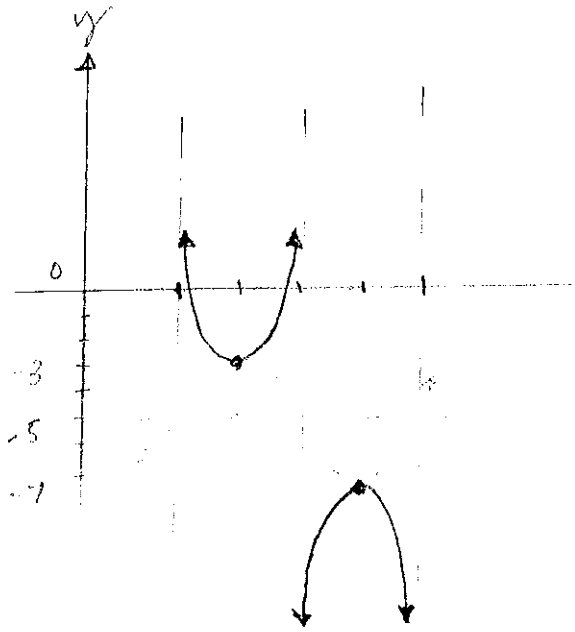
$\theta = \frac{\pi}{4}$

MIDLINE: -5

MAX: $-5 + 2 = -3$

MIN: $-5 - 2 = -7$

	θ	$\sin\theta$
	$\pi/4$	-5
$+\frac{\pi}{8}$	$3\pi/8$	-3
$+\frac{\pi}{8}$	$\pi/2$	-5
$+\frac{\pi}{8}$	$5\pi/8$	-7
$+\frac{\pi}{8}$	$3\pi/4$	-5



cos

EX 4: $y = \sec\left(\frac{2\theta}{3} + \frac{\pi}{6}\right) - 4$

period: $\frac{2\pi}{\frac{2}{3}} \rightarrow 2\pi \times \frac{3}{2} = 3\pi$

intervals: $\frac{3\pi}{4}$

phase shift: $\frac{2\theta}{3} + \frac{\pi}{6} = 0$

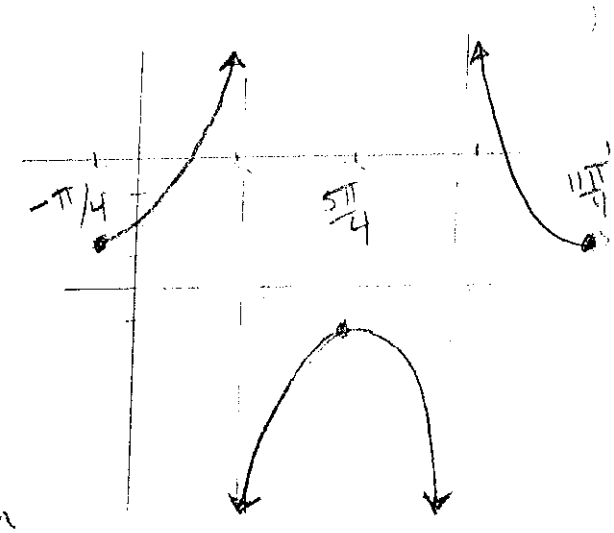
$\left(\frac{3}{2}\right)\frac{2\theta}{3} = -\frac{\pi}{6}\left(\frac{3}{2}\right)$

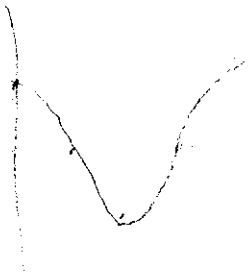
$\theta = \frac{-3\pi}{12} = \frac{-\pi}{4}$ begin x/y table

MIDLINE: -4

MAX: $-4 + 1 = -3$

MIN: $-4 - 1 = -5$





	θ	$\cos\theta$
	$-\pi/4$	-3
$+\frac{3\pi}{4}$	$\pi/2$	-4
$+\frac{3\pi}{4}$	$5\pi/4$	-5
$+\frac{3\pi}{4}$	2π	-4
$+\frac{3\pi}{4}$	$\frac{11\pi}{4}$	-3

Pre-Calculus

Name Key

4. $y = 5\csc\left(\frac{1}{2}\theta - 2\pi\right) - 3$

period: $\frac{2\pi}{\frac{1}{2}} = 4\pi$

intervals: $\frac{4\pi}{4} = \pi$

phase shift: $\frac{1}{2}\theta - 2\pi = 0$

$\frac{1}{2}\theta = 2\pi$

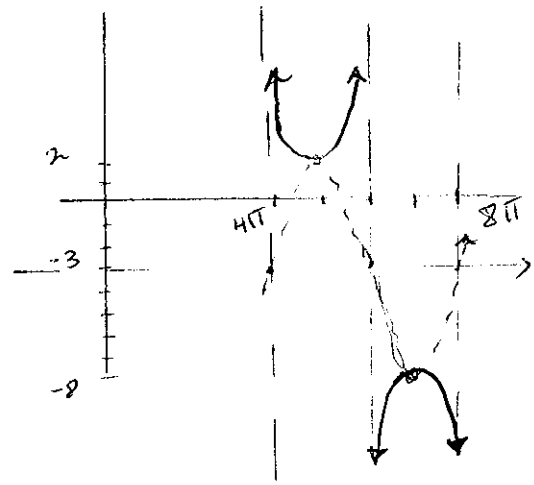
$\theta = 4\pi$

MIDLINE: -3

MAX: $-3+5=2$

MIN: $-3-5=-8$

θ	$\sin\theta$
4π	-3
5π	2
6π	-3
7π	-8
8π	2



5. $y = -2\csc(5\theta - \frac{3\pi}{2}) + 3$

period: $\frac{2\pi}{5}$

intervals: $\frac{2\pi}{5 \cdot 4} = \frac{2\pi}{20} = \frac{\pi}{10}$

phase shift: $5\theta - \frac{3\pi}{2} = 0$

$5\theta = \frac{3\pi}{2}$

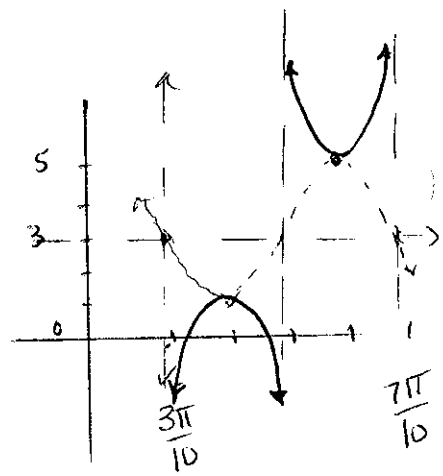
$\theta = \frac{3\pi}{10}$

MIDLINE: 3

MAX: $3+2=5$

MIN: $3-2=1$

θ	$-\sin\theta$	
$\frac{3\pi}{10}$	MID	3
$\frac{2\pi}{5}$	MIN	1
$\frac{\pi}{2}$	MID	3
$\frac{3\pi}{5}$	MAX	5
$\frac{7\pi}{10}$	MID	1



6. $y = 6 + 3\sec\left(\frac{2}{3}\theta - 4\pi\right)$

Period: $\frac{2\pi}{\frac{2}{3}} = 2\pi \times \frac{3}{2} = 3\pi$

intervals: $\frac{3\pi}{4}$

phase shift: $\frac{2}{3}\theta - 4\pi = 0$

$\frac{2}{3}\theta = 4\pi$

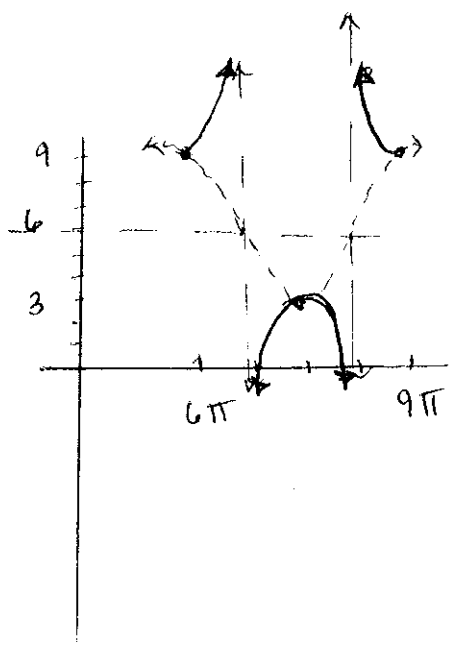
$\theta = 4\pi\left(\frac{3}{2}\right) = \frac{12\pi}{2} = 6\pi$

MIDLINE: 6

MAX $= 6+3 = 9$

MIN $= 6-3 = 3$

θ	$\cos\theta$
6π	9
$+\frac{3\pi}{4}$ $\frac{27\pi}{4}$	6
$+\frac{3\pi}{4}$ $15\pi/2$	3
$33\pi/4$	6
9π	9



Pre-Calculus Name _____

Sec and Csc Graphing

Sketch the graph of the function.

1. $y = -\frac{1}{2}\sec\theta$

period : 2π

intervals : $\frac{2\pi}{4} = \frac{\pi}{2}$

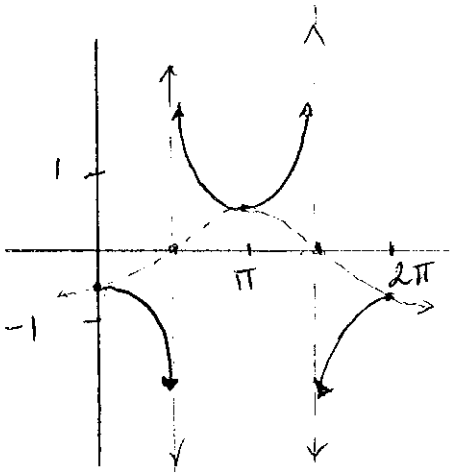
phase shift : NA

MIDLINE : 0

MAX : $\frac{1}{2}$

MIN : $-\frac{1}{2}$

θ	$-\cos\theta$	
0	MIN	$-1/2$
$\pi/2$	MID	0
π	MAX	$1/2$
$3\pi/2$	MID	0
2π	MIN	$-1/2$



2. $y = 4\csc 4\theta$

period : $\frac{2\pi}{4} = \frac{\pi}{2}$

intervals : $\frac{\pi}{2 \cdot 4} = \frac{\pi}{8}$

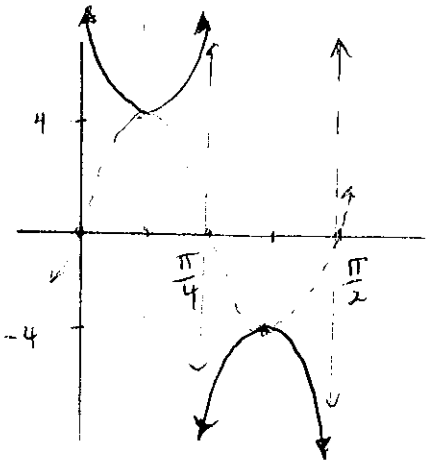
phase shift : NA

midline : 0

MAX : 4

MIN : -4

θ	$\sin\theta$	
0	MID	0
$\pi/8$	MAX	4
$\pi/4$	MID	0
$3\pi/8$	MIN	-4
$\pi/2$	MID	0



3. $y = -2 + 4\sec(4\theta - \pi)$

period : $\frac{2\pi}{4} = \frac{\pi}{2}$

intervals : $\frac{\pi}{2 \cdot 4} = \frac{\pi}{8}$

phase shift : $4\theta - \pi = 0$

$4\theta = \pi$

$\theta = \frac{\pi}{4}$

MIDLINE : -2

MAX : $-2 + 4 = 2$

MIN : $-2 - 4 = -6$

	θ	$\cos\theta$	
	$\pi/4$	MAX	2
$+\pi/8$	$3\pi/8$	MID	-2
$+\pi/8$	$\pi/2$	MIN	-6
$+\pi/8$	$5\pi/8$	MID	-2
$+\pi/8$	$3\pi/4$	MAX	2