The Parent Graphs of Sine and Cosine

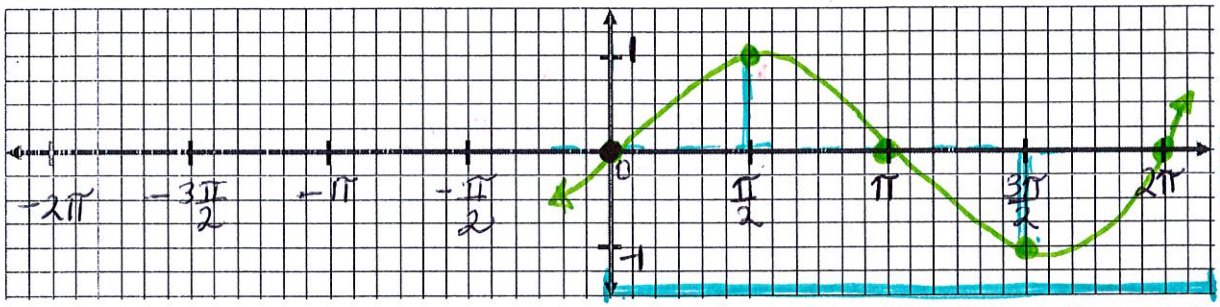
$y = \sin(x)$

5 critical values located at quadrantal angles

GRAPH 1 PERIOD

sin begins at the origin!

Degrees	0	30	45	60	90	120	135	150	180	210	225	240	270	300	315	330	360
Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π								
Exact	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0
Approx.	0	.5	.71	.87	1	.87	.71	.5	0	-.5	-.71	-.87	-1	-.87	-.71	-.5	0



x-axis
(θ-axis)
MIDLINE

$0 \rightarrow 2\pi$ = 1 period

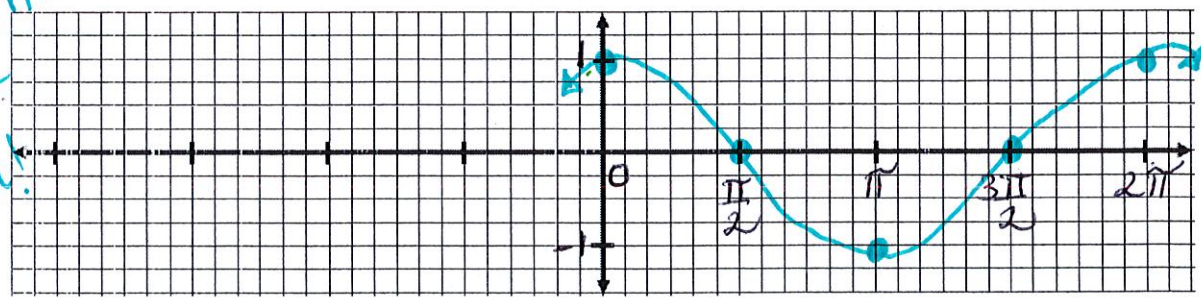
Domain: all $\mathbb{R}$ $(-\infty, +\infty)$ Range: $[-1, 1]$ (low, high) 5 critical points pattern: MID, MAX, MID, MIN, MID

MEMORIZE this!

$y = \cos(x)$

Degrees	0	30	45	60	90	120	135	150	180	210	225	240	270	300	315	330	360
Radians	0				$\frac{\pi}{2}$				π				3π				2π
Exact	1				0				-1				0				1
Approx.	1				0				-1				0				1

cosine does not begin at origin!

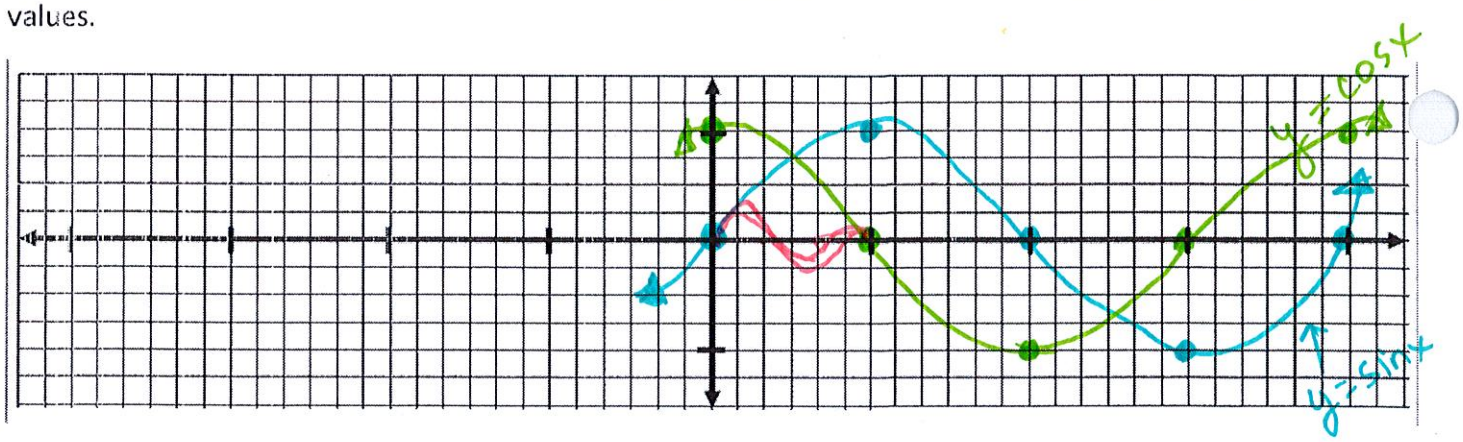


θ-axis

Inverted u-shape

Domain: all $\mathbb{R}$ Range: $[-1, 1]$ 5 critical points pattern: MAX, MID, MIN, MID, MAX

Let's graph two complete periods of sine and cosine together (but in different colors) using their quadrantal values.



How are they alike? How are they different?

Analyzing graphs of sine and cosine curves. ($y = A\sin B\theta$ and $y = A\cos B\theta$)

Amplitude $y = \lvert A \rvert \cos x$ or $y = \lvert A \rvert \sin x$ NEVER NEGATIVE amplitude of A; it tells how deep each peak/valley is; it's the distance to the midline from each MAX/MIN	**Period** If there is a number multiplying θ, there is a change in period: period $= \frac{2\pi}{B}$ Ex $y = 5\sin 4\theta \rightarrow$ period: $\frac{2\pi}{4} = \frac{\pi}{2}$
Midline the horizontal axis; if there is no constant, the midline = 0. The midline is a vertical shift.	**Intervals** $y = 2\cos\frac{1}{3}\theta \rightarrow \frac{2\pi}{\frac{1}{3}} \rightarrow 2\pi \times 3$ 6π Important for graphing! To determine spacing, find the period and divide by 4.
Maximum/Minimum If the midline is 0, then MAX = amplitude and MIN = − amplitude MAX = MIDLINE + amplitude MAX = MIDLINE − amplitude	**Phase Shift** horizontal translation Set argument equal to zero and solve; value is phase shift

Name

Sin/Cos Classwork

MAX → MIN
MIN → MAX

For each function, state the amplitude, period, intervals, and sinusoidal axis. Then graph one period.

1. $y = 8 \sin \theta$

+0 ← midline

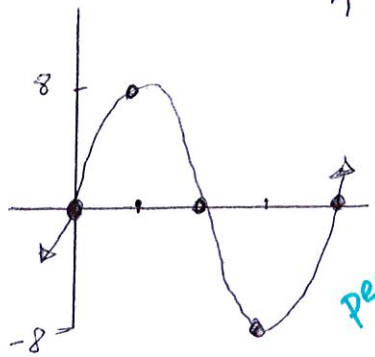
$A = 8$

period: 2π

find intervals: $\frac{2\pi}{4} = \frac{\pi}{2}$

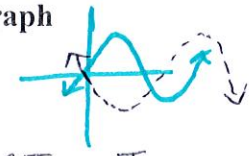
start at 0

	θ	$\sin\theta$
	0	MID = 0
$+\frac{\pi}{2}$	$\frac{\pi}{2}$	MAX = 8
$+\frac{\pi}{2}$	π	MID = 0
$+\frac{\pi}{2}$	$\frac{3\pi}{2}$	MIN = −8
$+\frac{\pi}{2}$	2π	MID = 0



2. $y = -5 \sin 4\theta$

Reflection

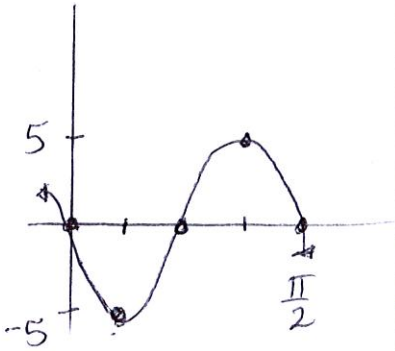


$A = 5$ period $= \frac{2\pi}{4} = \frac{\pi}{2}$

intervals: $\frac{\frac{\pi}{2}}{4} \rightarrow \frac{\pi}{2 \cdot 4} = \frac{\pi}{8}$

period

θ	$\sin\theta$
0	MID = 0
$\pi/8$	MIN = −5
$2\pi/8$	MID = 0
$3\pi/8$	MAX = 5
$4\pi/8$	MID = 0

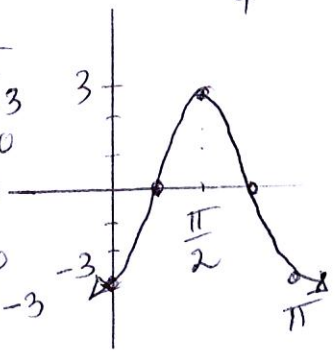


3. $y = -3 \cos 2\theta$ Reflection

$a = 3$ period $= \frac{2\pi}{2} = \pi$

intervals: $\frac{\pi}{4}$

	θ	$\cos\theta$
0	0	MIN −3
$\pi/4$	$\pi/4$	MID 0
$\pi/2$	$2\pi/4$	MAX 3
$\frac{3\pi}{4}$	$3\pi/4$	MID 0
π	$4\pi/4$	MIN −3

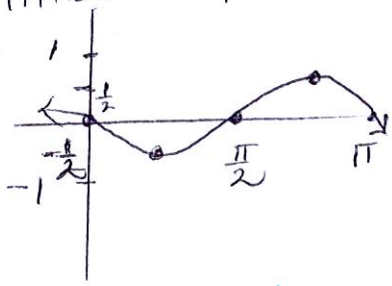


4. $y = -\frac{1}{2} \sin 2\theta$

period $= \frac{2\pi}{2} = \pi$

intervals: $\frac{\pi}{4}$

θ	$\sin\theta$
0	0
$\frac{\pi}{4}$	$-1/2$
$\frac{\pi}{2}$	0
$\frac{3\pi}{4}$	$1/2$
π	0

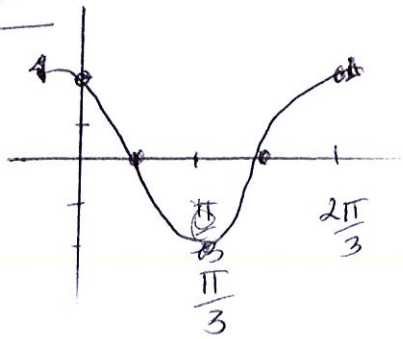


5. $y = 2 \cos 3\theta$

$A = 2$ period $= \frac{2\pi}{3}$

intervals $\frac{2\pi}{3 \cdot 4} = \frac{2\pi}{12} = \frac{\pi}{6}$

θ	$\cos\theta$
0	2
$\pi/6 \rightarrow \frac{\pi}{6}$	0
$\frac{2\pi}{6} \rightarrow \frac{\pi}{3}$	−2
$\frac{3\pi}{6} \rightarrow \frac{\pi}{2}$	0
$\frac{4\pi}{6} \rightarrow \frac{2\pi}{3}$	2



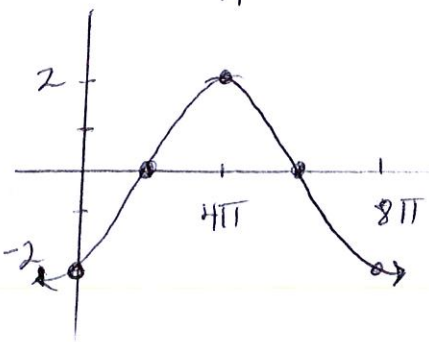
6. $y = -2 \cos(\frac{1}{4}\theta)$

MAX → MIN
MIN → MAX
$A = 2$

period $= \frac{2\pi}{\frac{1}{4}} = 2\pi \cdot \frac{4}{1} = 8\pi$

intervals: $\frac{8\pi}{4} = 2\pi$

θ	$\cos\theta$
0	MIN = −2
2π	MID = 0
4π	MAX = 2
6π	MID = 0
8π	MIN = −2



Sin/Cos Homework

For each function, state the amplitude, period, intervals, and Midline. Then graph one period.

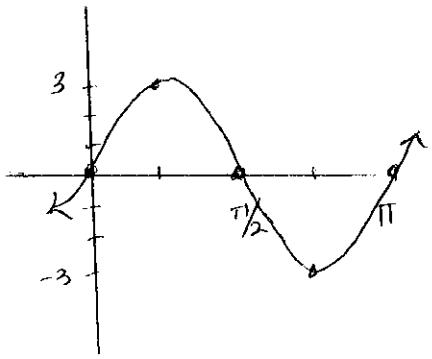
1. $y = 3\sin 2\theta$

amplitude = 3

period $= \frac{2\pi}{2} = \pi$

intervals: $\frac{\pi}{4}$

θ	$\sin\theta$
0	0
$\pi/4$	3
$\pi/2$	0
$3\pi/4$	-3
π	0



2. $y = -3\sin\frac{1}{3}\theta$

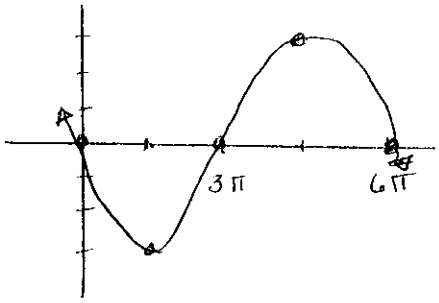
amplitude = 3

Reflection

intervals: $\frac{6\pi}{4} = \frac{3\pi}{2}$

period $= \frac{2\pi}{\frac{1}{3}} = 6\pi$

θ	$\sin\theta$
0	0
$3\pi/2$	-3
3π	6
$9\pi/2$	3
4π	-3



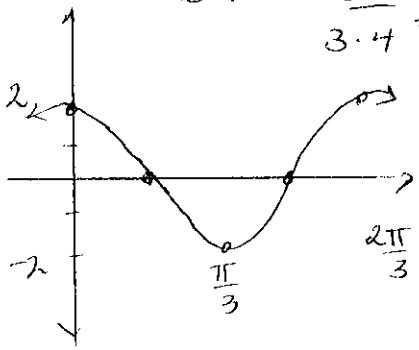
3. $y = 2\cos 3\theta$

amplitude = 2

period $= \frac{2\pi}{3}$

intervals $= \frac{2\pi}{3 \cdot 4} = \frac{2\pi}{12} = \frac{\pi}{6}$

θ	$\cos\theta$
0	2
$\pi/6$	0
$\pi/3$	-2
$\pi/2$	0
$2\pi/3$	2



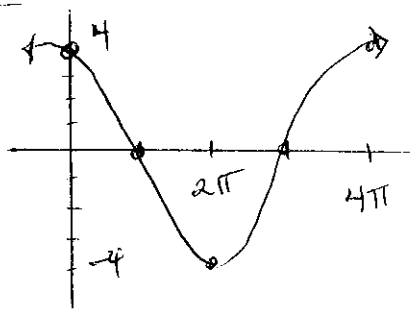
4. $y = 4\cos \frac{1}{2}\theta$

amplitude = 4

period $= \frac{2\pi}{\frac{1}{2}} = 4\pi$

intervals: $\frac{4\pi}{4} = \pi$

θ	$\cos\theta$
0	4
π	0
2π	-4
3π	0
4π	4



5. $y = -2\sin\frac{2}{3}\theta$

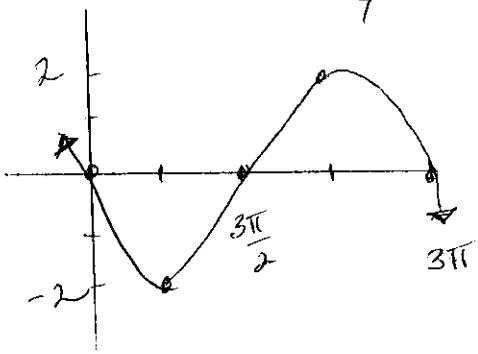
Reflection

amplitude = 2

period $= \frac{2\pi}{2/3} = 2\pi \cdot \frac{3}{2} = 3\pi$

intervals: $\frac{3\pi}{4}$

θ	$\sin\theta$
0	0
$3\pi/4$	-2
$3\pi/2$	0
$9\pi/4$	2
3π	0



6. $y = -5\cos\frac{1}{10}\theta$

amplitude = 5

reflection

period $= \frac{2\pi}{\frac{1}{10}} = 20\pi$

intervals: $\frac{20\pi}{4} = 5\pi$

θ	$\cos\theta$
0	-5
5π	0
10π	5
15π	0
20π	-5