How Can I Graph All the Transformations on a Sinusoidal Function?

divide $\frac{2\pi}{b}$ to find period, then divide by 4 to find intervals

The transformation equations for the sinusoidal functions are:

$y = a\sin(bx - c) + d$ and $y = a\cos(bx - c) + d$

amplitude

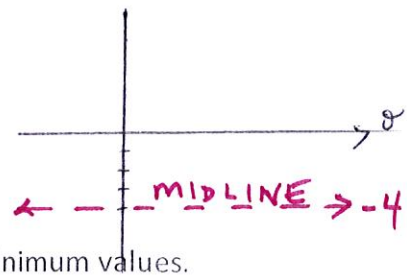
midline (vertical shift)

phase shift (set equal to zero and solve; begin x/y table here)

EX 1: Graph $y = 2\sin(3\theta - \frac{\pi}{2}) - 4$

Step 1: Determine the midline value.

$y = -4$ (the constant of the equation)



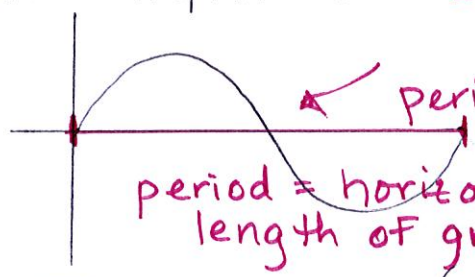
Step 2: Add and subtract the amplitude to the midline to get your maximum and minimum values.

MAXIMUM = MIDLINE + AMPLITUDE $\quad -4 + 2 = -2$

MINIMUM = MIDLINE − AMPLITUDE $\quad -4 - 2 = -6$

Step 3: Determine the period of the graph.

$\frac{2\pi}{b}$ or $\frac{2\pi}{3}$



period $= \frac{2\pi}{3}$

period = horizontal length of graph

Step 4: Determine the intervals.

$\frac{2\pi}{3} \div 4$ OR $\frac{2\pi}{3 \cdot 4} = \frac{2\pi}{12} = \frac{\pi}{6}$

(CRITICAL VALUES are spaced $\frac{\pi}{6}$ units apart)

Step 5: Determine the phase shift.

Set argument equal to zero and solve (θ is a variable) (same as x!)

$3\theta - \frac{\pi}{2} = 0$

$3\theta = \frac{\pi}{2}$

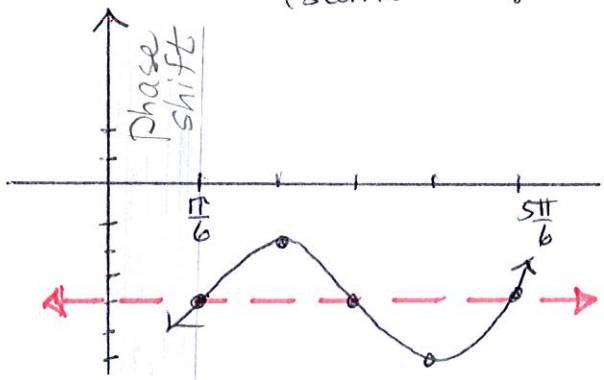
$\theta = \frac{\pi}{2 \cdot 3}$

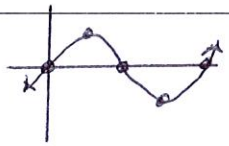
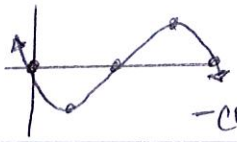
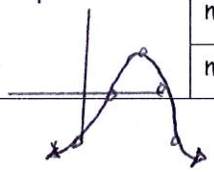
$\theta = \frac{\pi}{6}$

(begin graph here!)

Step 6: Construct the x/y table

	θ	$\sin\theta$	
phase shift	$\pi/6$	MID	-4
$+\pi/6$	$\pi/3$	MAX	-2
$+\pi/6$	$\pi/2$	MID	-4
$+\pi/6$	$2\pi/3$	MIN	-6
$+\pi/6$	$5\pi/6$	MID	-4



Interval	y-value (Depends on the graph) Memorize these patterns!	+ sin	-sin	+ cos	-cos
0 + phase shift	$\sin x$  $\cos x$ 	mid	mid	max	min
Int. 1 + phase shift		max	min	mid	mid
Int. 2 + phase shift		mid	mid	min	max
Int. 3 + phase shift	$-\sin x$  $-\cos x$ 	min	max	mid	mid
Int. 4 + phase shift		mid	mid	max	min

EX 2: Graph $y = -3\cos\left(\frac{1}{2}\theta + \pi\right) + 1$

Reflection

amplitude $= 3$

midline: $y = 1$

MAX: $1 + 3 = 4$

MIN: $1 - 3 = -2$

period: $\frac{2\pi}{\frac{1}{2}} = 4\pi$

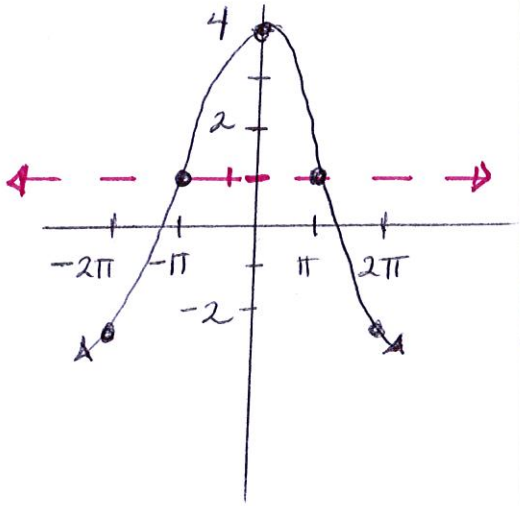
intervals: $\frac{4\pi}{4} = \pi$

phase shift: $\frac{1}{2}\theta + \pi = 0$

$\frac{\frac{1}{2}\theta}{\frac{1}{2}} = \frac{-\pi}{\frac{1}{2}}$

$\theta = -2\pi$ (start the x/y table)

	θ	$\cos\theta$
	-2π	-2
$+\pi$	$-\pi$	1
$+\pi$	0	4
$+\pi$	π	1
$+\pi$	2π	-2



EX 3: Graph $y = 6\sin\left(\frac{2}{3}\theta + \frac{\pi}{2}\right) + 0$

amplitude $= 6$

midline $= 0$

MAX $= 0 + 6 = 6$

MIN $= 0 - 6 = -6$

period: $\frac{2\pi}{\frac{2}{3}} \rightarrow 2\pi \times \frac{3}{2} = \frac{6\pi}{2} = 3\pi$

intervals: $\frac{3\pi}{4}$

phase shift: $\frac{2}{3}\theta + \frac{\pi}{2} = 0$

$\left(\frac{3}{2}\right)\frac{2}{3}\theta = -\frac{\pi}{2}\left(\frac{3}{2}\right)$

$\theta = -\frac{3\pi}{4}$ start your x/y table here

	θ	$\sin\theta$	
	$-\frac{3\pi}{4}$	MID	0
$+\frac{3\pi}{4}$	0	MAX	6
$+\frac{3\pi}{4}$	$\frac{3\pi}{4}$	MID	0
$+\frac{3\pi}{4}$	$\frac{6\pi}{4} \rightarrow \frac{3\pi}{2}$	MIN	-6
$+\frac{3\pi}{4}$	$\frac{9\pi}{4}$	MID	0