

Precalculus Worksheet
Section 4.7 – Inverse Trig Functions

Name _____
Period _____

Evaluate the given expression without the aid of a calculator.

1. $\sin^{-1}\left(\frac{1}{2}\right)$
 $30^\circ, \frac{\pi}{6}$

2. $\cos^{-1}\left(\frac{1}{2}\right)$
 $60^\circ, \frac{\pi}{3}$

3. $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right) \rightarrow \frac{\frac{1}{2} \cancel{y}}{\frac{\sqrt{3}}{2}} \rightarrow \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
 $30^\circ, \frac{\pi}{6}$

4. $\arccos\left(\frac{\sqrt{3}}{2}\right)$
 $30^\circ, \frac{\pi}{6}$

5. $\arcsin\left(\frac{\sqrt{2}}{2}\right)$
 $45^\circ, \frac{\pi}{4}$

6. $\arctan(1)$
 $45^\circ, \frac{\pi}{4}$

7. $\arcsin\left(-\frac{1}{2}\right)$
 $-30^\circ, -\frac{\pi}{6}$

8. $\arccos\left(-\frac{1}{2}\right)$
 $120^\circ, \frac{2\pi}{3}$

9. $\arctan\left(-\frac{\sqrt{3}}{3}\right)$
 $-30^\circ, -\frac{\pi}{6}$

10. $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$
 $150^\circ, \frac{5\pi}{6}$

11. $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$
 $-45^\circ, -\frac{\pi}{4}$

12. $\tan^{-1}(-1)$
 $-45^\circ, -\frac{\pi}{4}$

13. $\sin^{-1} 0$
 $0^\circ, 0$

14. $\cos^{-1} 0$
 ~~$0^\circ, 0$~~ ~~$90^\circ, \frac{\pi}{2}$~~

15. $\tan^{-1}(-\sqrt{3})$
 $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right) \rightarrow \frac{\sqrt{3}}{2}$
 $-60^\circ, -\frac{\pi}{3}$

16. $\arcsin(1)$
 $90^\circ, \frac{\pi}{2}$

17. $\arccos(1)$
 $0^\circ, 0$

18. $\tan^{-1} 0$
 $0^\circ, 0$

19. $\arcsin(-1)$
 $-90^\circ, -\frac{\pi}{2}$

20. $\arccos(-1)$
 $180^\circ, \pi$

Find the exact value without a calculator.

COMPOSITION FUNCTIONS

$$21. \cos \left[\sin^{-1} \left(\frac{1}{2} \right) \right]$$

\downarrow

$$\cos(30^\circ)$$
$$\boxed{\frac{\sqrt{3}}{2}}$$

$$22. \sin \left(\cos^{-1} \left(\frac{\sqrt{2}}{2} \right) \right)$$
$$\sin(45^\circ)$$
$$\boxed{\frac{\sqrt{2}}{2}}$$

$$23. \sin^{-1} \left(\cos \left(\frac{\pi}{3} \right) \right)$$
$$\sin^{-1} \left(\frac{1}{2} \right)$$
$$\boxed{30^\circ}$$

$$24. \cos^{-1} \left(\sin \left(\frac{\pi}{6} \right) \right)$$
$$\cos^{-1} \left(\frac{1}{2} \right)$$
$$\boxed{60^\circ}$$

$$25. \sin^{-1} \left(\sin \left(\frac{7\pi}{4} \right) \right)$$
$$\sin^{-1} \left(-\frac{\sqrt{2}}{2} \right)$$
$$\boxed{-45^\circ}$$

$$26. \arccos \left(\sin \left(\frac{\pi}{3} \right) \right)$$
$$\arccos \frac{\sqrt{3}}{2}$$
$$\boxed{30^\circ}$$

$$27. \sin \left(\tan^{-1} \left(\sqrt{3} \right) \right)$$

$\sin(60^\circ)$

$\cancel{\text{Stop here}}$

60°

$\frac{\sqrt{3}}{2}$
 $\frac{1}{2}$
OR
 $(\frac{1}{2}, \frac{\sqrt{3}}{2})$

$$\boxed{\frac{\sqrt{3}}{2}}$$

$$28. \cos \left(\tan^{-1} (-1) \right)$$
$$\cos(-45^\circ)$$
$$\boxed{\frac{\sqrt{2}}{2}}$$

$$29. \tan^{-1} (\cos(\pi))$$
$$\tan^{-1}(-1)$$
$$\boxed{-45^\circ}$$

$$30. \tan \left(\arccos \left(\frac{x}{3} \right) \right)$$

$$31. \sin(\arccos(x))$$

$$32. \sin \left(\arctan \left(\frac{x}{2} \right) \right)$$

Evaluate using your calculator to find the approximate value. Express your answer in degrees.

$$33. \sin^{-1}(.8621)$$

$$34. \tan^{-1}(.5893)$$

$$35. \cos^{-1}(-.3218)$$

$$36. \arcsin(-.6821)$$

$$37. \arctan(-1.6283)$$

$$38. \arccos(.2814)$$

Evaluate using your calculator to find the approximate value. Express your answer in radians.

$$39. \arcsin(.2618)$$

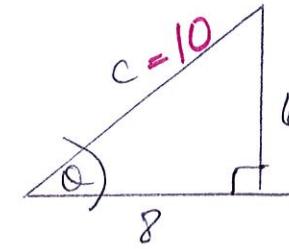
$$40. \cos^{-1}(-.8090)$$

$$41. \tan^{-1}(-1.7321)$$

Section
Assume all
values positive

Right Triangle Trig Day 1

1. $\tan \theta = \frac{6}{8}$; find $\sin \theta$



Remember, $\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$

- Label a Δ with an acute angle, θ
- Use Pythagorean Thm to find missing side measure

$$8^2 + 6^2 = c^2$$

$$64 + 36 = c^2$$

$$100 = c^2$$

$10 = c$ (we take only the positive square root as geometric measures cannot be negative.)

Now use found measure to write $\sin \theta$

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{6}{10} = \boxed{\frac{3}{5}} \quad \text{simplify, always}$$

2. $\sin \theta = \frac{5}{13}$; find $\sec \theta$

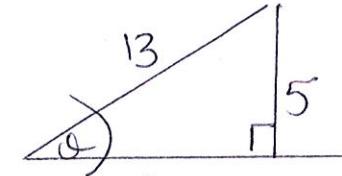
$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

$$5^2 + b^2 = 13^2$$

$$25 + b^2 = 169$$

$$b^2 = 144$$

$$b = 12 = 12$$



Recall that

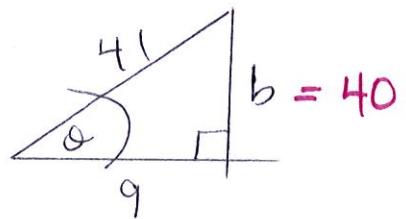
Secant is the reciprocal of cos

$$\therefore \sec \theta = \frac{1}{\cos} = \frac{\text{hyp}}{\text{adj}}$$

$$\boxed{\sec \theta = \frac{13}{12}}$$

3. $\cos \theta = \frac{9}{41}$; find $\cot \theta$

$$\begin{aligned} 9^2 + b^2 &= 41^2 \\ 81 + b^2 &= 1681 \\ b^2 &= 1600 \\ b &= 40 \end{aligned}$$

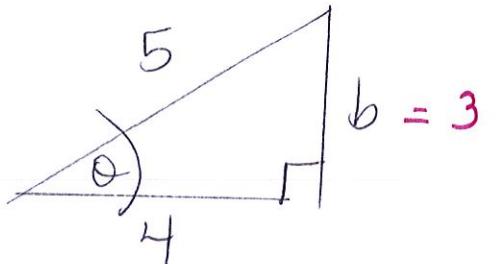


$$\cot \theta = \frac{1}{\tan \theta} = \frac{\text{adj}}{\text{opp}} = \boxed{\frac{9}{40}}$$

4. $\sec \theta = \frac{5}{4}$; find $\sin \theta$

$$\sec \theta = \frac{1}{\cos \theta} = \frac{\text{hyp}}{\text{adj}}$$

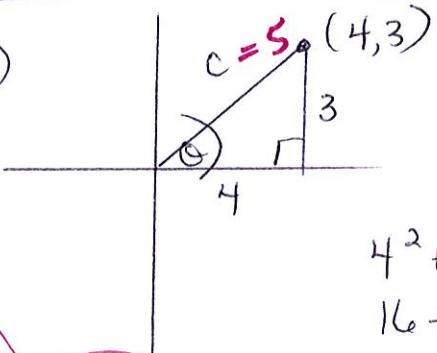
$$\begin{aligned} 4^2 + b^2 &= 5^2 \\ 16 + b^2 &= 25 \\ b^2 &= 9 \\ b &= 3 \end{aligned}$$



$$\sin \theta = \frac{\text{opp}}{\text{hyp}} = \boxed{\frac{3}{5}}$$

Section 2
Always
use "Trig"
Study Trig
Carefully
to determine
signs of trig functions

5. $(4, 3)$



$$\begin{aligned} 4^2 + 3^2 &= c^2 \\ 16 + 9 &= c^2 \\ 25 &= c^2 \\ 5 &= c \end{aligned}$$

$\sin \theta = \frac{3}{5}$

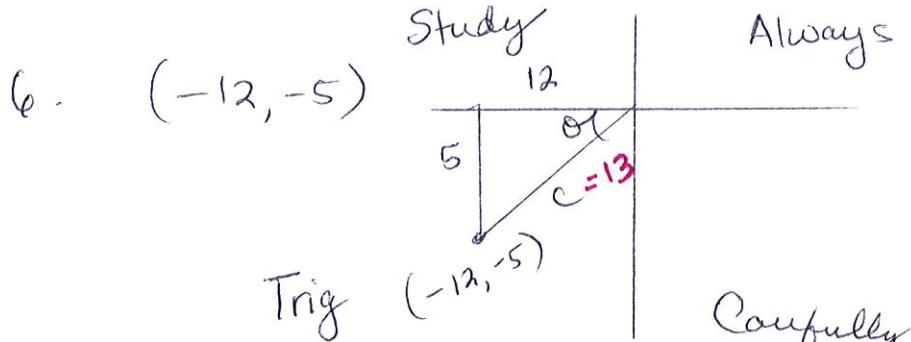
$\csc \theta = \frac{5}{3}$

$\cos \theta = \frac{4}{5}$

$\sec \theta = \frac{5}{4}$

$\tan \theta = \frac{3}{4}$

$\cot \theta = \frac{4}{3}$



$$12^2 + 5^2 = c^2$$

$$144 + 25 = c^2$$

$$169 = c^2$$

$$13 = c$$

$$\sin \theta = -\frac{5}{13}$$

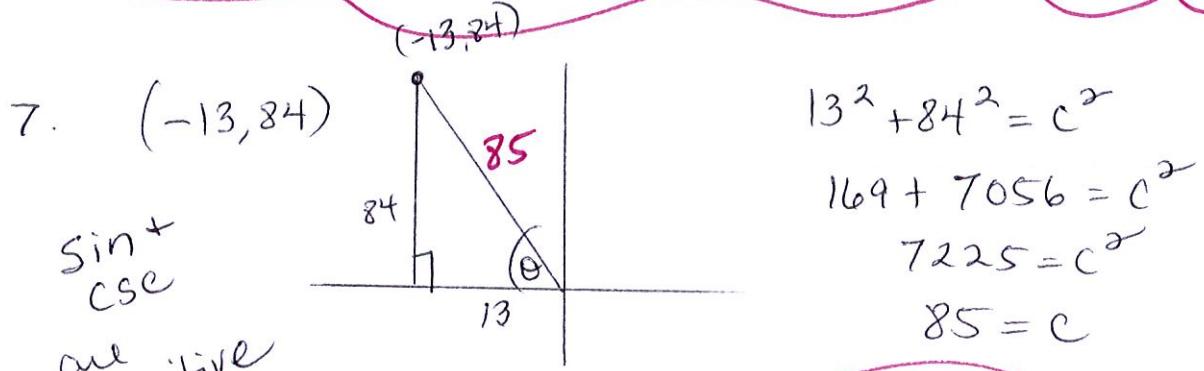
$$\cos \theta = -\frac{12}{13}$$

$$\tan \theta = \frac{5}{12}$$

$$\csc \theta = -\frac{13}{5}$$

$$\sec \theta = -\frac{13}{12}$$

$$\cot \theta = \frac{12}{5}$$



$$\sin \theta = \frac{84}{85}$$

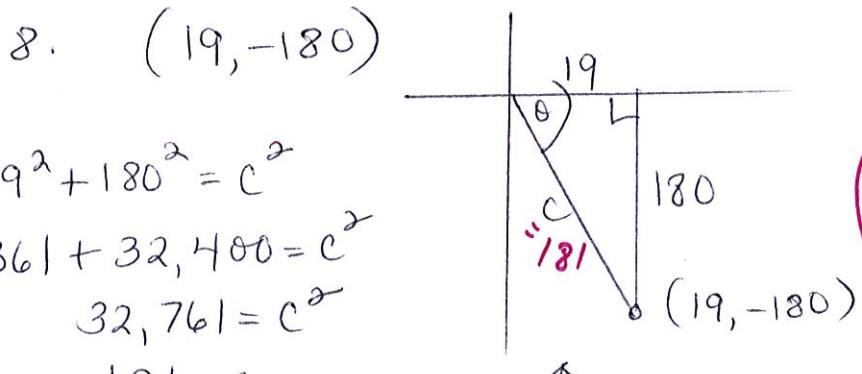
$$\cos \theta = -\frac{13}{85}$$

$$\tan \theta = \frac{84}{13}$$

$$\csc \theta = \frac{85}{84}$$

$$\sec \theta = -\frac{85}{13}$$

$$\cot \theta = -\frac{13}{84}$$



↑ cos + sec are positive in Q4

$$\sin \theta = -\frac{180}{181}$$

$$\csc \theta = -\frac{181}{180}$$

$$\cos \theta = \frac{19}{181}$$

$$\sec \theta = \frac{181}{19}$$

$$\tan \theta = -\frac{180}{19}$$

$$\cot \theta = \frac{19}{180}$$

Right Triangle Trig

Day 1 Practice
1-8

1. $\tan \theta = 5/8$, find $\sin \theta$

2. $\sin \theta = \frac{5}{13}$, find $\sec \theta$

3. $\cos \theta = \frac{9}{41}$, find $\cot \theta$

4. $\sec \theta = \frac{5}{4}$, find $\sin \theta$

In Exercises 5-8, the point given is on the terminal side of an angle in standard position. Determine the exact values of the six trigonometric functions of the angle.

5. $(4, 3)$

6. $(-12, -5)$

7. $(-\sqrt{3}, 8/4)$

8. $(19, -180)$

In Exercises 9-12, state the quadrant in which θ lies.

9. $\sin \theta < 0$ and $\cos \theta < 0$

Start here

10. $\sin \theta > 0$ and $\cos \theta > 0$

11. $\sin \theta > 0$ and $\tan \theta < 0$

12. $\sec \theta > 0$ and $\cot \theta < 0$

In Exercises 13–18, find the values of the six trigonometric functions of θ .

13. $\sin \theta = \frac{11}{61}$, constraint: θ lies in quadrant II.

14. $\cos \theta = -\frac{4}{5}$, constraint: θ lies in quadrant III.

15. $\tan \theta = -\frac{15}{8}$, constraint: $\sin \theta < 0$.

16. $\cos \theta = \frac{8}{17}$, constraint: $\tan \theta < 0$.

17. $\cot \theta = -3$, constraint: $\cos \theta > 0$.

18. $\csc \theta = 4$, constraint: $\cot \theta < 0$.