Solving Trig Equations WS 2

1. $2\cos 2\theta + 3 = 2$

$2\cos 2\theta = -1$

$\cos 2\theta = -\frac{1}{2}$

Solve same as $\cos\theta = -\frac{1}{2}$ and then divide by 2

$\cos^{-1}(\cos\theta) = \cos^{-1}\left(-\frac{1}{2}\right)$

$\theta = \frac{2\pi}{3} \div 2$

$\frac{2\pi}{6}$

or $\boxed{\frac{\pi}{3}}$

2. $3\tan^3\theta = \tan\theta$

Recognize GCF; Rewrite in standard form

$3\tan^3\theta - \tan\theta = 0$

$\tan\theta(3\tan^2\theta - 1) = 0$

$\tan^{-1}(\tan\theta) = \tan^{-1}(0)$

$\theta = 0$

$3\tan^2\theta - 1 = 0$

$3\tan^2\theta = 1$

$\tan^2\theta = \frac{1}{3}$

$\tan\theta = \pm\sqrt{\frac{1}{3}}$

$\tan^{-1}\sqrt{\frac{1}{3}}, \tan^{-1}-\sqrt{\frac{1}{3}}$

$\theta = \frac{\pi}{6}, \theta = -\frac{\pi}{6}$

$\boxed{\left\{0, \frac{\pi}{6}, -\frac{\pi}{6}\right\}}$

3. $2\cos 2\theta = 1$

$\cos^{-1}(\cos 2\theta) = \cos^{-1}\left(\frac{1}{2}\right)$

$\theta = \frac{\pi}{3} \div 2$

$\boxed{\theta = \frac{\pi}{6}}$

4. $2\sin 2\theta + \sqrt{3} = 0$

$2\sin 2\theta = -\sqrt{3}$

$\sin^{-1}(\sin 2\theta) = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

$\theta = -\frac{\pi}{3} \div 2$

$\boxed{\theta = -\frac{\pi}{6}}$

5. $2\cos^2\theta = 3\cos\theta - 1$

$2\cos^2\theta - 3\cos\theta + 1 = 0$

$2 \cdot 1 = 2$, $-2 - 1 = -3$

$(\cos\theta - \frac{2}{2})(\cos\theta - \frac{1}{2}) = 0$

$(\cos\theta - 1)(2\cos\theta - 1) = 0$

$\cos\theta - 1 = 0$

$\cos^{-1}(\cos\theta) = \cos^{-1}(1)$

$\theta = 0$

$2\cos\theta - 1 = 0$

$2\cos\theta = 1$

$\cos^{-1}(\cos\theta) = \cos^{-1}(\frac{1}{2})$

$\theta = \frac{\pi}{3}$

$\{0, \frac{\pi}{3}\}$

6. $3\cos^2\theta - 2\cos\theta = 1$

$3\cos^2\theta - 2\cos\theta - 1 = 0$

$3 \cdot -1 = -3$, $-3 + 1 = -2$

$(\cos\theta - \frac{3}{3})(\cos\theta + \frac{1}{3}) = 0$

$(\cos\theta - 1)(3\cos\theta + 1) = 0$

$\cos\theta - 1 = 0$

$\cos\theta = 1$

$3\cos\theta + 1 = 0$

$3\cos\theta = -1$

$\cos\theta = -\frac{1}{3}$

$\cos^{-1} 1 = 0$

$\cos^{-1}(-\frac{1}{3}) \approx 1.91$

$\{0, 1.91\}$

7. $2\csc^2\theta - 4 = 0$

$2\csc^2\theta = 4$

$\csc^2\theta = 2$

$\therefore \sin^2\theta = \frac{1}{2}$

and $\sin\theta = \pm\sqrt{\frac{1}{2}}$

$\sin^{-1}\sqrt{\frac{1}{2}}$ $\quad$ $\sin^{-1}\pm\left(\sqrt{\frac{1}{2}}\right)$

$\theta = \frac{\pi}{4}$ $\quad$ $\theta = -\frac{\pi}{4}$

$\left\{\frac{\pi}{4}, -\frac{\pi}{4}\right\}$

8. $\sec\left(\frac{3\theta}{2}\right) + 2 = 0$ $\quad$ $B = \frac{3}{2}$

$\sec\left(\frac{3\theta}{2}\right) = -2$

$\therefore \cos\left(\frac{3\theta}{2}\right) = -\frac{1}{2}$

$\theta = \cos^{-1}\left(-\frac{1}{2}\right)$

$\frac{3}{2}$

$\theta = \frac{2\pi}{3} \cdot \frac{2}{3}$

$\boxed{\theta = \frac{4\pi}{9}}$

9. $\cot\theta\sec\theta + \cot\theta = 0$

GCF! $\cot\theta(\sec\theta + 1) = 0$

$\cot\theta = \frac{0}{1}$ $\quad$ $\sec\theta + 1 = 0$

$\therefore \tan\theta = \frac{1}{0}$ $\quad$ $\sec\theta = -1$

$\therefore \cos\theta = -1$

$\theta = \pi$

this value occurs at $\frac{\pi}{2}$

$\boxed{\left\{\frac{\pi}{2}, \pi\right\}}$

10. $2 + \sec\theta = 0$

$\sec\theta = -2$

$\therefore \cos\theta = -\frac{1}{2}$

$\theta = \cos^{-1}\left(-\frac{1}{2}\right)$

$\boxed{\theta = \frac{2\pi}{3}}$

11. $\csc^2\theta + 2\csc\theta = 0$

$\csc\theta(\csc\theta + 2) = 0$

$\csc\theta = \frac{0}{1}$

$\therefore \sin\theta = \frac{1}{0}$

sin is never undefined on unit circle; this part of the equation yields no solution

$\csc\theta + 2 = 0$

$\csc\theta = -2$

$\therefore \sin\theta = -\frac{1}{2}$

$\boxed{\theta = -\frac{\pi}{6}}$

12. $1 - \cot^2\theta = 0$

$1 = \cot^2\theta$

$\pm\sqrt{1} = \cot\theta$

$\cot\theta = 1$ and $\cot\theta = -1$

$\therefore \tan\theta = 1$ and $\tan\theta = -1$

$\theta = \frac{\pi}{4}$ and ~~$\theta = -\frac{\pi}{4}$~~ NO!

We must add $\frac{4\pi}{4}$ to obey domain restriction

$\theta = -\frac{\pi}{4} + \frac{4\pi}{4} = \frac{3\pi}{4}$

same as π

$\boxed{\left\{\frac{\pi}{4}, \frac{3\pi}{4}\right\}}$

13. $2\sin\theta\sec\theta = \sec\theta$

$2\sin\theta\sec\theta - \sec\theta = 0$

$\sec\theta(2\sin\theta - 1) = 0$

$\sec\theta = \frac{0}{1}$

where $\cos\theta = \frac{1}{0}$ UND

$2\sin\theta - 1 = 0$

$2\sin\theta = 1$

$\sin\theta = \frac{1}{2}$

$\boxed{\theta = \frac{\pi}{6}}$

Remember, when using inverses to solve cotangent, add π to negative solutions