Precalculus Notes: Solving Trig Equations

Warm-up/EXTENSION LESSON

FOR SIN, COS, SEC + CSC ADD $2\pi K$ or $360°K$

FOR TAN, COT ADD πK OR $180°K$

Solve the following trig equations. Write a **general solution** in radian form.

EX 1: $2\cos\theta = \sqrt{2}$	EX 2: $\csc^2\theta - 4 = 0$	EX 3: $\tan\theta = -1$
$\cos\theta = \frac{\sqrt{2}}{2}$ $\cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4} + 2\pi K$	$\csc^2\theta = 4$ $\sin^2\theta = \frac{1}{4}$ $\sin\theta = \pm\frac{1}{2}$ $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6} + 2\pi K$ $\sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6} + 2\pi K$	$\tan^{-1}(-1) =$ $-\frac{\pi}{4} + \pi K$

This is a way to express all solutions coterminal

Solve the following trig equations. Write a **general solution** in degree form.

EX 4: $\cos x - \cos^2 x = 0$	EX 5: $2\sec^2 x + \sec x - 1 = 0$	EX 6: $-3\cot x = \sqrt{3}$
$\cos x(1 - \cos x) = 0$ $\cos x = 0$ $\quad$ $1 - \cos x = 0$ $\cos^{-1}(0) = 90°$ $\quad$ $1 = \cos x$ $\cos^{-1}(1) = 0°$ $\theta = 90° + 360°K$ $\quad$ $\theta = 0° + 360°K$	$\sec\theta = -1$ $\cos\theta = -1$ $\cos^{-1}(-1) = 180° + 360°K$ $\cos^{-1}(2) = \text{UND}$ -2, $\frac{2}{2}$, -1, 1, -1, $\frac{1}{2}$, $\frac{1}{2}$	$\cot x = -\frac{\sqrt{3}}{3}$ $x = \cancel{-60°} + 180°K$ $\cancel{+180°}$ $120° + 180°$

How to Solve Trig Equations with Transformations

EX 1: Find a general solution in radian form for the equation $\tan 3\theta = \sqrt{3}$

DIVIDE SOLUTIONS by 3

SOLVE $\tan\theta = \sqrt{3}$

$\tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$ $\cdot 3$

$\boxed{\theta = \frac{\pi}{9} + \frac{\pi}{3K}}$

$\frac{\pi}{3 \cdot 3} + \frac{\pi}{K \cdot 3}$

EX 2: Find a general solution in degree form for the equation $2\cos 5x + \sqrt{3} = 0$

DIVIDE SOLUTIONS BY 5

$2\cos x = -\sqrt{3}$

$\cos x = -\frac{\sqrt{3}}{2}$

$x = \frac{150°}{5} + \frac{360°K}{5}$

$x = 30° + 72°K$

EX 3: Solve $\sqrt{2}\sec\left(\frac{x}{6}\right) - 2 = 0$

Same as $B = \frac{1}{6}$ (multiply solutions by 6)

$\sqrt{2}\sec x - 2 = 0$

$\sqrt{2}\sec x = 2$

$\sec x = \frac{2}{\sqrt{2}}$

$\cos x = \frac{\sqrt{2}}{2}$

$x = 45° \times 6$

$x = 270°$

EX 4: Solve $5\cos^2\left(\theta - \frac{\pi}{4}\right) = 5$

$\cos^2\theta = 1$

$\cos\theta = \pm 1$

$\cos^{-1}(1) = 0°$ $\quad$ $\cos^{-1}(-1) = 180°$

$\theta - \frac{\pi}{4} = 0°$ $\quad$ $\theta - \frac{\pi}{4} = 180°$

$\theta = 45°$ $\quad$ $\theta = 225°$

EX 5: Solve $\sin(\frac{x}{2} - \frac{\pi}{6}) = 1$

SOLVE $\sin x = 1$

$\sin^{-1}(1) = 90°$

Set argument equal to $90°$ (or $\pi/2$)

$\frac{x}{2} - \frac{\pi}{6} = \frac{\pi}{2}$

$\frac{x}{2} = \frac{4\pi}{6} \rightarrow x = \frac{8\pi}{6} = \frac{4\pi}{3}$

EX 6: Solve $\csc(\pi - 4x) - 2 = 0$

$\csc x - 2 = 0$

$\csc x = 2$

$\sin x = \frac{1}{2}$

$\sin^{-1}(\frac{1}{2}) = \frac{\pi}{6}$

$\pi - 4x = \frac{\pi}{6}$

$-4x = \frac{\pi}{6} - \frac{6\pi}{6}$

$-4x = -\frac{5\pi}{6}$

$x = \frac{5\pi}{24}$

You try:

$4\sin(\theta + 60°) = -2\sqrt{3}$

$\sqrt{3} = 3\tan(\frac{\theta}{3} + \frac{7\pi}{4})$

$4\cos(2\theta + \frac{3\pi}{4}) = 4$

SOLVING TRIG. EQUATIONS WORKSHEET 2

Solve using an inverse. Write you answer in degree form. You may use a calculator or the unit circle.

1. $2\cos 2\theta + 3 = 2$

2. $3\tan^3\theta = \tan\theta$

3. $2\cos 4\theta = 1$

4. $2\sin 2\theta + \sqrt{3} = 0$

5. $2\cos^2(\theta - \frac{\pi}{6}) = 3\cos(\theta - \frac{\pi}{6}) - 1$

6. $3\cos^2 5\theta - 2\cos 5\theta = 1$

7. $2\csc^2(\theta + \pi) - 4 = 0$

8. $\sec\left(\frac{3\theta}{2}\right) + 2 = 0$

9. $2 + \sec(\theta - 47°) = 0$

10 $\csc^2(4\theta + 12°) + 2\csc(4\theta + 12°) = 0$

11. $1 - \cot^2 9\theta = 0$

1. $2\cos 2\theta + 3 = 2$

$2\cos 2\theta = -1$

$\cos 2\theta = -\frac{1}{2}$

$\cos^{-1}\left(-\frac{1}{2}\right) = \frac{120^\circ}{2} = 60^\circ$

DIVIDE SOLUTIONS BY 2

2. $3\tan^3\theta = \tan\theta$

CUBIC equation → 3 solutions

$3\tan^3\theta - \tan\theta = 0$

$\tan\theta\,(3\tan^2\theta - 1) = 0$

Factor using GCF

$\tan\theta = 0$

$\tan^{-1}(0) = \boxed{0^\circ}$

$3\tan^2\theta - 1 = 0$

$3\tan^2\theta =$

$\tan^2\theta = \frac{1}{3}$

$\tan\theta = \pm\sqrt{\frac{1}{3}} = \pm\frac{\sqrt{3}}{3}$

$\tan^{-1}\left(\frac{\sqrt{3}}{3}\right) = \boxed{30^\circ}$, $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right) = \boxed{-30^\circ}$

3. $2\cos 4\theta = 1$

$\cos 4\theta = \frac{1}{2}$

$\cos^{-1}\left(\frac{1}{2}\right) = \frac{60^\circ}{4} = \boxed{15^\circ}$

Divide solutions by 4

4. $2\sin 2\theta + \sqrt{3} = 0$

$2\sin 2\theta = -\sqrt{3}$

$\sin 2\theta = -\frac{\sqrt{3}}{2}$

$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \frac{-60^\circ}{2} = -30^\circ$

5. $2\cos^2\left(\theta - \frac{\pi}{6}\right) = 3\cos\left(\theta - \frac{\pi}{6}\right) - 1$ QUADRATIC BI C.X

Same as $2\cos^2\theta = 3\cos\theta - 1$

SIMPLIFY APPEARANCE

$2\cos^2\theta - 3\cos\theta + 1 = 0$

2

$-\frac{2}{2}$ $-\frac{1}{2}$

-3

1 $\frac{1}{2}$

$\cos^{-1}(1) = 0^\circ$ $\cos^{-1}\left(\frac{1}{2}\right) = 60^\circ$

$\theta - \frac{\pi}{6} = 0^\circ$ $\theta - \frac{\pi}{6} = 60^\circ$

$\theta - 30^\circ = 0^\circ$ $\theta - 30^\circ = 60^\circ$

$\boxed{\theta = 30^\circ}$ $\boxed{\theta = 90^\circ}$

Set argument equal to each solution and solve

6. $3\cos^2 5\theta - 2\cos 5\theta = 1$

$3\cos^2 5\theta - 2\cos 5\theta - 1 = 0$

-3

$-\frac{3}{3}$ $+\frac{1}{3}$

-2

$+1$ $-\frac{1}{3}$

$\cos^{-1}(1) = \frac{0^\circ}{5}$ $\cos^{-1}\left(-\frac{1}{3}\right) \approx \frac{109.47^\circ}{5}$

$\theta = 0^\circ$ and $\theta \approx 21.9^\circ$

same as #5

not on unit circle, but value exists

7. $2\csc^2(\theta + \pi) - 4 = 0$

$2\csc^2(\theta + \pi) = 4$

$\csc^2(\theta + \pi) = 2$

$\sin^2(\theta + \pi) = \frac{1}{2}$

$\sin(\theta + \pi) = \pm\sqrt{\frac{1}{2}} = \pm\frac{\sqrt{2}}{2}$

Solve by square root principle; you must use $\pm$ to get both solutions!

$\sin^{-1}\left(\sqrt{\frac{2}{2}}\right) = 45^\circ$

$\theta + \pi = 45^\circ$

$\theta + 180^\circ = 45^\circ$

$\boxed{\theta = -135^\circ}$

$\sin^{-1}\left(-\sqrt{\frac{2}{2}}\right) = -45^\circ$

$\theta + \pi = -45^\circ$

$\boxed{\theta = -225^\circ}$

8. $\sec\left(\frac{3\theta}{2}\right) + 2 = 0$

$\sec\left(\frac{3\theta}{2}\right) = -2$

$\cos\left(\frac{3\theta}{2}\right) = -\frac{1}{2}$

$\cos^{-1}\left(-\frac{1}{2}\right) = \frac{120^\circ}{\frac{3}{2}} = 120^\circ \times \frac{2}{3} = \boxed{80^\circ}$

multiply by reciprocal

9. $2 + \sec(\theta - 47^\circ) = 0$

$\sec(\theta - 47^\circ) = -2$

$\cos(\theta - 47^\circ) = -\frac{1}{2}$

$\cos^{-1}\left(-\frac{1}{2}\right) = 120^\circ$

$\theta - 47^\circ = 120^\circ$

$\boxed{\theta = 167^\circ}$

10. $\csc^2(4\theta + 12^\circ) + 2\csc(4\theta + 12)^\circ = 0$

$\cancel{\csc(4\theta + 12^\circ)(\csc}$

same as

$\csc^2\theta + 2\csc\theta = 0$ ← simplify problem

$\csc(\csc\theta + 2) = 0$ ← Factor GCF

$\csc\theta = \frac{0}{1}$

$\sin\theta = \text{UND}$

NO SOLUTION

$\csc\theta + 2 = 0$ APPLY the ZPP

$\csc\theta = -2$

$\sin\theta = -\frac{1}{2} \rightarrow \sin^{-1}\left(-\frac{1}{2}\right)$

$\theta = -30^\circ$

$4\theta + 12^\circ = -30^\circ$

$4\theta = -42^\circ$

$\boxed{\theta = -10.5^\circ}$

11. $1 - \cot^2 9\theta = 0$

$1 = \cot^2 9\theta$ take square roots; 2 solutions!

$\cot 9\theta = \pm 1$

$\cot^{-1}(1) = 45^\circ, \quad \cot^{-1}(-1) = 135^\circ$

$\quad 9 \qquad\qquad\qquad 9$

$\boxed{\theta = 5^\circ, \quad \theta = 15^\circ}$