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Trig. Equations Worksheet 1

Solve for θ in the interval: $0 \leq \theta < 360$

1. $\cos\theta + 1 = 0$

$$\theta = 180^\circ$$

2. $\sin^2\theta = 0$

$$\theta = 0^\circ$$

3. $2\cos\theta - \sqrt{3} = 0$

$$\theta = 30^\circ$$

4. $\cos\theta(\tan^2\theta - 3) = 0$

$$\theta = 90^\circ, \theta = 60^\circ, \theta = -60^\circ$$

5. $2\cos^2\theta = 3\cos\theta - 1$

$$\theta = 0^\circ, \theta = 60^\circ$$

6. $2\cos^2\theta - \cos\theta = 1$

$$\theta = 0^\circ, \theta = 120^\circ$$

Solve for ALL values of θ using degrees. *← technical language. We will discuss next week.*

7. $3\tan^2\theta - 1 = 0$

$$\theta = 30^\circ + 180^\circ k, \theta = -30^\circ + 180^\circ k$$

8. $4\cos^2\theta - 1 = 2$

$$\theta = 30^\circ + 360^\circ k, \theta = -30^\circ + 360^\circ k$$

9. $6\sin^2\theta + 5 = 8$

$$\theta = 45^\circ + 360^\circ k, \theta = -45^\circ + 360^\circ k$$

Solve for θ in $[0, 2\pi)$

10. $2\cos\theta + 4 = 5$

$$\theta = \frac{\pi}{3}$$

11. $5\sin\theta - \sqrt{3} = 3\sin\theta$

$$\theta = \frac{\pi}{3}$$

12. $4\sin^2\theta - 2 = 0$

$$\theta = \frac{\pi}{4}, \theta = -\frac{\pi}{4}$$

13. $6\sin^2\theta = 5\sin\theta + 4$

$$\theta = \frac{\pi}{6}$$

Solve for ALL values of θ using radians. *← technical language. We will discuss next week.*

14. $\sin^2\theta - 4\sin\theta - 5 = 0$

$$\theta = \frac{3\pi}{2} + 2\pi k$$

15. $16\cos^2\theta - 8 = 0$

$$\theta = \frac{\pi}{3} + 2\pi k, \theta = -\frac{\pi}{3} + 2\pi k$$

Let's now use the strategies learned in Algebra 2 for solving linear, quadratic and cubic equations to for solving trigonometric equations.

<p>EX 1: Solve $2\cos\theta = -1$.</p> <p>$\cos\theta = -\frac{1}{2}$ $\theta = \text{an unknown angle}$</p> <p>$\cos^{-1}\left(-\frac{1}{2}\right) = 120^\circ$</p>	<p>You try: Solve $2\sin x + \sqrt{3} = 0$</p> <p>$2\sin x = -\sqrt{3}$ $\sin x = \frac{-\sqrt{3}}{2}$ $x = -60^\circ$</p>
<p>EX 2: Solve $2\sin^2 x = 1$. QUADRATIC = 2 ANSWERS</p> <p>$\sqrt{\sin^2 x} = \pm \sqrt{\frac{1}{2}}$ take square root both both</p> <p>$\sin x = \pm \frac{\sqrt{1} \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}}$</p> <p>$\sin x = \pm \frac{\sqrt{2}}{2}$</p> <p>$x = 45^\circ$ $x = -45^\circ$</p>	<p>You try: $4\cos^2 x = 1$</p> <p>$\sqrt{\cos^2 x} = \sqrt{\frac{1}{4}} = \frac{\sqrt{1}}{\sqrt{4}}$</p> <p>$\cos x = \pm \frac{1}{2}$</p> <p>$\therefore x = \cos^{-1}\left(\frac{1}{2}\right)$</p> <p>$x = 60^\circ$ $x = -60^\circ$</p>
<p>EX 3: Solve $2\cos^2\theta - \sqrt{3}\cos\theta = 0$</p> <p>Alternate Strategy $2x^2 - \sqrt{3}x = 0$ GCF!</p> <p>$x(2x - \sqrt{3}) = 0$ ZPP</p> <p>$x = 0$ $2x - \sqrt{3} = 0$ $\cos\theta = 0$ $2x = \sqrt{3}$ $\cos^{-1}(0) = 90^\circ$ $x = \frac{\sqrt{3}}{2}$ $\cos\theta = \frac{\sqrt{3}}{2}$ $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = 30^\circ$</p>	<p>You try: $3\tan^2\theta - \sqrt{3}\tan\theta = 0$ GCF!</p> <p>$\tan\theta(3\tan\theta - \sqrt{3}) = 0$</p> <p>$\tan\theta = 0$ $3\tan\theta - \sqrt{3} = 0$ $\tan^{-1}(0) = 0^\circ$ $3\tan\theta = \sqrt{3}$ $\tan\theta = \frac{\sqrt{3}}{3}$ $\theta = 30^\circ$</p>
<p>EX 4: Solve $2\sin^2 x + 7\sin x = 4$</p> <p>$2\sin^2 x + 7\sin x - 4 = 0$ Put in standard form.</p> <p>$(\sin x + \frac{8}{2})(\sin x - \frac{1}{2})$</p> <p>$\sin x = -4$ $\sin x = \frac{1}{2}$</p> <p>$\sin^{-1}\left(\frac{1}{2}\right) = 30^\circ$</p>	<p>You try: $5\cos^2 x - 8\cos x = -3$</p> <p>$5\cos^2 x - 8\cos x + 3 = 0$</p> <p>$\cos^{-1}(1) = 0^\circ$ $\cos^{-1}\left(\frac{3}{5}\right) = 53.1^\circ$</p> <p>$\cos x = 1$ $\cos x = \frac{3}{5}$</p>
<p>EX 5: Solve $\sin\theta\cos\theta(\tan\theta - 1) = 0$ USE the ZPP</p> <p>$\sin\theta = 0$ $\cos\theta$ $\tan\theta - 1 = 0$</p> <p>$\sin^{-1}(0)$ $\cos^{-1}(0)$ $\tan^{-1}(1)$</p> <p>0° 90° 45°</p> <p>This equation is already factored! Apply the Zero Product Property</p>	<p>You try: $(2\cos\theta - 1)(2\sin\theta + \sqrt{3}) = 0$</p> <p>$2\cos\theta - 1 = 0$ $2\sin\theta + \sqrt{3} = 0$</p> <p>$2\cos\theta = 1$ $2\sin\theta = -\sqrt{3}$</p> <p>$\cos\theta = \frac{1}{2}$ $\sin\theta = \frac{-\sqrt{3}}{2}$</p> <p>$\cos^{-1}\left(\frac{1}{2}\right) = 60^\circ$ $\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right)$</p> <p>$-60^\circ$</p>

$$1. \cos \theta + 1 = 0$$

$$\cos \theta = -1$$

$$\cos^{-1}(-1) = \boxed{180^\circ}$$

$$2. \sin^2 \theta = 0$$

$$\sin \theta = 0$$

$$\sin^{-1}(0) = \boxed{0^\circ}$$

$$3. 2 \cos \theta - \sqrt{3} = 0$$

$$2 \cos \theta = \sqrt{3}$$

$$\cos \theta = \frac{\sqrt{3}}{2} = \boxed{30^\circ}$$

$$4. \cos \theta (\tan^2 \theta - 3) = 0$$

$$\cos \theta = 0 \quad \tan^2 \theta = 3$$

$$\cos^{-1}(0) = \boxed{90^\circ} \quad \tan \theta = \pm \sqrt{3}$$

$$\boxed{\begin{array}{l} \theta = 60^\circ, \\ -60^\circ \end{array}}$$

$$5. 2 \cos^2 \theta = 3 \cos \theta - 1$$

$$2 \cos^2 \theta - 3 \cos \theta + 1 = 0$$

$$\begin{array}{r} \cancel{2} \\ -2 \quad \cancel{-1} \\ \cancel{2} \quad -3 \quad \cancel{2} \end{array}$$

$$\cos \theta = 1 \quad \cos \theta = \frac{1}{2}$$

$$\cos^{-1}(1) = \boxed{0^\circ} \quad \cos^{-1}\left(\frac{1}{2}\right) = \boxed{60^\circ}$$

$$\begin{array}{r} \cancel{-2} \\ -2 \quad \cancel{+1} \\ \cancel{2} \quad -1 \quad \cancel{2} \end{array}$$

$$\cos \theta = 1 \quad \cos \theta = \frac{1}{2}$$

$$\theta = \cos^{-1}(1) \quad \theta =$$

$$\boxed{\theta = 0^\circ} \quad \cos^{-1}\left(\frac{1}{2}\right)$$

$$\boxed{\theta = 120^\circ}$$

$$7. 3 \tan^2 \theta - 1 = 0$$

$$3 \tan^2 \theta = 1$$

$$\tan^2 \theta = \frac{1}{3}$$

$$\tan \theta = \pm \sqrt{\frac{1}{3}}$$

$$\theta = \pm \tan^{-1}\left(\sqrt{\frac{1}{3}}\right)$$

$$\boxed{\theta = 30^\circ, -30^\circ}$$

$$8. 4 \cos^2 \theta - 1 = 2$$

$$4 \cos^2 \theta = 3$$

$$\cos^2 \theta = \frac{3}{4}$$

$$\cos \theta = \pm \sqrt{\frac{3}{4}}$$

$$\theta = \pm \cos^{-1}\sqrt{\frac{3}{4}}$$

$$\boxed{\theta = 30^\circ, -30^\circ}$$

$$9. \quad 6 \sin^2 \theta + 5 = 8$$

$$6 \sin^2 \theta = 3$$

$$\sin^2 \theta = \frac{3}{6}$$

$$\sin^2 \theta = \frac{1}{2}$$

$$\sin \theta = \pm \sqrt{\frac{1}{2}}$$

$$\theta = \pm \sin^{-1} \sqrt{\frac{1}{2}}$$

$$\boxed{\theta = 45^\circ, -45^\circ}$$

$$10. \quad 2 \cos \theta + 4 = 5$$

$$2 \cos \theta = 1$$

$$\cos \theta = \frac{1}{2}$$

$$\theta = \cos^{-1} \left(\frac{1}{2} \right)$$

$$\boxed{\theta = \frac{\pi}{3}}$$

$$11. \quad 5 \sin \theta - \sqrt{3} = 3 \sin \theta$$

$$2 \sin \theta - \sqrt{3} = 0$$

$$2 \sin \theta = \sqrt{3}$$

$$\sin \theta = \frac{\sqrt{3}}{2}$$

$$\theta = \sin^{-1} \left(\frac{\sqrt{3}}{2} \right)$$

$$\boxed{\theta = \frac{\pi}{3}}$$

$$12. \quad 4 \sin^2 \theta - 2 = 0$$

$$4 \sin^2 \theta = 2$$

$$\sin^2 \theta = \frac{2}{4}$$

$$\sin^2 \theta = \frac{1}{2}$$

$$\sin \theta = \pm \sqrt{\frac{1}{2}}$$

$$\theta = \pm \sin^{-1} \sqrt{\frac{1}{2}}$$

$$\theta = \pm \sin^{-1} \frac{\sqrt{2}}{2}$$

$$\boxed{\theta = \frac{\pi}{4} \text{ and } \theta = -\frac{\pi}{4}}$$

$$13. \quad 6 \sin^2 \theta = 5 \sin \theta + 4$$

$$6 \sin^2 \theta - 5 \sin \theta - 4 = 0$$

$$\begin{array}{r} \begin{array}{r} 8 \\ \frac{8}{6} \end{array} \begin{array}{r} -24 \\ -5 \end{array} \begin{array}{r} -3 \\ \frac{-3}{6} \end{array} \\ \hline = \frac{4}{3} \quad = -\frac{1}{2} \end{array}$$

$$\sin \theta = -\frac{4}{3} \quad \sin \theta = \frac{1}{2}$$

$$\theta = \sin^{-1} \left(-\frac{4}{3} \right) \quad \theta = \sin^{-1} \left(\frac{1}{2} \right)$$

UND

$$\boxed{\theta = \frac{\pi}{6}}$$

$$14. \sin^2 \theta - 4 \sin \theta - 5 = 0$$

A hand-drawn diagram showing a cross with numbers. The top-left and bottom-right quadrants contain '-5', the top-right and bottom-left quadrants contain '+1', and the bottom-center contains '-4'.

$$(\sin \theta - 5)(\sin \theta + 1) = 0$$

$$\sin \theta - 5 = 0$$

$$\sin \theta + 1 = 0$$

$$\sin \theta = 5$$

$$\sin \theta = -1$$

$$\theta = \sin^{-1}(5)$$

UND

$$\theta = \frac{3\pi}{2}$$

$$15. 16 \cos^2 \theta - 8 = 0$$

$$16 \cos^2 \theta = 8$$

$$\cos^2 \theta = \frac{8}{16}$$

$$\cos^2 \theta = \frac{1}{2}$$

$$\cos \theta = \pm \sqrt{\frac{1}{2}}$$

$$\theta = \pm \cos^{-1} \sqrt{\frac{1}{2}}$$

$$\theta = \frac{\pi}{3} \text{ and } \theta = -\frac{\pi}{3}$$

Precalculus Notes – Solving Trig Equations

Let's now use the strategies learned in Algebra 2 for solving linear, quadratic and cubic equations to for solving trigonometric equations.

EX 1: Solve $2\cos\theta = -1$.	You try: Solve $2\sin x + \sqrt{3} = 0$
EX 2: Solve $2\sin^2 x = 1$.	You try: $4\cos^2 x = 1$
EX 3: Solve $2\cos^2\theta - \sqrt{3}\cos\theta = 0$	You try: $3\tan^2\theta - \sqrt{3}\tan\theta = 0$
EX 4: Solve $2\sin^2 x + 7\sin x = 4$	You try: $5\cos^2 x - 8\cos x = -3$
EX 5: Solve $\sin\theta\cos\theta(\tan\theta - 1) = 0$	You try: $(2\cos\theta - 1)(2\sin\theta + \sqrt{3}) = 0$

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Solve for ALL values of θ using degrees

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